Sensitivity Analysis

1. Oak Products is a furniture company located in High Point. This month they have plans to make two kinds of chairs, Captain and Mate which bring profits of $56 and $40 per unit respectively. These are generic products and the company can sell as many as they can produce. There is a minimum requirement of 100 chairs in any combination. The inventory of the components from which these chairs are assembled can not be increased since there is a longer than a month’s production lead time. The solution and the sensitivity report from Excel Solver is given;

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
|  |  | Oak Products Model: |  |  |  |
|  |  |  |  |  |  |  |
| Chair Style | Captain | Mate | Profit |  |  |  |
| Profit/chair | 56 | 40 |  |  |  |  |
| Quantity | 130 | 60 | 9680 |  |  |  |
|  |  |  |  |  |  |  |
|  | Chair componentRequirements |  Total  usage |  |  startInventory |  |
|  |  |  |
| Long Dowels | 8 | 4 | 1280 | <= | 1280 |  |
| Short Dowels | 4 | 12 | 1240 | <= | 1600 |  |
| Legs | 4 | 4 | 760 | <= | 760 |  |
| Heavy Seats | 1 | 0 | 130 | <= | 140 |  |
| Light Seats | 0 | 1 | 60 | <= | 120 |  |
|  |  |  |  |  |  |  |
| Chair Production | 1 | 1 | 190 | >= | 100 |  |
|  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Adjustable Cells |  |  |  |  |  |  |
|  |  |  | **Final** | **Reduced** | **Objective** | **Allowable** | **Allowable** |
|  | **Cell** | **Name** | **Value** | **Cost** | **Coefficient** | **Increase** | **Decrease** |
|  | $B$6 |  Quantity Captain | 130 | 0 | 56 | 24 | 16 |
|  | $C$6 |  Quantity Mate | 60 | 0 | 40 | 16 | 12 |
|  |  |  |  |  |  |  |  |
| Constraints |  |  |  |  |  |  |
|  |  |  | **Final** | **Shadow** | **Constraint** | **Allowable** | **Allowable** |
|  | **Cell** | **Name** | **Value** | **Price** | **R.H. Side** | **Increase** | **Decrease** |
|  | $D$10 | Long Dowels usage | 1280 | 4 | 1280 | 40 | 180 |
|  | $D$11 | Short Dowels usage | 1240 | 0 | 1600 | 1E+30 | 360 |
|  | $D$12 | Legs usage | 760 | 6 | 760 | 72 | 40 |
|  | $D$13 | Heavy Seats usage | 130 | 0 | 140 | 1E+30 | 10 |
|  | $D$14 | Light Seats usage | 60 | 0 | 120 | 1E+30 | 60 |
|  | $D$16 | Chair production  | 190 | 0 | 100 | 90 | 1E+30 |

Answer each of the following questions independently.

1. Suppose 30 more Short Dowels are available. What will be the change in OV?

No effect. The short dowels constraint is not binding. They’ll simply have 390 unused short dowels

1. Suppose there are 300 fewer short dowels. What will be the change in OV?

Nothing again, 360 extra short dowels can absorb this short-fall

1. The shadow price on Short Dowels is valid for what range?

1240 and up (allowable increase is infinity, allowable decrease is 360, value of the slack)

1. Suppose the right hand side of the Legs constraint is changed to 750. Will the optimal solution change? Will the shadow price change? What is the effect of this change on the OV, if any?

This is a binding constraint. The Optimal solution will have to change--the current optimal solution needs 760 legs. However the 10 unit reduction is in the allowable range thus the **Shadow Prices** will not change. The change in OV = (-10)\*SP = -10\*6 = -$60.

1. By how much can the total chair production constraint be tightened before the shadow price for this constraint possibly changes?

Can be tightened (increased) by the value of the surplus of 90, *i.e*. x1 + x2 >= 190.

1. Suppose that 50 more legs are available. By how much will the OV change?

Legs constraint is binding and the change is in the allowable range thus SP do not change OV changes by (50)\*SP = 50 \* 6 = $300.

1. Suppose the profit per unit of Mates is reduced to $30. What is the resulting optimal solution? Optimal value?

The reduction is in the allowable range, thus the optimal solution does not change (still 130 captains and 60 Mates. OV however changes by -10 \* 60 = $ -600.

1. Suppose the profit per unit of Captains increase to $80. What is the resulting optimal solution? Optimal value?

Δc = $24, in the allowable rage, thus solution remains optimal, OV increases by 130\*24

= $3120. There will also be an alternative optimum with more Captains.

1. Suppose Captain and Mate chair profits increased by $10 and $5 simultaneously. Will the optimal solution change? What will be the value of the OV?

|  |  |  |  |
| --- | --- | --- | --- |
| Variable | Δc | All. Inc. | % |
| Captain | $10 | $24 | 41.67% |
| Mate | 5 | 16 | 31.25% |

 Since the sum of the % does not exceed 100%, OS stays the same and OV increases by 10\*130 + 5\* 60 = $1,600.

