INSIDE LAB 3: KEPLER'S LAWS

OBJECTIVE: *To become familiar with the properties of the orbits of the Moon and planets.*

DISCUSSION:

As seen from the Earth the motions of the Sun, Moon and planets are rather complicated when studied in detail. Sometimes they move faster through the sky with respect to the stars, sometimes slower. Sometimes a planet will stop moving with respect to the stars, reverse its direction for awhile, stop again and resume its usual course. When moving "backwards" with respect to the stars we say the planet is undergoing **retrograde** motion.

All of the orbits of these objects are elliptical and each object obeys Kepler's three laws. Kepler's first law states that the orbits of the planets about the Sun are ellipses with the Sun at one focus. His second law says that the line between a planet and the Sun sweeps out equal areas of the ellipse in equal times. His third law states that for a planet orbiting the Sun, $P^2 = a^3$ with P the orbital period in years and "a" the semimajor axis in AU. (For the Moon the law reads $P^2 = 1/3 \times 10^6 a^3$ with P in years and a in AU. This is because the Moon orbits the Earth rather than the Sun.)

In this lab you will become more familiar with the properties of ellipses and use a computer program which demonstrates Kepler's second law and can be used to deduce his third law.
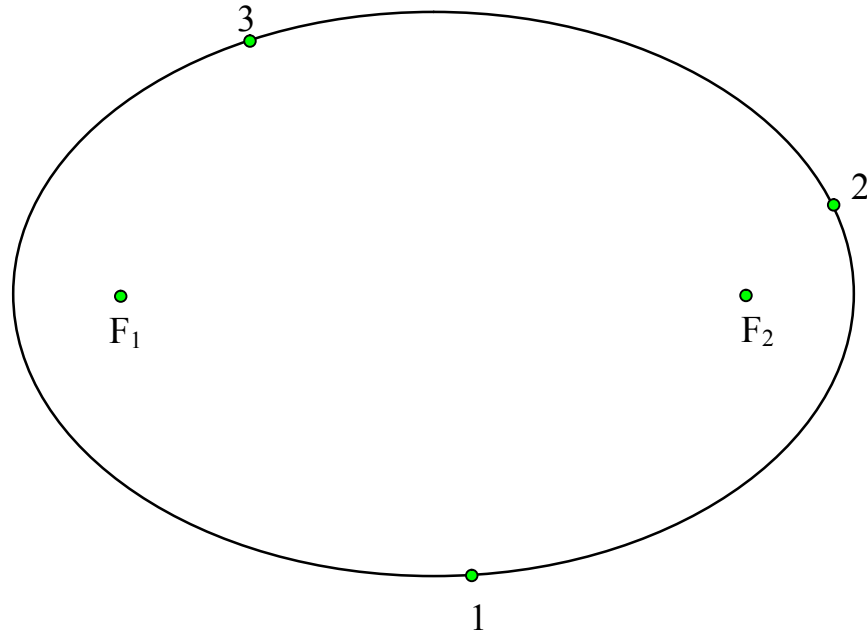
Figure 1: Planetary orbit. The planet's position is shown at three different times.

It turns out that all planetary orbits obey the same physical principles. For example, the orbit of every body in the solar system has two special points located inside the orbit. Consider Figure 2 above. It shows a planet at three points in its orbit (labeled 1, 2, and 3) and the special interior points of the orbits (labeled F_1 and F_2). These interior points are called the **foci** of the orbit (the singular is **focus**).

EXERCISE 1:

For each point in the orbit draw a straight line to each focus, and then measure its length. What do you notice about the measurements? Compare the sum of the distances for each point in the orbit.

Answer:

