OUTSIDE LAB 5: Finding the Diameters of Celestial Objects

OBJECT: To measure the angular diameters of various celestial objects and to convert these angular measures into linear diameters.

DISCUSSION:

The most straightforward method of measuring the size of an astronomical object is to measure its angular diameter and its distance. Once these are known, its linear (or actual) diameter can be determined using the formula

$$\frac{D}{2\pi d} = \frac{\alpha}{360} \tag{1}$$

where D is the linear diameter, d is the distance to the object, and α is the angular diameter measured in degrees.

A simple way to measure the angular size of a spherical object is to use the apparent motion of the object through the field of view of the telescope. An object at the celestial equator appears to move a distance of about one degree every four minutes of time. Away from the celestial equator, the apparent angular speed v of a celestial object is given by

$$v = \tfrac{1}{240}(\cos\delta) \text{ degrees/second} \tag{2}$$

where δ is the declination of the object. To see that this formula is reasonable, we note that as the declination of the object increases from 0^o to 90^o, $\cos\delta$ decreases from 1 to 0. For the north celestial pole, δ is 90^o, $\cos\delta = 0$, and so $v = 0$. That is, the north celestial pole is stationary on the celestial sphere.

The angular size α of the object is given by

$$\alpha = vt,$$

where t is the lapse of time during which the object moves across the crosshairs in the eyepiece.

EXERCISES:

1. Record the Telescope number and tool box number in the box provided here.

Telescope # ____

Tool Box # ____

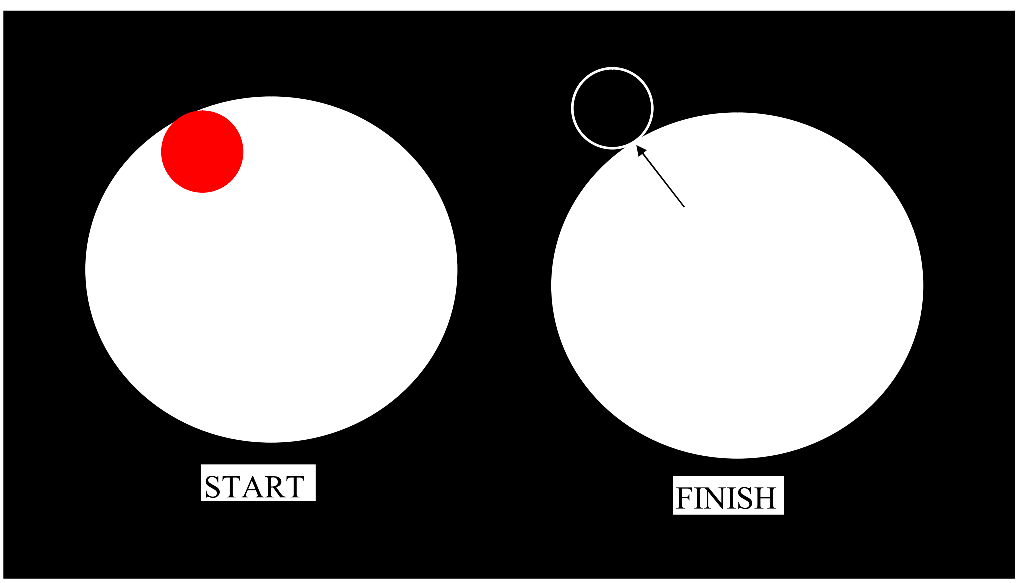
2. Locating the celestial object designated by your instructor, use the slow-motion controls of the telescope so that the object is near the edge of the field of view.

3. Sketch the object(s)

First Object	Second Object

4. Note the direction of motion of the object as it crosses the field of view.

5. Position the telescope so that the object moves directly out of the field of view



6. Start the stopwatch when the object just touches the edge of the field of view. Stop the watch when the object just exits the field of view.

7. Repeat part 6 several times for practice. When you can take the measurements accurately, make a series of measurements. Have your lab partner record them as you read them out.

Object	**Trial:**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
	Time										
	Time										

8. Perform Exercises 2-7 for other objects specified by your instructor.

9. Record the Right Ascension and Declination of each object. If the object is a planet, you will also need to know the right ascension and declination of the Sun.

