The Use of PN Diagnostics in Evaluating Initial-Data Sets

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Feb. 10, 2007
Overview

• Why do we need “diagnostics” for initial data?
• What quantities do we test in BH initial data?
• How have PN diagnostics been used so far?
• Eccentricity in the initial data.
• Spin issues.
• What needs to be done?
Why do we need diagnostics for ID?

- Initial data is a combination of prescribed and constrained quantities.
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• Choices are made for parameters that control:
  1) Separation of holes  
  2) Relative sizes of holes
  3) Linear Momenta of holes  
  4) Spin of holes

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• In most cases, physically important properties of the initial data are not directly controlled, and can only be determined after the full initial data has been constructed.
What Quantities Do We Test?

- Total ADM energy, linear & angular momentum \textit{Gauge indep.}
- Apparent horizon area, proper separation \textit{Slicing dep.}
- Orbital angular velocity \textit{Gauge indep./Method dep.}
- BH spin, total mass \textit{Quasilocal}
- Orbital eccentricity(\textit{Komar mass}) \textit{Conservative dyn./Def. dep.}
– Earliest Uses of PN Diagnostics –

$(E_b, \Omega, J)$ for Circular Orbits

- 2PN circular orbits,
  Cook (1994)[7]
  Tichy and Brügmann (2004)[15].

- 2PN ISCO,
  Baumgarte (2000)[3].

- 2PN+spin circular orbits,
  Pfeiffer, Teukolsky, Cook (2000)[14].

- 3PN ISCO,
  Grandclément, Gourgoulhon, and Bonazzola (2002)[10]
  Hannam (2005)[11].

- 3PN circular orbits,
  Damour, Gourgoulhon, and Grandclément (2002)[9]
  Cook and Pfeiffer (2004)[8]
  Yo, J.Cook, Shapiro, and Baumgarte (2004)[16]
  Caudill, Cook, Grigsby, and Pfeiffer (2006)[6].
Eccentricity Diagnostics

- Conservative dynamics:
  - Circular ID have some eccentricity – Mora and Will (2004)[13]; Berti, Iyer, and Will (2006)[4]

\[
e = \frac{\sqrt{\Omega_p} - \sqrt{\Omega_a}}{\sqrt{\Omega_p} + \sqrt{\Omega_a}}
\]

\[
\zeta = \left( \frac{m\Omega_p}{(1 + e)^2} \right)^{2/3} = \left( \frac{m\Omega_a}{(1 - e)^2} \right)^{2/3}
\]

“eccentricity” due to tangential velocity
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“eccentricity” due to tangential velocity

• Full dynamics:
  – Circular ID have \( \dot{r} = 0 \) –
    Miller (2004)[12]

\[
\frac{e}{1 + e^2/2} = \frac{\Delta r_{1,2}}{2\langle r \rangle_{1,2}}
\]

“eccentricity” due to radial velocity
What produces the eccentricity
- Absence of initial radial momentum
- Errors in initial energy or angular momentum
  $\Rightarrow$ error in frequency

How to reduce the eccentricity
- Introduce a small amount of radial velocity in initial data

Numerical Relativity Data Analysis meeting 17

[Caltech/Cornell University]
Eccentricity – ID Parameters

[Brügmann, González, Hannam, Husa, Sperhake & Tichy (2006)]
Eccentricity – ID Questions

- How much of the conservative eccentricity in ID is due to conformal flatness and how much to 3PN?

- Can we use PN parameters to set $\Omega_0$ and $\dot{r}$?
  - How much bias will conformal flatness introduce?
  - How much bias will $\dot{\gamma}_{ij} = 0$/Bowen-York/$\sum$ Kerr introduce?