It turns out that the sum of the distances you just measured equals a number *that never changes* throughout the orbit. Hopefully, you measurements in Exercise 1 were accurate enough to help convince you of this. The type of curve that obeys this property is called an **ellipse**.
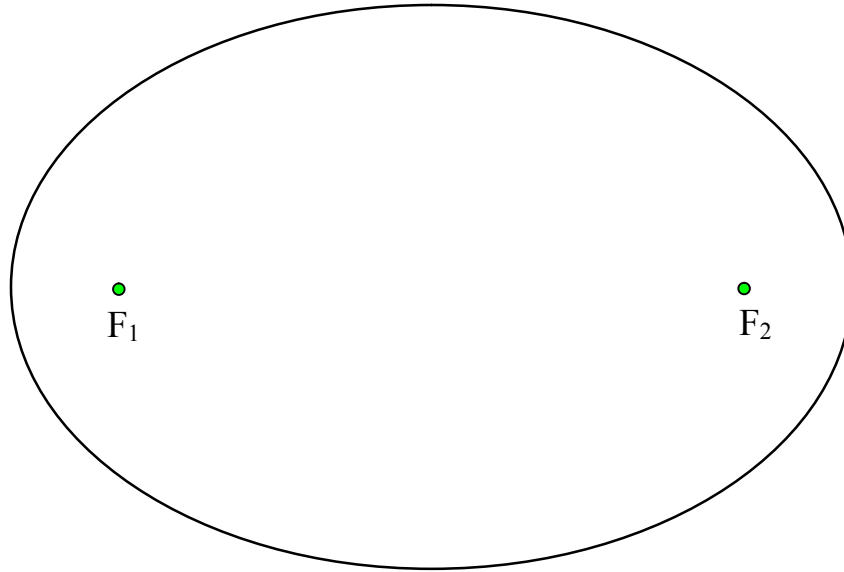

Figure 3: An ellipse.

EXERCISE 2:

Carefully draw a straight line through the widest part of the ellipse in Figure 3 (through the foci) and measure its length. How does its length compare to the sums you computed in Exercise 1?

Answer:

We call the line you have just drawn the **major axis** of the orbit. We usually work with half of this length, called the **semi-major axis**. Astronomers have found that for any planet, the length of the major axis is equal to the constant sum of the distances discussed above. You're measurement may not show them to be *exactly* equal (why not?), but hopefully it was close. All of this is **Kepler's First Law**: the orbit of every planet is an ellipse with the Sun located at one focus. There is nothing located at the other focus.

EXERCISE 3:

In Figure 3, carefully bisect the major axis and draw in a line that is perpendicular to it, from the top of the ellipse to the bottom of it. This line is called the **minor axis**. We usually work half of this length, called the **semi-minor axis**. The point where the major and minor axes intersect is the **center** of the ellipse (this should be obvious from your figure). Label the semi-major axis a, the semi-minor axis b, and the distance from the center of the ellipse to the left-hand focus f. (Is the right-hand focus the same distance from the center of the ellipse as the left-hand focus? It should be.) Finally, carefully draw a straight line from the left-hand focus to the top of the ellipse, where the minor axis meets the edge of the curve. Measure this length. How does it compare to the length of the semi-major axis?

Answer:

EXERCISE 4:

Looking at Figure 3, the last line you've just drawn should be the same length as the semi-major axis. Based on what you've discovered so far, you can write down a geometric formula that connects *a, b,* and *f.* By the way, *f* is called the **focal length** of the ellipse.

The formula: _____

An ellipse is characterized by its <u>size</u> and <u>shape</u>. The size of the orbit is given by the semi-major axis, which you have labeled *a*. The shape of the orbit is given by the ratio of the focal length to the semi-major axis. This ratio is called the **eccentricity, *e.***

EXERCISE 5:

Determine the ratio of the focal length to the semi-major axis in Figure 3.

Answer:

Kepler's Third Law relates the time needed for the planet to complete one full orbit to the geometry of the ellipse that describes its orbit. To discover Kepler's Third Law, we first have to determine whether the size or shape of the orbit, or both, affects the period. In the next exercise you will devise and complete a simple experiment to determine whether the semi-major axis or the eccentricity, or both, affects the period of a planet's orbit.

EXERCISE 6:

Open the program *Kepler*, which can be found online at
http://users.wfu.edu/yipcw/phy/kepler2/
The software requires two inputs; the first is the length of the semi-major axis in Astronomical Units (A.U.)[1] and the second is the eccentricity of the orbit. Eccentricities must be between 0 and 1 (why?). *Kepler* shows the elapsed time in the lower right hand corner. It's measured in days. When the orbit is finished, the time shown is the period of the orbit. Devise and complete an experiment to determine which parameter, *a, e*, or both, affects the period of the orbit. Write in a few sentences your procedure and results. Use the space below and the next page for your results. If you need to, continue on the back of these pages.

_____

[1] An "Astronomical Unit" is the average distance of between the Earth and the Sun. It is approximately 93,000,000 miles or 150,000,000 kilometers.

