In[1]:=
```
(* This notebook computes the data used for the two plots
 that ended up in the final version of the paper JHEP 01, (2022) 192.

    I am letting r = x/H  *)
```

In[2]:=
```
Vctilde = ((x - xc) / (xc - xb)) *
   ((x + xc + xb) / (xc + 2 * xb)) ^ (xb / (2 * xc)) * (xb / x) ^ (xb * (2 * xc + xb) / 2)
```

Out[2]=
$$\frac{\left(\frac{xb}{x}\right)^{\frac{1}{2}\,xb\,(xb+2\,xc)}\,(x-xc)\,\left(\frac{x+xb+xc}{2\,xb+xc}\right)^{\frac{xb}{2\,xc}}}{-xb+xc}$$

In[3]:=
```
Ubtilde = - ((x - xb) / (xc - xb)) *
   ((x + xc + xb) / (2 * xc + xb)) ^ (xc / (2 * xb)) * (xc / x) ^ (xc * (xc + 2 * xb) / 2)
```

Out[3]=
$$-\frac{(x-xb)\,\left(\frac{xc}{x}\right)^{\frac{1}{2}\,xc\,(2\,xb+xc)}\,\left(\frac{x+xb+xc}{xb+2\,xc}\right)^{\frac{xc}{2\,xb}}}{-xb+xc}$$

In[4]:=
```
DelUbtilde = (Ubtilde /. {x → x1}) - (Ubtilde /. {x → x2})
```

Out[4]=
$$-\frac{(x1-xb)\,\left(\frac{xc}{x1}\right)^{\frac{1}{2}\,xc\,(2\,xb+xc)}\,\left(\frac{x1+xb+xc}{xb+2\,xc}\right)^{\frac{xc}{2\,xb}}}{-xb+xc}+\frac{(x2-xb)\,\left(\frac{xc}{x2}\right)^{\frac{1}{2}\,xc\,(2\,xb+xc)}\,\left(\frac{x2+xb+xc}{xb+2\,xc}\right)^{\frac{xc}{2\,xb}}}{-xb+xc}$$

In[5]:=
```
DelVctilde = (Vctilde /. {x → x1}) - (Vctilde /. {x → x2})
```

Out[5]=
$$\frac{\left(\frac{xb}{x1}\right)^{\frac{1}{2}\,xb\,(xb+2\,xc)}\,(x1-xc)\,\left(\frac{x1+xb+xc}{2\,xb+xc}\right)^{\frac{xb}{2\,xc}}}{-xb+xc}-\frac{\left(\frac{xb}{x2}\right)^{\frac{1}{2}\,xb\,(xb+2\,xc)}\,(x2-xc)\,\left(\frac{x2+xb+xc}{2\,xb+xc}\right)^{\frac{xb}{2\,xc}}}{-xb+xc}$$

In[6]:=
```
(* Gbar(x1,x2) =
 2 pi G(x1,x2) - T(kappa_b+kappa_c) + Log[omega_0^2/(kappa_b*kappa_c)] + 2*gamma_E  *)
```

In[7]:=
```
Gbar = - Log[Abs[DelUbtilde * DelVctilde]]
```

Out[7]=
$$-\mathrm{Log}\Bigg[\mathrm{Abs}\Bigg[\left(\frac{\left(\frac{xb}{x1}\right)^{\frac{1}{2}\,xb\,(xb+2\,xc)}\,(x1-xc)\,\left(\frac{x1+xb+xc}{2\,xb+xc}\right)^{\frac{xb}{2\,xc}}}{-xb+xc}-\frac{\left(\frac{xb}{x2}\right)^{\frac{1}{2}\,xb\,(xb+2\,xc)}\,(x2-xc)\,\left(\frac{x2+xb+xc}{2\,xb+xc}\right)^{\frac{xb}{2\,xc}}}{-xb+xc}\right)$$
$$\left(-\frac{(x1-xb)\,\left(\frac{xc}{x1}\right)^{\frac{1}{2}\,xc\,(2\,xb+xc)}\,\left(\frac{x1+xb+xc}{xb+2\,xc}\right)^{\frac{xc}{2\,xb}}}{-xb+xc}+\frac{(x2-xb)\,\left(\frac{xc}{x2}\right)^{\frac{1}{2}\,xc\,(2\,xb+xc)}\,\left(\frac{x2+xb+xc}{xb+2\,xc}\right)^{\frac{xc}{2\,xb}}}{-xb+xc}\right)\Bigg]\Bigg]$$

In[8]:=
```
(* Now solve for xc in terms of xb using the identity xb^2 + xb*xc + xc^2 = 1 *)
```

In[9]:=
```
A = xb^2 + xb * xc + xc^2 - 1
```

Out[9]=
$$-1+xb^2+xb\,xc+xc^2$$

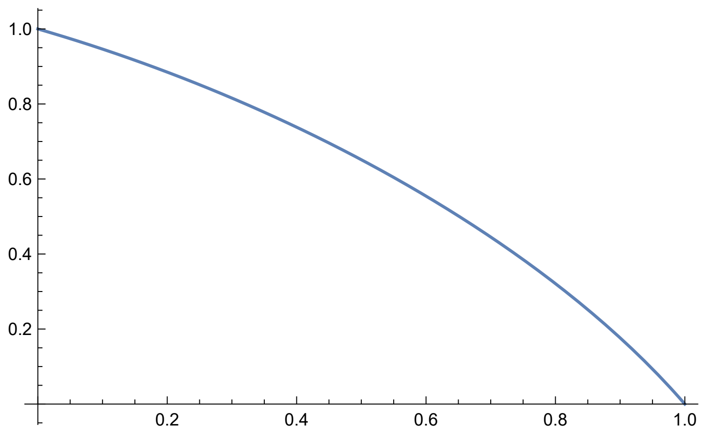
In[10]:=
```
sol = Solve[A == 0, xc]
```
Out[10]=

$$\left\{\left\{xc \to \frac{1}{2}\left(-xb - \sqrt{4 - 3\,xb^2}\right)\right\}, \left\{xc \to \frac{1}{2}\left(-xb + \sqrt{4 - 3\,xb^2}\right)\right\}\right\}$$

