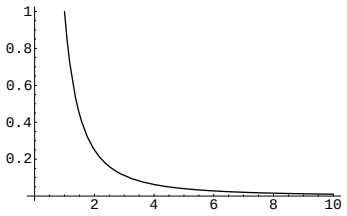
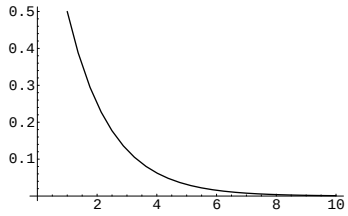
Power law distributions Many man-made and naturally occurring phenomena, including city sizes, incomes, word frequencies, and earthquake magnitudes, are distributed according to a power-law distribution. A power-law implies that small occurrences are extremely common, whereas large instances are extremely rare. The Boltzmann-Gibbs distribution also has this characteristic, that large energy values are very rare. So how do the two distributions differ?



Power Law Distribution x^{-2}



Negative Exponential Distribution 2^{-x}

Look at the two graphs above. Negative Exponential distributions, e.g. the Boltzmann-Gibbs, fall off more sharply compared to a power law distribution.

A power law distribution has a longer 'tail' to the right. If personal wealth is governed by a power law it means that there will be more very rich people than there would be if wealth was governed by a negative exponential distribution.

A power law has the general form of probability density function $P[X = x] = Cx^{-a}$, which is a formula that gives the probability that a person's income is exactly £20,000 (e.g., $P[X = 20000]$). The Pareto distribution is the cumulative distribution function corresponding to a power law. It is generally written as $P[X > x] = x^{-k}$, for example the probability that a person's income is greater than £20,000. It is related to the power law distribution by the formula $a = k + 1$.