1. At Eastern Steel, the ore from four different locations is blended to make a steel alloy. Each ore contains three essential elements, denoted for simplicity as *A, B,* and *C,* that must appear in the final blend at minimum threshold levels. Eastern pays a different price per ton for the ore from each location. The cost-minimizing blend is obtained by solving the following LP model, where *Ti* = the fraction of a ton of ore from location *i* in one ton of the blend. The Solver solution to this LP model is given along with its Sensitivity Report.

Min 800*T*1 + 400*T*2 + 600*T*3 *+* 500*T4* (total cost)
s.t. 10*T*1 + 3*T*2 + 8*T*3, + 2*T*4≥ 5 (requirement on *A*)
90 *T*1 + 150 *T*2 + 75*T*3 *+* 175 *T*4 ≥ 100 (requirement on *B)*45 *T*1 + 25 *T*2 + 20 *T*3 + 37 *T*4 ≥ 30 (requirement on *C)
T*1 + *T2 + T*3 + *T*4 = 1 (blend condition)
*Ti* ≥ 0 *i= 1,2,3,4*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | Ore Blending Problem |  |  |
| **Mine** | **T1** | **T2** | **T3** | **T4** |  |  |
| **Ton Fraction** | 0.259259 | 0.703704 | 0.037037 | 0 |  **Cost** |  |
| **Cost/Ton** | $800 | $400 | $600 | $500 | $511.11 |  |
| **Constraints:** |  **Composition per Ton** |  | **Total Elements** | **Required** |
| **A** | 10 | 3 | 8 | 2 | 5.00 | 5 |
| **B** | 90 | 150 | 75 | 175 | 131.67 | 100 |
| **C** | 45 | 25 | 20 | 37 | 30.00 | 30 |
| **Composition** | 1 | 1 | 1 | 1 | 1.00 | 1 |
| Adjustable Cells |  |  |  |  |  |
|  |  | **Final** | **Reduced** | **Objective** | **Allowable** | **Allowable** |
| **Cell** | **Name** | **Value** | **Cost** | **Coefficient** | **Increase** | **Decrease** |
| $C$3 | Ton Fraction T1 | 0.259 | 0.000 | 800.000 | 223.636 | 120.000 |
| $D$3 | Ton Fraction T2 | 0.704 | 0.000 | 400.000 | 66.848 | 300.000 |
| $E$3 | Ton Fraction T3 | 0.037 | 0.000 | 600.000 | 85.714 | 118.269 |
| $F$3 | Ton Fraction T4 | 0.000 | 91.111 | 500.000 | 1.000E+30 | 91.111 |
|  |  |  |  |  |  |  |
| Constraints |  |  |  |  |  |
|  |  | **Final** | **Shadow** | **Constraint** | **Allowable** | **Allowable** |
| **Cell** | **Name** | **Value** | **Price** | **R.H. Side** | **Increase** | **Decrease** |
| $G$6 | A Total Elements | 5.000 | 44.444 | 5 | 2.375 | 0.250 |
| $G$7 | B Total Elements | 131.667 | 0.000 | 100 | 31.667 | 1.000E+30 |
| $G$8 | C Total Elements | 30.000 | 4.444 | 30 | 0.714 | 7.000 |
| $G$9 | Composition  | 1.000 | 155.556 | 1 | 0.250 | 0.043 |

Answer each independently

1. How much would the price per ton of ore from location 4 have to decrease in order for it to become attractive to purchase it?

T4 = 0, RC = $91.111. Thus per ton price of T4 must go down by at least 91.111 (or down to $408.89) before ore T4 is a viable substitute for other ores.

1. Suppose that the price of ore from location 1 decreases by $80 per ton. Is there any change in the optimal solution or in the OV?

$ -80 is within the allowable decrease of $120. Solution stays the same OV (cost of per ton of the blend) changes by -80\*.259 = $- 20.72 to $490.39

1. Suppose that the price of ore from location 1 increases by $100 per ton. Is there any change in the optimal solution? What, if any, is the associated change in the cost of an optimally blended ton?

Change is in the allowable range. Thus solution is intact, OV (cost of the blend) changes by $100\*0.259 = $25.90.

1. Suppose that the price of ore from location 3 increases by $50 per ton. Is there any change in the optimal solution? What, if any, is the associated change in the OV?

$ 50 is in the allowable increase. Thus, no change in the optimal solution. OV changes by = $50\* .037 = $1.85/ ton.

1. Analyze the effect on the optimal solution of decreasing the cost of ore from location 3 by exactly $118.269 per ton. (For example, does the present solution remain optimal? Is there an additional optimal solution? And if so, how can it be characterized?)

Since the decrease is not greater than the allowable decrease the current solution will remain optimal. However since the change is equal to the allowable decrease (T3 is “almost” too cheap not to use more of it), there is another optimal solution in which more of T3 (and thus less of T1 and T2) is used

(f) For the change described above in part (e), what is the new OV?

OV going down by $118.269 \* .037 = $4.38.