In[11]:=
```
xcc = Part[sol, 2, 1, 2]
```
Out[11]=

$$\frac{1}{2}\left(-xb + \sqrt{4 - 3\,xb^2}\right)$$

In[12]:=
```
Plot[xcc, {xb, 0, 1}]
```
Out[12]=



In[13]:=
```
Gbar = Gbar /. {xc → xcc}
```

Out[13]=

$$
\begin{aligned}
-\mathrm{Log}\Bigg[\mathrm{Abs}\Bigg[\Bigg(&-\frac{1}{-xb+\frac{1}{2}\left(-xb+\sqrt{4-3\,xb^2}\right)}\,2^{-\frac{1}{4}\left(-xb+\sqrt{4-3\,xb^2}\right)\left(2\,xb+\frac{1}{2}\left(-xb+\sqrt{4-3\,xb^2}\right)\right)} \\
&\quad (x1-xb)\left(\frac{-xb+\sqrt{4-3\,xb^2}}{x1}\right)^{\frac{1}{4}\left(-xb+\sqrt{4-3\,xb^2}\right)\left(2\,xb+\frac{1}{2}\left(-xb+\sqrt{4-3\,xb^2}\right)\right)} \\
&\quad \left(\frac{x1+xb+\frac{1}{2}\left(-xb+\sqrt{4-3\,xb^2}\right)}{\sqrt{4-3\,xb^2}}\right)^{\frac{-xb+\sqrt{4-3\,xb^2}}{4\,xb}} + \\
&\frac{1}{-xb+\frac{1}{2}\left(-xb+\sqrt{4-3\,xb^2}\right)}\,2^{-\frac{1}{4}\left(-xb+\sqrt{4-3\,xb^2}\right)\left(2\,xb+\frac{1}{2}\left(-xb+\sqrt{4-3\,xb^2}\right)\right)}\,(x2-xb) \\
&\quad \left(\frac{-xb+\sqrt{4-3\,xb^2}}{x2}\right)^{\frac{1}{4}\left(-xb+\sqrt{4-3\,xb^2}\right)\left(2\,xb+\frac{1}{2}\left(-xb+\sqrt{4-3\,xb^2}\right)\right)} \\
&\quad \left(\frac{x2+xb+\frac{1}{2}\left(-xb+\sqrt{4-3\,xb^2}\right)}{\sqrt{4-3\,xb^2}}\right)^{\frac{-xb+\sqrt{4-3\,xb^2}}{4\,xb}}\Bigg) \\
&\Bigg(\frac{\left(\frac{xb}{x1}\right)^{\frac{1}{2}\,xb\,\sqrt{4-3\,xb^2}}\left(x1+\frac{1}{2}\left(xb-\sqrt{4-3\,xb^2}\right)\right)\left(\frac{x1+xb+\frac{1}{2}\left(-xb+\sqrt{4-3\,xb^2}\right)}{2\,xb+\frac{1}{2}\left(-xb+\sqrt{4-3\,xb^2}\right)}\right)^{\frac{xb}{-xb+\sqrt{4-3\,xb^2}}}}{-xb+\frac{1}{2}\left(-xb+\sqrt{4-3\,xb^2}\right)} - \\
&\quad \frac{\left(\frac{xb}{x2}\right)^{\frac{1}{2}\,xb\,\sqrt{4-3\,xb^2}}\left(x2+\frac{1}{2}\left(xb-\sqrt{4-3\,xb^2}\right)\right)\left(\frac{x2+xb+\frac{1}{2}\left(-xb+\sqrt{4-3\,xb^2}\right)}{2\,xb+\frac{1}{2}\left(-xb+\sqrt{4-3\,xb^2}\right)}\right)^{\frac{xb}{-xb+\sqrt{4-3\,xb^2}}}}{-xb+\frac{1}{2}\left(-xb+\sqrt{4-3\,xb^2}\right)}\Bigg)\Bigg]\Bigg]
\end{aligned}
$$

In[14]:=
```
(* Now locate the cosmological horizon for various values of xb. *)
```

In[15]:=
```
xcxbp1 = N[xcc /. {xb → 1 / 10}]
```

Out[15]=
```
0.946243
```

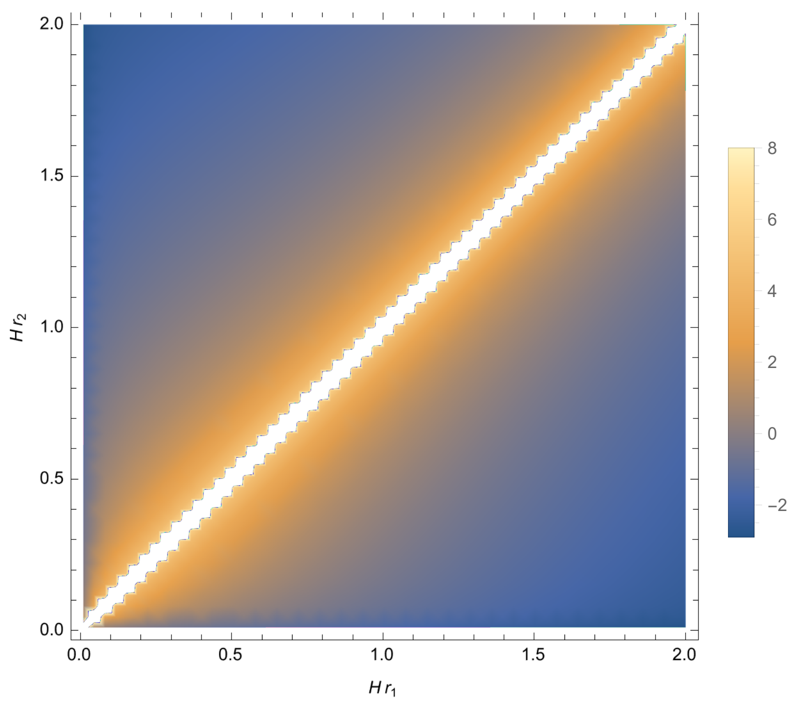
In[16]:=
```
xcxbp2 = N[xcc /. {xb → 2 / 10}]
```