Procedure and Results:

EXERCISE 7:

Now, use the numbers given in the table below as the inputs for *Kepler*. As you have seen, *Kepler* gives you the period of the orbit in days. For each object, write this value in the table below and convert the same to years. Fill out the remaining columns with the indicated quantity. For example, in the P^2 column you would enter the square of the period in years. (In each of the P-columns you are working with P in <u>years</u>.)

Notice that last three columns are labeled for either the eccentricity *e* or the semi-major axis *a*. Fill in the values for the one that you determined to have an affect on the period in Exercise 6. Make sure you were correct!

Object	*a* (AU)	*e*	*P* (days)	*P* (years)	P^2	P^3	*e* or *a*	e^2 or a^2	e^3 or a^3
Earth	1.0000	0.017							
Mars	1.5237	0.093							
Jupiter	5.202	0.048							
Ceres[2]	2.77	0.097							

Table 1 Orbital data. Fill in the values for the period and semi-major axis from *Kepler*. From the data, what is Kepler's Third Law?

Examining the last six columns of data, what must be Kepler's Third Law? Express the law in both words and an equation.
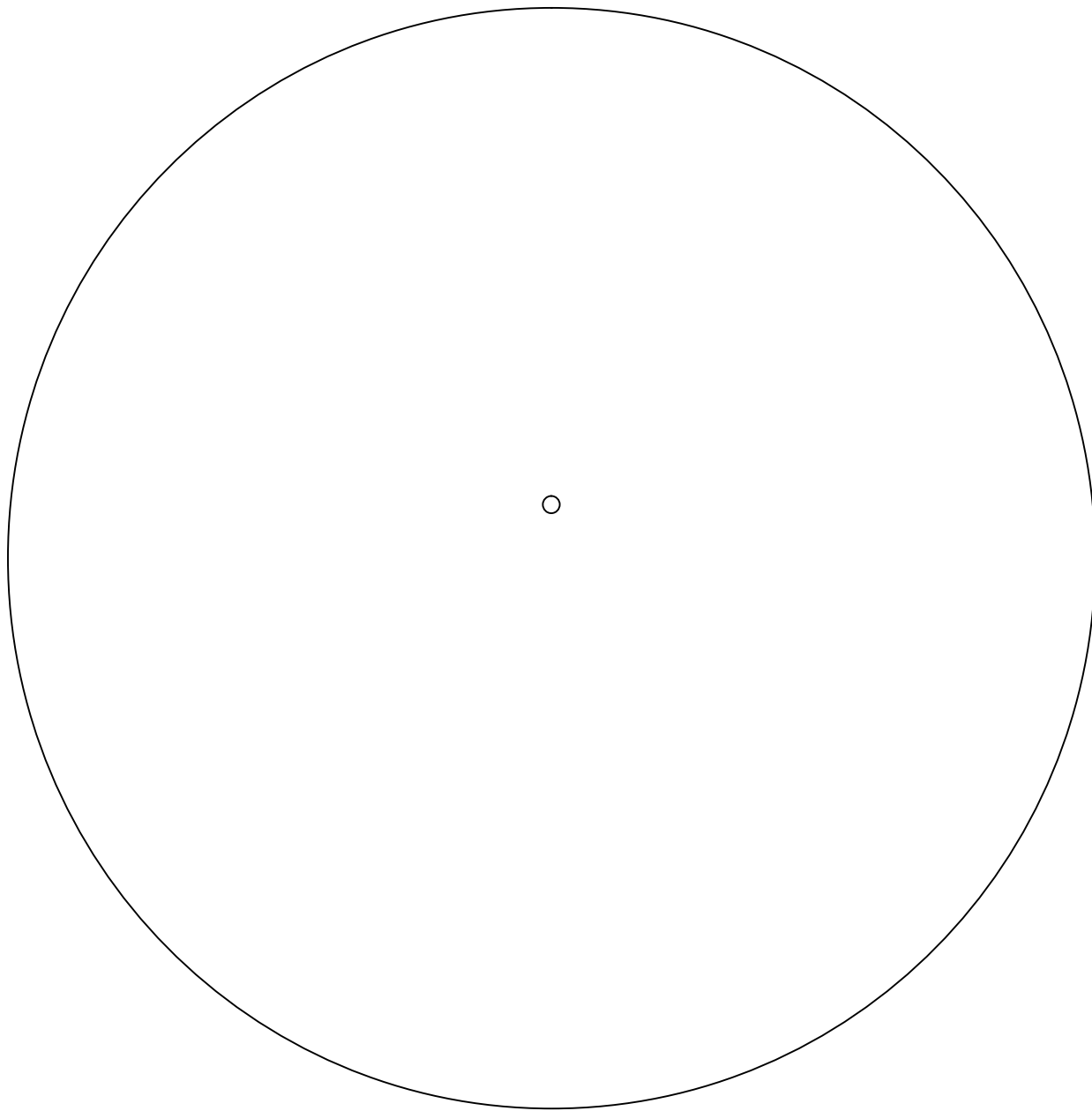
Answer:

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[2] Ceres is a large asteroid. After this exercise, you will know whether Kepler's Third Law applies to objects <u>other than planets</u>. (Kepler himself didn't know, in part because he didn't know there were any asteroids.)
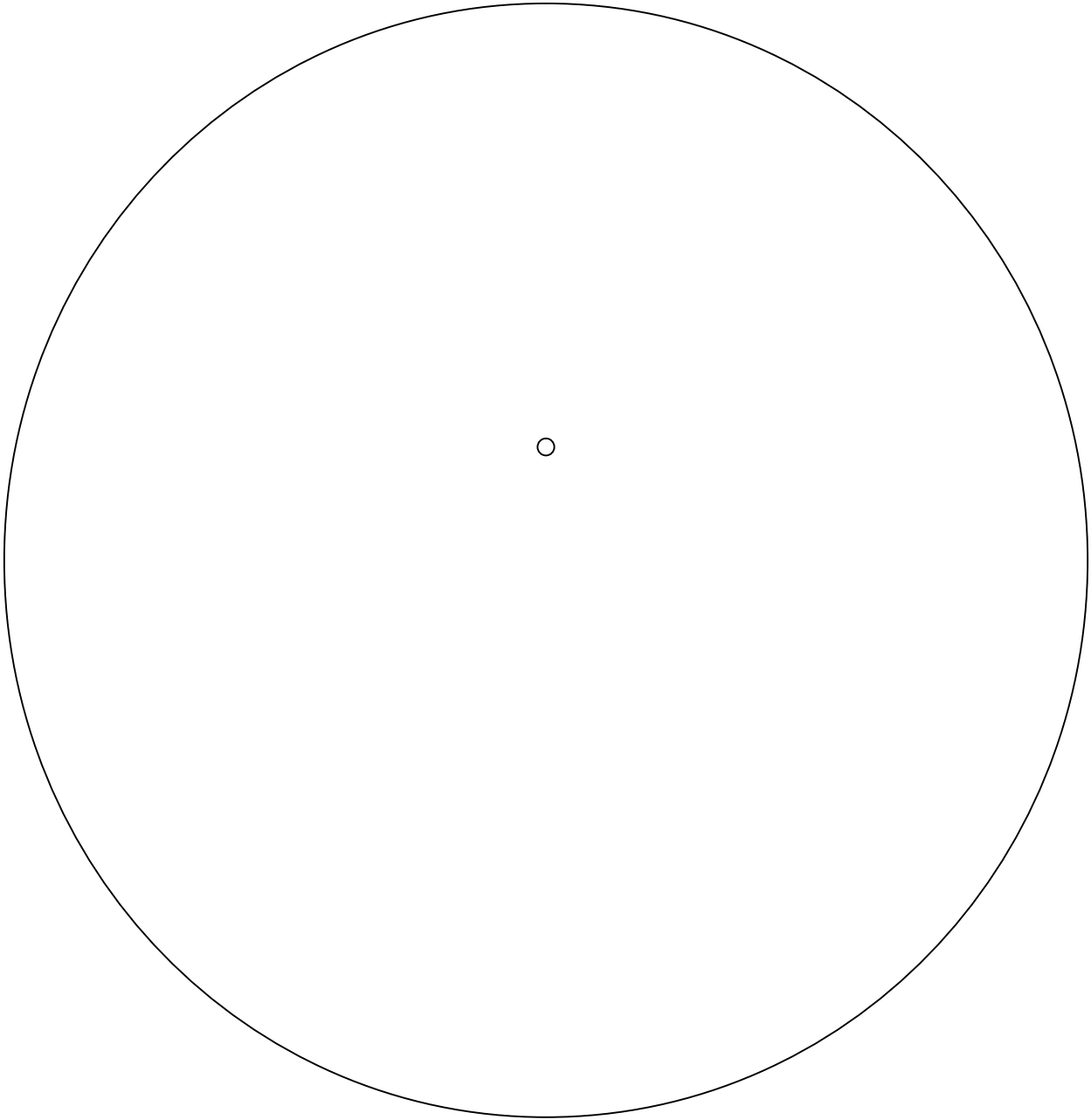
EXERCISE 8:

On the next several pages are scale drawings of an actual planetary ellipses. The small circle in the interior of the orbit is the Sun. (Do you think the size of the Sun is to scale? How could you tell?) Complete the following using the ellipses your TA designates:

(a) Find and draw in the major and minor axes and the positions of the two focal points of the ellipse (you should know where one is right away!).

(b) Find and measure the semi-major axis a, the semi-minor axis b, and the focal length f. Fill in these values below.

(c) Check that, approximately, the formula you derived above relating f, b, and a is correct. Use the 'f (from the formula)' space below for the value from your formula.

(d) Compute the eccentricity of the orbit and the period of the planet using Kepler's Third Law, which you discovered earlier. Place these values in the spaces provided.
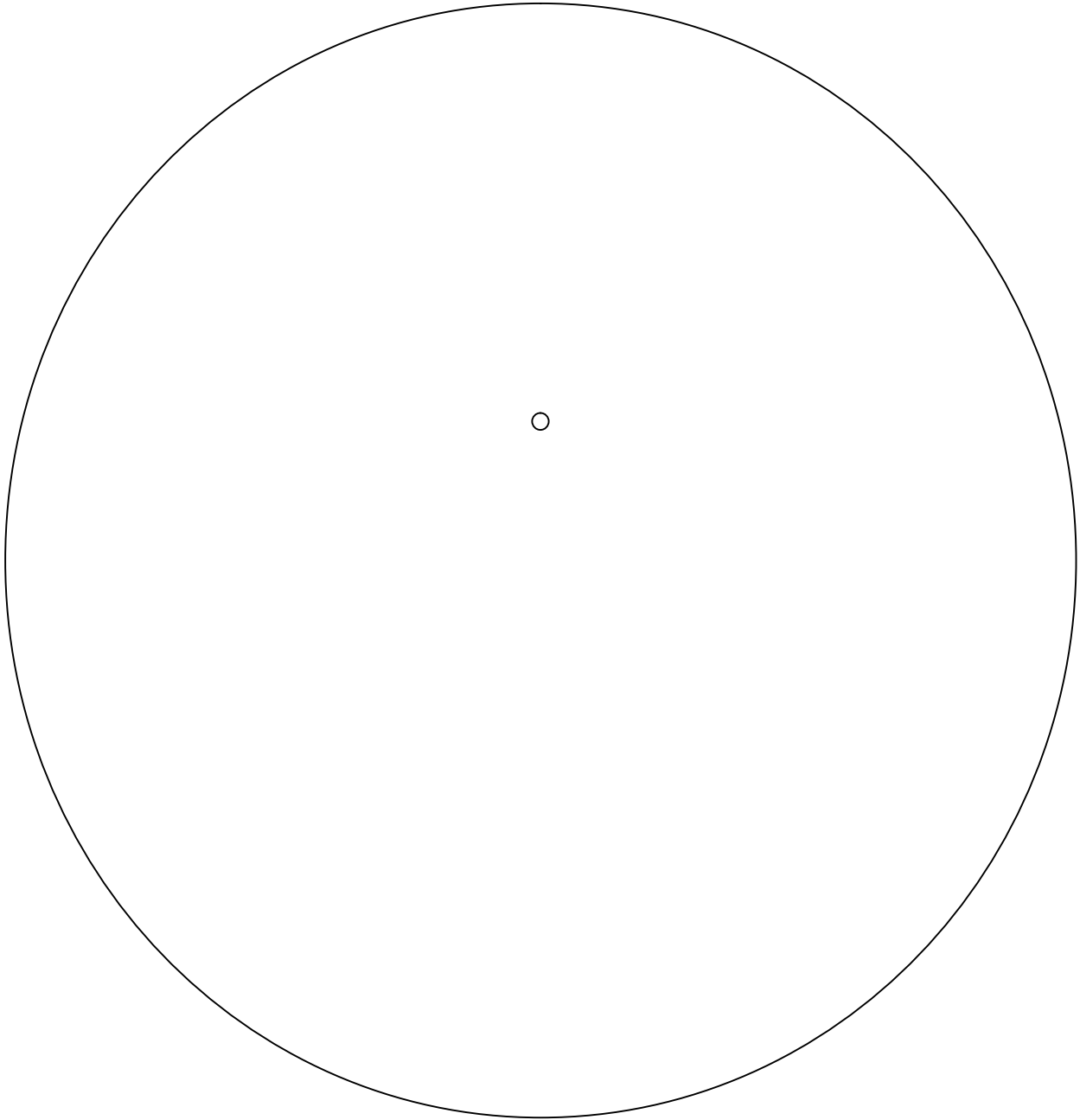
(e) Use the table of eccentricities and periods below to determine which planet's orbit is described by the ellipse.