- In order to provide template waveforms, how accurately do we need to specify the $\dot{r}$ and $\Omega_0$?
Tidal Field Rotation Rate

From the local inertial frame of one black hole, the tidal field of the second black hole is rotating with an angular velocity $\Omega_T[1]$

$$\Omega_T = \Omega_0 \left[ 1 - \eta \left( \frac{m}{b} \right) + O \left( \frac{m}{b} \right)^{3/2} \right]$$

To 1PN order

$$\frac{m}{b} = (m\Omega_0)^{2/3} \left[ 1 + (1 - \frac{1}{3}\eta) (m\Omega_0)^{2/3} + O (m\Omega_0)^{4/3} \right]$$

$$\frac{\Omega_T}{\Omega_0} = 1 - \eta (m\Omega_0)^{2/3} + \Lambda (m\Omega_0) - \left[ \eta \left( 1 - \frac{1}{3}\eta \right) - \Gamma \right] (m\Omega_0)^{4/3} + \ldots$$
Ratio of Spin and Orbital Angular Velocities; $\omega_{\text{BH}}/\Omega_0$ vs $m\Omega_0$
Eccentricity of Corotating data –Spin Corrected–

\[ \omega_{\text{BH}} = \Omega_0 \left(1 - \frac{1}{4}(m\Omega_0)^{2/3}\right) \]

[Grigsby & Cook]
Measuring the Spin of a Black Hole

- Spin is only rigorously defined at spatial/null infinity.

- Must use quasi-local definition: e.g. Brown & York\cite{brown_york} or Ashtekar & Krishnan\cite{ashtekar_krishnan}

\[
S = -\frac{1}{8\pi} \oint_{\text{BH}} K_{ij} \xi^i s^j \sqrt{h} d^2x
\]

\[\xi^i = \begin{cases} 
\xi^i_{\text{CK}} : \text{Killing vector of } \tilde{h}_{ij} \Rightarrow \text{conformal Killing vector of } h_{ij} \\
\xi^i_{\text{KT}} : \text{Approximate Killing vector of } h_{ij}
\end{cases}\]
Measuring the Spin of a Black Hole

• Spin is only rigorously defined at spatial/null infinity.

• Must use *quasi-local* definition: e.g. Brown & York[5] or Ashtekar & Krishnan[2]

\[
S = - \frac{1}{8\pi} \oint_{BH} K_{ij} \xi^i s^j \sqrt{h}d^2x
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\[\xi^i = \begin{cases} 
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“Best” approximate Killing vector(s) of \( h_{ij} \).

[Cook & Whiting 2007]
New Approximate Killing Vectors

Advantages over Killing Transport method:

- Minimizes, in a least-squares sense, deviation of approximate Killing vector from being a true Killing vector.

- Guarantees $\xi^i$ is divergence free.

- Guarantees $L = \frac{1}{2}\epsilon_{ij}D^i\xi^j$.

- Does not depend on “choice of initial path”.

\[
\begin{align*}
\xi^i & \equiv \epsilon^{ij}D_j\nu, \\
D^iD_i\nu + 2L & = 0, \\
D^iD_iL - (1 - \Theta) \left[\frac{1}{2}(D^iD\nu)D_i\nu - 2RL\right] & = 0.
\end{align*}
\]
New Approximate Killing Vectors

\[ \langle S_{ij} S_{ij} \rangle \]

- **CKV**
- **KT**
- **AKV**
- **Θ**

[Cook & Whiting]
New Approximate Killing Vectors

[Graph showing two curves: $\phi_1$ and $\phi_2 - \pi/2$. The graph plots $\Delta \phi$ (radians) against $m\Omega_0$. The inset graph shows degrees against $m\Omega_0$.]

[Cook & Whiting]
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• Understand effect of uncertainty in physical parameters of ID on “confidence” in NR waveforms.
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— Know what you are evolving —

• We should be evolving truly eccentric data!

• Understand effect of uncertainty in physical parameters of ID on “confidence” in NR waveforms.

• Improve our choices for $\tilde{\gamma}_{ij}$ (& $\dot{\tilde{\gamma}}_{ij}$).

• Deal with $\dot{r}$ — Quasilocal linear momentum?

• Make sure we are doing unequal masses “correctly”.

– Greg Cook – (WFU Physics)
References