Out[16]=
```
0.884886
```

In[17]:=
```
xcxbp3 = N[xcc /. {xb → 3 / 10}]
```
Out[17]=
```
0.81566
```
In[18]:=
```
xcxbp4 = N[xcc /. {xb → 4 / 10}]
```
Out[18]=
```
0.738083
```
In[19]:=
```
xcxbp5 = N[xcc /. {xb → 5 / 10}]
```
Out[19]=
```
0.651388
```

The following is the plot used in Fig. 1.

In[20]:=
```
p1 = DensityPlot[Gbar /. {xb → 2 / 10, xc → xcxbp2}, {x1, 1 / 100, 2}, {x2, 1 / 100, 2},
  PlotLegends → Automatic, Frame → True, FrameLabel → {H Subscript[r, 1], H Subscript[r, 2]}]
```
Out[20]=



In[21]:=
```
(* Now I'll put in the velocity correlation function plot. *)
```
In[22]:=
```
(* So xb goes from 0 to 1/Sqrt[3]   and xc then goes from 1 to 1/Sqrt[3]   *)
```
In[23]:=
```
xstar = Log[Abs[x - xb] / (xc - xb)] / (2 * kapb) -
  Log[Abs[x - xc] / (xc - xb)] / (2 * kapc) + Log[(x + xb + xc) / (xc + 2 * xb)] / (2 * kapN) -
  (xc / (4 * xb * kapb)) * Log[(2 * xc + xb) / (xc + 2 * xb)] - (xb * xc / (2 * (xc - xb))) * Log[xb / xc]
```
Out[23]=

$$-\frac{xb\ xc\ \text{Log}\left[\frac{xb}{xc}\right]}{2\ (-xb+xc)}+\frac{\text{Log}\left[\frac{x+xb+xc}{2\ xb+xc}\right]}{2\ \text{kapN}}-\frac{xc\ \text{Log}\left[\frac{xb+2\ xc}{2\ xb+xc}\right]}{4\ \text{kapb}\ xb}+\frac{\text{Log}\left[\frac{\text{Abs}[x-xb]}{-xb+xc}\right]}{2\ \text{kapb}}-\frac{\text{Log}\left[\frac{\text{Abs}[x-xc]}{-xb+xc}\right]}{2\ \text{kapc}}$$

In[24]:=
```
h = Log[Abs[x - xb] / (xc - xb)] / (2 * kapb) + Log[Abs[x - xc] / (xc - xb)] / (2 * kapc) +
   (1 / 2) * (xc / (xb * kapb) - 1 / kapN) * Log[(x + xb + xc) / (xc + 2 * xb)] -
   (xb * xc / (2 * (xc - xb))) * Log[x^2 / (xb * xc)] +
   (xc / (4 * (xb * kapb))) * Log[(xc + 2 * xb) / (2 * xc + xb)]
```

Out[24]=

$$-\frac{xb\ xc\ Log\left[\frac{x^2}{xb\ xc}\right]}{2\ (-xb+xc)}+\frac{1}{2}\left(-\frac{1}{kapN}+\frac{xc}{kapb\ xb}\right)Log\left[\frac{x+xb+xc}{2\ xb+xc}\right]+\frac{xc\ Log\left[\frac{2\ xb+xc}{xb+2\ xc}\right]}{4\ kapb\ xb}+\frac{Log\left[\frac{Abs[x-xb]}{-xb+xc}\right]}{2\ kapb}+\frac{Log\left[\frac{Abs[x-xc]}{-xb+xc}\right]}{2\ kapc}$$

In[25]:=
```
(* Let's check these.  *)
```

In[26]:=
```
u = Expand[T - h - xstar]
```

Out[26]=

$$T+\frac{xb\ xc\ Log\left[\frac{x^2}{xb\ xc}\right]}{2\ (-xb+xc)}+\frac{xb\ xc\ Log\left[\frac{xb}{xc}\right]}{2\ (-xb+xc)}-\frac{xc\ Log\left[\frac{x+xb+xc}{2\ xb+xc}\right]}{2\ kapb\ xb}-\frac{xc\ Log\left[\frac{2\ xb+xc}{xb+2\ xc}\right]}{4\ kapb\ xb}+\frac{xc\ Log\left[\frac{xb+2\ xc}{2\ xb+xc}\right]}{4\ kapb\ xb}-\frac{Log\left[\frac{Abs[x-xb]}{-xb+xc}\right]}{kapb}$$

In[27]:=
```
v = Expand[T - h + xstar]
```

Out[27]=

$$T+\frac{xb\ xc\ Log\left[\frac{x^2}{xb\ xc}\right]}{2\ (-xb+xc)}-\frac{xb\ xc\ Log\left[\frac{xb}{xc}\right]}{2\ (-xb+xc)}+\frac{Log\left[\frac{x+xb+xc}{2\ xb+xc}\right]}{kapN}-\frac{xc\ Log\left[\frac{x+xb+xc}{2\ xb+xc}\right]}{2\ kapb\ xb}-\frac{xc\ Log\left[\frac{2\ xb+xc}{xb+2\ xc}\right]}{4\ kapb\ xb}-\frac{xc\ Log\left[\frac{xb+2\ xc}{2\ xb+xc}\right]}{4\ kapb\ xb}-\frac{Log\left[\frac{Abs[x-xc]}{-xb+xc}\right]}{kapc}$$

