

INSIDE LAB 4: Determining the Masses of the Earth and Jupiter

OBJECTIVE: *To determine the mass of the Earth by measuring the acceleration of gravity on the surface of the Earth and to determine the mass of Jupiter using the orbits of its four Galilean moons.*

MEASURING THE EARTH'S MASS:

Newton's law of gravity states that the acceleration of gravity, g , at a distance d from the center of a spherically symmetric sphere of matter whose mass is M , is given by $g = GM/d^2$. Here G is a universal constant equal to $6.673 \times 10^{-11} \text{ m}^3/\text{kg}/\text{s}^2$. To a good approximation, the Earth is a sphere, so near its surface the distance d is just the radius of the Earth, R_E . We can solve the above equation for the mass M_E of the Earth:

$$M_E = gR_E^2/G$$

Since we know the value of G and we know the radius of the Earth (it was first measured by Eratosthenes), then if we measure g , we can compute the mass of the Earth.

One way to measure g is by measuring the acceleration of a falling object.

MEASURING THE MASS OF THE EARTH:

1. Obtain an iPad, ball and meter stick. After finding a suitable location in front of a blank wall, place the meter stick parallel to the iPad and in the same plane of motion as the falling ball.
2. Open the *Video Physics* app from the home screen. From the *Videos* screen, tap the + icon in the upper right corner of the screen and select *Take a Video*.
3. Have one lab partner stand beside the meter stick and hold the ball up above their head. Be certain that you can see the entire meter stick, the ball and the floor before tapping the red record button.
4. After recording has begun, have your partner release the ball. Be sure to hold the iPad steady and parallel to the falling ball. Do not pan or tilt the iPad during recording.
5. Tap the record button again to stop recording. Once you are satisfied with your video, select *Use*.
6. The video transport controls should be visible at the bottom of the screen. If not, drag up from the bottom to expose the controls. Jog the video to find the frame that contains the beginning of the motion.
7. Select the *Coordinate* axis tool and move the crosshairs to your origin.
8. Next, select the *Scale* tool. Move the crosshairs to one end of the meter stick and tap. (Note that you do not need to place your finger directly over the crosshairs. You may place your finger anywhere on the screen allowing you to easily see the location of

the marker.) Now, move the crosshairs to the other end and tap again to set the scale length. Be sure to enter the length and units into the gray box near the top of the screen.

9. Select the *Points* tool and drag the crosshairs over the object. Tap once anywhere on screen to mark the position of the ball. The video will automatically advance one frame and you will again move center the crosshairs on the object and tap.
10. Repeat this process until the ball reaches the floor. You will notice the ball will begin to blur as it accelerates. Be consistent in choosing the leading section of the ball.
11. If you make a mistake, simply go back one frame, adjust the crosshairs and tap again. To delete a point, move to the frame such that the point is highlighted yellow, tap and hold until you see the *Delete* pop-up.
12. Once you have completed the position marking, tap the *Share* menu and choose *Open Data in...*
13. Choose *Open in Graphical* and the Graphical Analysis app will open with two graphs visible.
14. Tap the *Views* menu icon and select *1 Graph*.
15. Tap the vertical axis label and uncheck *X Velocity*. Next, tap the axis again to slide the graph back to fullscreen.
16. Tap and drag across the data to select the region of interest. Then tap inside this region to bring up the *Selection Options* menu.
17. Scroll through the *Curve Fit* menu and select *Linear*. Next, tap *Add*.
18. The slope of your velocity vs. time graph will give your average acceleration. Record this value in the blank below. Compare this value to the accepted acceleration due to gravity value (9.8 m/s²).

Gravitational acceleration: $g = \underline{\hspace{2cm}}$ m/s²

$$\text{Fractional error} = \frac{g - 9.80 \text{ m/s}^2}{9.80 \text{ m/s}^2} \times 100\% = \underline{\hspace{2cm}} \%$$

19. Calculate the mass of the Earth using $M_E = gR_E^2/G$, where $R_E = 6.371 \times 10^6$ m, and $G = 6.673 \times 10^{-11}$ m³/kg/s². Compare with the actual value M_{actual} from your text or some other source.

$$\text{Earth's Mass } M_E = \frac{gR_E^2}{G} = \underline{\hspace{2cm}} \text{ kg}$$

$$\text{Fractional error} = \frac{M_E - M_{\text{actual}}}{M_{\text{actual}}} \times 100\% = \underline{\hspace{2cm}} \%$$

20. Delete from the iPad the video you have created during this lab.

MEASURING THE MASS OF JUPITER:

Newton's version of Kepler's third law states that

$$(M_1 + M_2)P^2 = \frac{4\pi^2}{G}(a_1 + a_2)^3.$$

This formula applies to any two objects which are in orbit about their common center of mass. Recall that the center of mass is the point at which you would place the fulcrum of a "seesaw" if the two objects were on either end of it and you wanted to balance them. F

We can use this formula to determine the mass of Jupiter from the orbits of its four Galilean moons. We will apply it to each moon separately.

1. First we must simplify the formula. If $M_1 = M_J =$ mass of Jupiter and M_2 is the mass of one of its moons, what do you know to be true about the relationship between these two masses? How can you use this to simplify the above formula? Given your result about the relationship between the two masses, what do you think will be true about the motion of Jupiter versus the motion of its moon? How can you use that to simplify the above formula?
2. Start up the program *Starry Night*. Select **Favourites, C-The Planets, Outer Planets, Galilean Moons**. If the animation starts running, stop it. Then change the time step to hours.
3. Pick one of the moons, say Io. We wish to carefully record exactly how long it takes Io to orbit Jupiter. To determine this distance, note carefully the day and time when Io first passes behind Jupiter (or as close to it as possible). You may need to stop the animation to get this time accurately. Continue the animation until Io passes behind Jupiter again, and again record the day and time. Take the difference to determine the total number of days, hours, and minutes. Record all in the table below.
4. Repeat part 3 for the other three Galilean moons. Convert these periods into seconds.
5. Look up the semi-major axes of these moon's orbits in the table below and convert these distances into m.
6. For each moon, calculate the mass of Jupiter using the formula you found in part 1. Use $G = 6.673 \times 10^{-11} \text{ m}^3/\text{kg}/\text{s}^2$.

Name of Moon	Semimajor Axis (km)	Semimajor Axis (m)	Period (d, h m)	Period (s)	Mass of Jupiter (kg)
Io	421,800				
Europa	671,100				
Ganymede	1,070,400				
Callisto	1,882,700				

7. What is the average mass that you find for Jupiter? Compare this with the value of Jupiter's mass from your textbook or some other source.

Jupiter's Mass $M_J =$ _____ kg

$$\text{Fractional error} = \frac{M_J - M_{\text{actual}}}{M_{\text{actual}}} \times 100\% = \text{_____} \%$$