a = _____   a = _____

b = _____   b = _____

f = _____   f = _____

f (from the formula) = _____   f (from the formula) = _____

e = f/a = _____   e = f/a = _____

period = _____   period = _____

Planet:_____   *Planet*:_____
(use the table below and compare your eccentricity and period)

Planet/asteroid/comet	Period (years)	Eccentricity
Mercury	0.2436	0.206
Venus	0.615	0.007
Earth	1.00	0.017
Mars	1.88	0.093
Jupiter	11.86	0.048
Saturn	29.46	0.056
Uranus	84.01	0.046
Neptune	164.8	0.010
Pluto	248.5	0.248
Icarus (asteroid)	1.12	0.830
Adonis (asteroid)	2.76	0.780
Halley's comet	76.0	0.967

Table 2 Period (in years) and eccentricity of the nine planets.

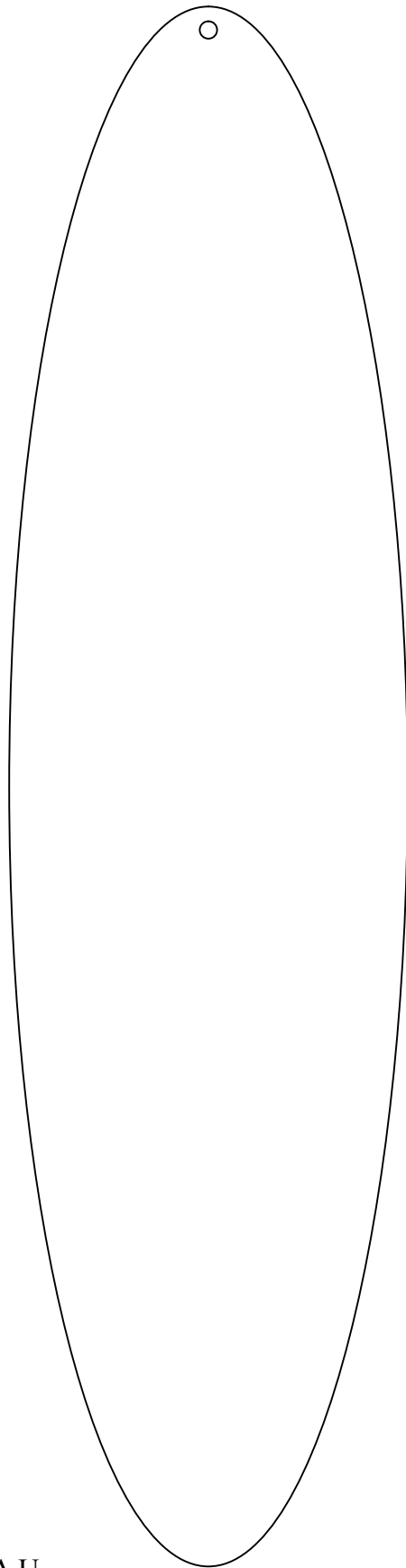Note: 1cm = 0.186 A.U.

Planet A

Scale: 1cm = 0.0467 A.U.

Planet B

Scale: 1cm = 4.64 A.U.

Planet C

Scale: 1cm = 1.58 A.U.

Object D

Inside 3-5