In[28]:=
```
uHc = Expand[u /. {x → xc}]
```

Out[28]=

$$T+\frac{xb\ xc\ Log\left[\frac{xb}{xc}\right]}{2\ (-xb+xc)}+\frac{xb\ xc\ Log\left[\frac{xc}{xb}\right]}{2\ (-xb+xc)}-\frac{xc\ Log\left[\frac{2\ xb+xc}{xb+2\ xc}\right]}{4\ kapb\ xb}-\frac{xc\ Log\left[\frac{xb+2\ xc}{2\ xb+xc}\right]}{4\ kapb\ xb}-\frac{Log\left[\frac{Abs[-xb+xc]}{-xb+xc}\right]}{kapb}$$

In[29]:=
```
Assuming[xc > xb > 0, FullSimplify[uHc]]
```

Out[29]=

T

In[30]:=
```
vHb = Expand[v /. {x → xb}]
```

Out[30]=

$$T-\frac{xc\ Log\left[\frac{2\ xb+xc}{xb+2\ xc}\right]}{4\ kapb\ xb}-\frac{xc\ Log\left[\frac{xb+2\ xc}{2\ xb+xc}\right]}{4\ kapb\ xb}-\frac{Log\left[\frac{Abs[xb-xc]}{-xb+xc}\right]}{kapc}$$

In[31]:=
```
Assuming[xc > xb > 0, FullSimplify[vHb]]
```

Out[31]=

T

In[32]:=
```
kapb = (xc - xb) * (xc + 2 * xb) / (2 * xb)
```
Out[32]=

$$\frac{(-xb + xc)\ (2\,xb + xc)}{2\,xb}$$

In[33]:=
```
kapc = (xc - xb) * (2 * xc + xb) / (2 * xc)
```
Out[33]=

$$\frac{(-xb + xc)\ (xb + 2\,xc)}{2\,xc}$$

In[34]:=
```
kapN = (2 * xc + xb) * (xc + 2 * xb) / (2 * (xc + xb))
```
Out[34]=

$$\frac{(2\,xb + xc)\ (xb + 2\,xc)}{2\ (xb + xc)}$$

In[35]:=
```
(* Now solve for xc in terms of xb using the identity xb^2 + xb*xc + xc^2 = 1 *)
```

In[36]:=
```
A = xb^2 + xb * xc + xc^2 - 1
```
Out[36]=

$$-1 + xb^2 + xb\,xc + xc^2$$

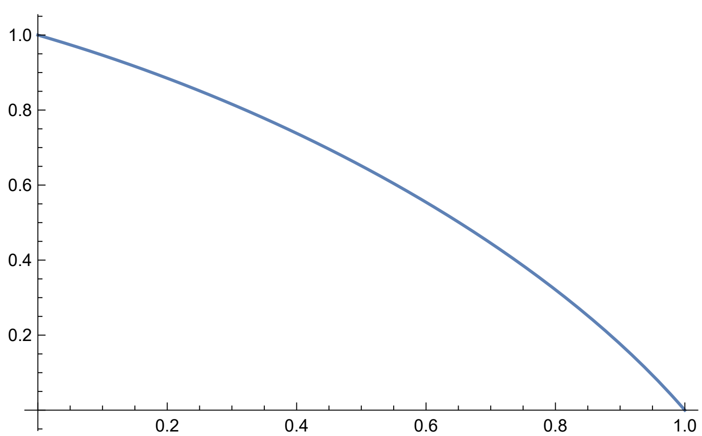
In[37]:=
```
sol = Solve[A == 0, xc]
```
Out[37]=

$$\left\{\left\{xc \to \frac{1}{2}\left(-xb - \sqrt{4 - 3\,xb^2}\right)\right\},\ \left\{xc \to \frac{1}{2}\left(-xb + \sqrt{4 - 3\,xb^2}\right)\right\}\right\}$$

In[38]:=
```
xcc = Part[sol, 2, 1, 2]
```
Out[38]=

$$\frac{1}{2}\left(-xb + \sqrt{4 - 3\,xb^2}\right)$$

In[39]:=
```
Plot[xcc, {xb, 0, 1}]
```
Out[39]=



In[40]:=
```
N[1 / Sqrt[3]]
```
Out[40]=

```
0.57735
```

In[41]:=
```
N[xcc /. {xb → 1 / Sqrt[3]}]
```
Out[41]=

0.57735

In[42]:=
```
(* So xb goes from 0 to 1/sqrt[3] and xc goes from 1 to 1/sqrt[3]  *)
```

In[43]:=
```
u1 = u /. {x → x1}
```
Out[43]=

$$T+\frac{xb\,xc\,\text{Log}\left[\frac{x1^2}{xb\,xc}\right]}{2\,(-xb+xc)}+\frac{xb\,xc\,\text{Log}\left[\frac{xb}{xc}\right]}{2\,(-xb+xc)}-\frac{xc\,\text{Log}\left[\frac{x1+xb+xc}{2\,xb+xc}\right]}{(-xb+xc)\,(2\,xb+xc)}-\frac{xc\,\text{Log}\left[\frac{2\,xb+xc}{xb+2\,xc}\right]}{2\,(-xb+xc)\,(2\,xb+xc)}+\frac{xc\,\text{Log}\left[\frac{xb+2\,xc}{2\,xb+xc}\right]}{2\,(-xb+xc)\,(2\,xb+xc)}-\frac{2\,xb\,\text{Log}\left[\frac{\text{Abs}[x1-xb]}{-xb+xc}\right]}{(-xb+xc)\,(2\,xb+xc)}$$