Object	**Right Ascension (γ)**	**Declination (δ)**
Sun		

10. Calculate the average of the lapses of times during which the object moves out of the field of view. Record the results below.

11. Determine the angular diameter of the object using $\alpha = vt$. We will fill out the remainder of this table in the rest of these exercises.

Object	**Average Time**	**Angular Size**

12. If the object is the Moon, your instructor will provide the distance. Use this to determine diameter.

13. If the object is a planet then you will have to calculate the distance from the Earth to the planet.

(a) Determine the distance to the object. You will have to get the distance using the law of cosines as illustrated in the figures below.

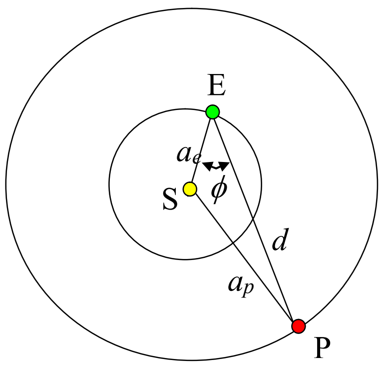


Figure 1. Outer planet

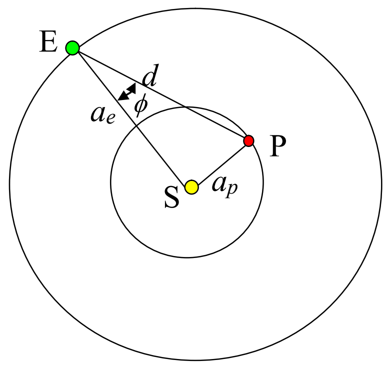


Figure 2a. Inner Planet

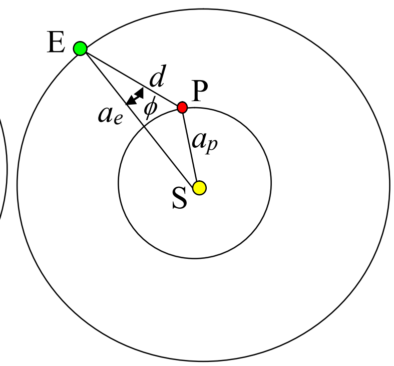


Figure 2b. Inner Planet

The law of cosines relates the distance we want to the Earth-Sun and Planet-Sun distance according to the formula:

$$a_p^2 = a_e^2 + d^2 - 2\, a_e\, d \cos\phi$$

This can be solved for d using the quadratic formula to give

$$d = a_e \cos\phi \pm \sqrt{a_p^2 - a_e^2(1 - \cos^2\phi)}$$

Here a_p is the semimajor axis of the orbit of the planet, a_e is the semimajor axis of the orbit of the earth, and φ is the angle between the planet and the Sun subtended at the earth. While for an outer planet there is only one positive solution for d, for an inner planet there are two positive solutions, one corresponding to Fig. 2a and the other to Fig. 2b. Your instructor will tell you which solution is appropriate. Work out the radical in the worksheet above, and then put the distance in the table on the previous page.

(a) Work out the cosine of the angle between the Sun and the object using the formula

$$\cos\phi = \sin\delta_S \sin\delta_P + \cos\delta_S \cos\delta_P \cos(\gamma_S - \gamma_P)$$

where δ_S and δ_P are the declinations of the Sun and planet, and (γ_S and γ_P are their corresponding right ascensions (in degrees). Hint: It is much easier to get the answer right if you compute certain terms separately, write the numbers down, and then put them together for the final answer.

i. $\sin \delta_S \sin \delta_P =$ ____________

ii. $\cos \delta_S \cos \delta_P =$ ____________

iii. $\cos(\gamma_S - \gamma_P) =$ ____________

iv. $\cos \phi =$ ____________

v. $d =$ ____________

i. $\sin \delta_S \sin \delta_P =$ ____________

ii. $\cos \delta_S \cos \delta_P =$ ____________

iii. $\cos(\gamma_S - \gamma_P) =$ ____________

iv. $\cos \phi =$ ____________

v. $d =$ ____________

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