In[44]:=
```
u2 = u /. {x → x2}
```
Out[44]=

$$T+\frac{xb\,xc\,\text{Log}\left[\frac{x2^2}{xb\,xc}\right]}{2\,(-xb+xc)}+\frac{xb\,xc\,\text{Log}\left[\frac{xb}{xc}\right]}{2\,(-xb+xc)}-\frac{xc\,\text{Log}\left[\frac{x2+xb+xc}{2\,xb+xc}\right]}{(-xb+xc)\,(2\,xb+xc)}-\frac{xc\,\text{Log}\left[\frac{2\,xb+xc}{xb+2\,xc}\right]}{2\,(-xb+xc)\,(2\,xb+xc)}+\frac{xc\,\text{Log}\left[\frac{xb+2\,xc}{2\,xb+xc}\right]}{2\,(-xb+xc)\,(2\,xb+xc)}-\frac{2\,xb\,\text{Log}\left[\frac{\text{Abs}[x2-xb]}{-xb+xc}\right]}{(-xb+xc)\,(2\,xb+xc)}$$

In[45]:=
```
v1 = v /. {x → x1}
```
Out[45]=

$$T+\frac{xb\,xc\,\text{Log}\left[\frac{x1^2}{xb\,xc}\right]}{2\,(-xb+xc)}-\frac{xb\,xc\,\text{Log}\left[\frac{xb}{xc}\right]}{2\,(-xb+xc)}-\frac{xc\,\text{Log}\left[\frac{x1+xb+xc}{2\,xb+xc}\right]}{(-xb+xc)\,(2\,xb+xc)}+\frac{2\,(xb+xc)\,\text{Log}\left[\frac{x1+xb+xc}{2\,xb+xc}\right]}{(2\,xb+xc)\,(xb+2\,xc)}-\frac{xc\,\text{Log}\left[\frac{2\,xb+xc}{xb+2\,xc}\right]}{2\,(-xb+xc)\,(2\,xb+xc)}-\frac{xc\,\text{Log}\left[\frac{xb+2\,xc}{2\,xb+xc}\right]}{2\,(-xb+xc)\,(2\,xb+xc)}-\frac{2\,xc\,\text{Log}\left[\frac{\text{Abs}[x1-xc]}{-xb+xc}\right]}{(-xb+xc)\,(xb+2\,xc)}$$

In[46]:=
```
v2 = v /. {x → x2}
```
Out[46]=

$$T+\frac{xb\,xc\,\text{Log}\left[\frac{x2^2}{xb\,xc}\right]}{2\,(-xb+xc)}-\frac{xb\,xc\,\text{Log}\left[\frac{xb}{xc}\right]}{2\,(-xb+xc)}-\frac{xc\,\text{Log}\left[\frac{x2+xb+xc}{2\,xb+xc}\right]}{(-xb+xc)\,(2\,xb+xc)}+\frac{2\,(xb+xc)\,\text{Log}\left[\frac{x2+xb+xc}{2\,xb+xc}\right]}{(2\,xb+xc)\,(xb+2\,xc)}-\frac{xc\,\text{Log}\left[\frac{2\,xb+xc}{xb+2\,xc}\right]}{2\,(-xb+xc)\,(2\,xb+xc)}-\frac{xc\,\text{Log}\left[\frac{xb+2\,xc}{2\,xb+xc}\right]}{2\,(-xb+xc)\,(2\,xb+xc)}-\frac{2\,xc\,\text{Log}\left[\frac{\text{Abs}[x2-xc]}{-xb+xc}\right]}{(-xb+xc)\,(xb+2\,xc)}$$

In[47]:=
```
(* Every quantity depends on either u2 - u1 or v2 -
  v1 and once the derivatives are computed we set T2 = T1 = T so
    the T dependence goes away.  Further they are symmetric in u2-u1 or v2-v1.   *)
```

In[48]:=
```
Delu = u2 - u1
```
Out[48]=

$$-\frac{xb\ xc\ \mathrm{Log}\left[\frac{x1^2}{xb\ xc}\right]}{2\ (-xb+xc)}+\frac{xb\ xc\ \mathrm{Log}\left[\frac{x2^2}{xb\ xc}\right]}{2\ (-xb+xc)}+\frac{xc\ \mathrm{Log}\left[\frac{x1+xb+xc}{2\ xb+xc}\right]}{(-xb+xc)\ (2\ xb+xc)}-\frac{xc\ \mathrm{Log}\left[\frac{x2+xb+xc}{2\ xb+xc}\right]}{(-xb+xc)\ (2\ xb+xc)}+\frac{2\ xb\ \mathrm{Log}\left[\frac{\mathrm{Abs}[x1-xb]}{-xb+xc}\right]}{(-xb+xc)\ (2\ xb+xc)}-\frac{2\ xb\ \mathrm{Log}\left[\frac{\mathrm{Abs}[x2-xb]}{-xb+xc}\right]}{(-xb+xc)\ (2\ xb+xc)}$$

In[49]:=
```
Delv = v2 - v1
```
Out[49]=

$$-\frac{xb\ xc\ \mathrm{Log}\left[\frac{x1^2}{xb\ xc}\right]}{2\ (-xb+xc)}+\frac{xb\ xc\ \mathrm{Log}\left[\frac{x2^2}{xb\ xc}\right]}{2\ (-xb+xc)}+\frac{xc\ \mathrm{Log}\left[\frac{x1+xb+xc}{2\ xb+xc}\right]}{(-xb+xc)\ (2\ xb+xc)}-\frac{2\ (xb+xc)\ \mathrm{Log}\left[\frac{x1+xb+xc}{2\ xb+xc}\right]}{(2\ xb+xc)\ (xb+2\ xc)}-\frac{xc\ \mathrm{Log}\left[\frac{x2+xb+xc}{2\ xb+xc}\right]}{(-xb+xc)\ (2\ xb+xc)}+\frac{2\ (xb+xc)\ \mathrm{Log}\left[\frac{x2+xb+xc}{2\ xb+xc}\right]}{(2\ xb+xc)\ (xb+2\ xc)}+\frac{2\ xc\ \mathrm{Log}\left[\frac{\mathrm{Abs}[x1-xc]}{-xb+xc}\right]}{(-xb+xc)\ (xb+2\ xc)}-\frac{2\ xc\ \mathrm{Log}\left[\frac{\mathrm{Abs}[x2-xc]}{-xb+xc}\right]}{(-xb+xc)\ (xb+2\ xc)}$$

In[50]:=
```
(* Now I'll compute the parts of \partial_T \partial_T' G
  that depend on Delu and Delv separately.  I'll use lt for less than
  in the names and gt for greater than.  I'll start with 0 < x1 < x2 < xb  *)
```

In[51]:=
```
GTTUbxxxb =
 Assuming[{x1 > 0, x2 > 0, xb > 0}, Simplify[-kapb^2 / (8 * Pi * Sinh[kapb * Delu / 2] ^2)]]
```
Out[51]=

$$-\frac{1}{32\,\pi\,xb^2}\,(xb-xc)^2\,(2\,xb+xc)^2\,\mathrm{Csch}\left[\frac{1}{4\,xb}\left(-xb\,xc\,(2\,xb+xc)\,\mathrm{Log}[x1]+xb\,xc\,(2\,xb+xc)\,\mathrm{Log}[x2]+xc\,\mathrm{Log}\left[\frac{x1+xb+xc}{2\,xb+xc}\right]-xc\,\mathrm{Log}\left[\frac{x2+xb+xc}{2\,xb+xc}\right]+2\,xb\,\mathrm{Log}[\mathrm{Abs}[x1-xb]]-2\,xb\,\mathrm{Log}[\mathrm{Abs}[x2-xb]]\right)\right]^2$$

In[52]:=
```
(* This is for one point less than xb and the other greater than xb. *)
```

In[53]:=
```
GTTUbxxbx =
 Assuming[{x1 > 0, x2 > 0, xb > 0} , Simplify[kapb^2 / (8 * Pi * Cosh[kapb * Delu / 2] ^2)]]
```
Out[53]=

$$\frac{1}{32\,\pi\,xb^2}\,(xb-xc)^2\,(2\,xb+xc)^2\,\mathrm{Sech}\left[\frac{1}{4\,xb}\left(-xb\,xc\,(2\,xb+xc)\,\mathrm{Log}[x1]+xb\,xc\,(2\,xb+xc)\,\mathrm{Log}[x2]+xc\,\mathrm{Log}\left[\frac{x1+xb+xc}{2\,xb+xc}\right]-xc\,\mathrm{Log}\left[\frac{x2+xb+xc}{2\,xb+xc}\right]+2\,xb\,\mathrm{Log}[\mathrm{Abs}[x1-xb]]-2\,xb\,\mathrm{Log}[\mathrm{Abs}[x2-xb]]\right)\right]^2$$

In[54]:=
```
(* This is for both points greater than xb. *)
```

In[55]:=
```
GTTUbxbxx =
 Assuming[{x1 > 0, x2 > 0, xb > 0} , Simplify[-kapb^2 / (8 * Pi * Sinh[kapb * Delu / 2] ^2)]]
```

Out[55]=

$$-\frac{1}{32\,\pi\,xb^2}\,(xb-xc)^2\,(2\,xb+xc)^2\,\text{Csch}\Big[\frac{1}{4\,xb}\Big(-xb\,xc\,(2\,xb+xc)\,\text{Log}[x1]+xb\,xc\,(2\,xb+xc)\,\text{Log}[x2]+xc\,\text{Log}\Big[\frac{x1+xb+xc}{2\,xb+xc}\Big]-xc\,\text{Log}\Big[\frac{x2+xb+xc}{2\,xb+xc}\Big]+2\,xb\,\text{Log}[\text{Abs}[x1-xb]]-2\,xb\,\text{Log}[\text{Abs}[x2-xb]]\Big)\Big]^2$$

In[56]:=
```
(* This is for both points less than xc. *)
```

In[57]:=
```
GTTVcxxxc =
 Assuming[{x1 > 0, x2 > 0, xc > 0}, Simplify[-kapc^2 / (8 * Pi * Sinh[kapc * Delv / 2] ^2)]]
```

Out[57]=

$$-\frac{1}{32\,\pi\,xc^2}\,(xb-xc)^2\,(xb+2\,xc)^2\,\text{Csch}\Big[\frac{1}{4\,xc}\Big(-xb\,xc\,(xb+2\,xc)\,\text{Log}[x1]+xb\,xc\,(xb+2\,xc)\,\text{Log}[x2]+xb\,\text{Log}\Big[\frac{x1+xb+xc}{2\,xb+xc}\Big]-xb\,\text{Log}\Big[\frac{x2+xb+xc}{2\,xb+xc}\Big]+2\,xc\,\text{Log}[\text{Abs}[x1-xc]]-2\,xc\,\text{Log}[\text{Abs}[x2-xc]]\Big)\Big]^2$$

This is the component used for Figure 2.

In[58]:=
```
(* This is for one point less than and one point greater than xc. *)
```

In[59]:=
```
GTTVcxxcx =
 Assuming[{x1 > 0, x2 > 0, xc > 0}, Simplify[kapc^2 / (8 * Pi * Cosh[kapc * Delv / 2] ^2)]]
```

Out[59]=

$$\frac{1}{32\,\pi\,xc^2}\,(xb-xc)^2\,(xb+2\,xc)^2\,\text{Sech}\Big[\frac{1}{4\,xc}\Big(-xb\,xc\,(xb+2\,xc)\,\text{Log}[x1]+xb\,xc\,(xb+2\,xc)\,\text{Log}[x2]+xb\,\text{Log}\Big[\frac{x1+xb+xc}{2\,xb+xc}\Big]-xb\,\text{Log}\Big[\frac{x2+xb+xc}{2\,xb+xc}\Big]+2\,xc\,\text{Log}[\text{Abs}[x1-xc]]-2\,xc\,\text{Log}[\text{Abs}[x2-xc]]\Big)\Big]^2$$

In[60]:=
```
(* This is for both points greater than xc. *)
```

In[61]:=
```
GTTVcxcxx =
  Assuming[{x1 > 0, x2 > 0, xc > 0}, Simplify[-kapc^2 / (8 * Pi * Sinh[kapc * Delv / 2]^2)]]
```

Out[61]=

$$-\frac{1}{32\,\pi\,xc^2}(xb-xc)^2\,(xb+2\,xc)^2\,\text{Csch}\Big[\frac{1}{4\,xc}\Big(-xb\,xc\,(xb+2\,xc)\,\text{Log}[x1]+xb\,xc\,(xb+2\,xc)\,\text{Log}[x2]+xb\,\text{Log}\Big[\frac{x1+xb+xc}{2\,xb+xc}\Big]-xb\,\text{Log}\Big[\frac{x2+xb+xc}{2\,xb+xc}\Big]+2\,xc\,\text{Log}[\text{Abs}[x1-xc]]-2\,xc\,\text{Log}[\text{Abs}[x2-xc]]\Big)\Big]^2$$

In[62]:=
```
(* Now locate the cosmological horizon for various values of xb. *)
```

In[63]:=
```
xcxbp1 = N[xcc /. {xb → 1 / 10}]
```

Out[63]=
```
0.946243
```

In[64]:=
```
xbdivxcp1 = 1 / (10 * xcxbp1)
```

Out[64]=
```
0.105681
```

In[65]:=
```
xcdivxbp1 = 1 / xbdivxcp1
```

Out[65]=
```
9.46243
```

In[66]:=
```
xcxbp1p1 = N[xcc /. {xb → 11 / 100}]
```

Out[66]=
```
0.940452
```

In[67]:=
```
xcxbp1p2 = N[xcc /. {xb → 12 / 100}]
```

Out[67]=
```
0.934585
```

In[68]:=
```
xbdivxcp1p2 = 12 / (100 * xcxbp1p2)
```

Out[68]=
```
0.128399
```

In[69]:=
```
xcdivxbp1p2 = 1 / xbdivxcp1p2
```

Out[69]=
```
7.78821
```

In[70]:=
```
xcxbp1p5 = N[xcc /. {xb → 15 / 100}]
```

Out[70]=
```
0.916527
```

In[71]:=
```
xcxbp2 = N[xcc /. {xb → 2 / 10}]
```

Out[71]=
```
0.884886
```

```
In[72]:= xbdivxcp2 = 2 / (10 * xcxbp2)
Out[72]=
0.226018

In[73]:= xcdivxbp2 = 1 / xbdivxcp2
Out[73]=
4.42443

In[74]:= xcxbp3 = N[xcc /. {xb → 3 / 10}]
Out[74]=
0.81566

In[75]:= xbdivxcp3 = 3 / (10 * xcxbp3)
Out[75]=
0.3678

In[76]:= xcdivxbp3 = 1 / xbdivxcp3
Out[76]=
2.71887

In[77]:= xcxbp4 = N[xcc /. {xb → 4 / 10}]
Out[77]=
0.738083

In[78]:= xbdivxcp4 = 4 / (10 * xcxbp4)
Out[78]=
0.541944

In[79]:= xcdivxbp4 = 1 / xbdivxcp4
Out[79]=
1.84521

In[80]:= xcxbp5 = N[xcc /. {xb → 5 / 10}]
Out[80]=
0.651388

In[81]:= xbdivxcp5 = 5 / (10 * xcxbp5)
Out[81]=
0.767592

In[82]:= xcdivxbp5 = 1 / xbdivxcp5
Out[82]=
1.30278

In[83]:= (* If one point is on xb then GTTUb =
 0 so long as the other point is elsewhere so I just need to plot GTTVc.  Also in order
   to plot more than one value of xb on the same plot I will change variables to x =
  xc*y so that y = 1 is always the location of the cosmological horizon.  The
    reason for this is that I get a peak
    when the other point is outside the cosmological horizon.   *)
```

In[84]:=
```
GTTVcxxxcy = GTTVcxxxc /. {x1 → xc * y1, x2 → xc * y2}
```

Out[84]=

$$-\frac{1}{32\,\pi\,xc^2}\,(xb-xc)^2\,(xb+2\,xc)^2\,\mathrm{Csch}\left[\frac{1}{4\,xc}\left(-xb\,xc\,(xb+2\,xc)\,\mathrm{Log}[xc\,y1]+xb\,\mathrm{Log}\left[\frac{xb+xc+xc\,y1}{2\,xb+xc}\right]+xb\,xc\,(xb+2\,xc)\,\mathrm{Log}[xc\,y2]-xb\,\mathrm{Log}\left[\frac{xb+xc+xc\,y2}{2\,xb+xc}\right]+2\,xc\,\mathrm{Log}[\mathrm{Abs}[-xc+xc\,y1]]-2\,xc\,\mathrm{Log}[\mathrm{Abs}[-xc+xc\,y2]]\right)\right]^2$$

In[85]:=
```
GTTVcxxcxy = GTTVcxxcx /. {x1 → xc * y1, x2 → xc * y2}
```

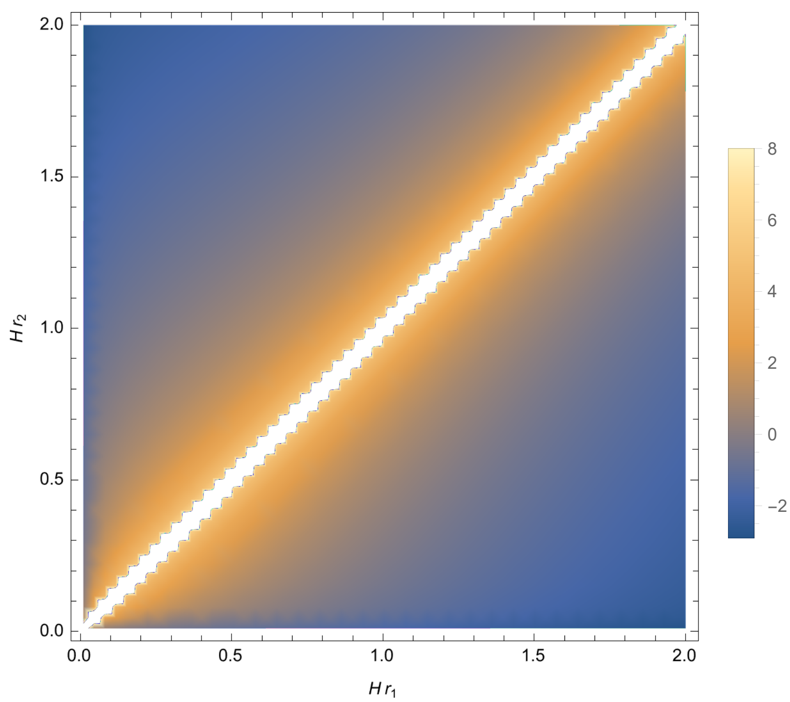
Out[85]=

$$\frac{1}{32\,\pi\,xc^2}\,(xb-xc)^2\,(xb+2\,xc)^2\,\mathrm{Sech}\left[\frac{1}{4\,xc}\left(-xb\,xc\,(xb+2\,xc)\,\mathrm{Log}[xc\,y1]+xb\,\mathrm{Log}\left[\frac{xb+xc+xc\,y1}{2\,xb+xc}\right]+xb\,xc\,(xb+2\,xc)\,\mathrm{Log}[xc\,y2]-xb\,\mathrm{Log}\left[\frac{xb+xc+xc\,y2}{2\,xb+xc}\right]+2\,xc\,\mathrm{Log}[\mathrm{Abs}[-xc+xc\,y1]]-2\,xc\,\mathrm{Log}[\mathrm{Abs}[-xc+xc\,y2]]\right)\right]^2$$

In[86]:=
```
(* I'll reproduce p1 here so both the plots are at the end. *)
```

In[87]:=
```
p1
```

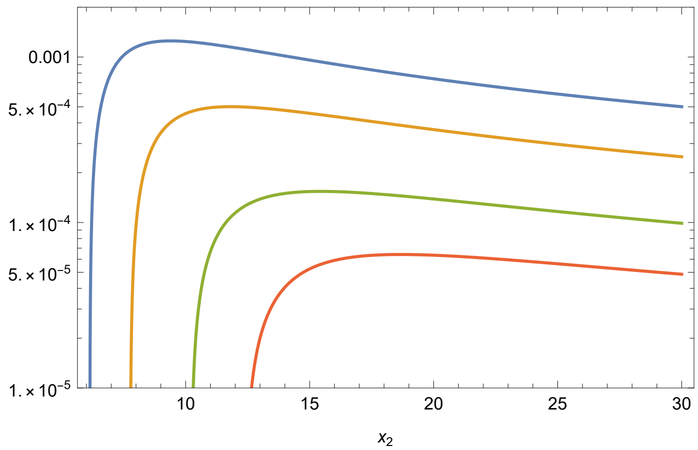
Out[87]=



This is the second plot for the published version of the paper. However, that actual plot was produced by a different program from the data that is computed below. The original plot was labeled p7.

In[88]:=
```
p2 = LogPlot[{(GTTUbxbxx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.9 * xcxbp1}) +
     (GTTVcxxcx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.9 * xcxbp1}),
   (GTTUbxbxx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.95 * xcxbp1}) +
     (GTTVcxxcx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.95 * xcxbp1}),
   (GTTUbxbxx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.98 * xcxbp1}) +
     (GTTVcxxcx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.98 * xcxbp1}),
   (GTTUbxbxx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.99 * xcxbp1}) +
     (GTTVcxxcx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.99 * xcxbp1})}, {x2, xcxbp1, 30},
  PlotRange → {10^(-5), 2 * 10^(-3)}, Frame → True, FrameLabel → Subscript[x, 2]]
```

Out[88]=



In[89]:=
```
(* This is the data that I used for the plot in
 Fig. 2 of the paper.  The file is called Fig_p7.png.  *)
```

In[90]:=
```
T7 = Table[
   {N[i / 200], N[(GTTUbxbxx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.9 * xcxbp1, x2 → i / 200}) +
       (GTTVcxxcx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.9 * xcxbp1, x2 → i / 200})],
    N[(GTTUbxbxx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.95 * xcxbp1, x2 → i / 200}) +
       (GTTVcxxcx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.95 * xcxbp1, x2 → i / 200})],
    N[(GTTUbxbxx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.98 * xcxbp1, x2 → i / 200}) +
       (GTTVcxxcx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.98 * xcxbp1, x2 → i / 200})],
    N[(GTTUbxbxx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.99 * xcxbp1, x2 → i / 200}) +
       (GTTVcxxcx /. {xb → 1 / 10, xc → xcxbp1, x1 → 0.99 * xcxbp1, x2 → i / 200})]}, {i, 6000}];
```

In[91]:=
```
(* Export["C:\\Users\\anderson\\Tex\\Research - Current\\BH in
   Cosmology\\2D Stress Tensor\\Numerical_data_for_plots\\Plot7.dat",T7] *)
```