

A NOTE ON RATIONING, PRICE CONTROL, AND
CONSUMER WELFARE

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1. *Introduction*

In intermediate microeconomics courses, I present a simple argument comparing two regimes: one in which the quantity purchased of a good X is subject to an upper limit per period of time (a rationing constraint), while its price is also subject to control; and the complementary case in which both quantity and price are uncontrolled. In Figure 1, the good subject to rationing and price control is X ; Y is "all other goods." The ration quantity is \bar{X} . The Figure shows the situation of two consumers, Poor (P) and Rich (R), with "R" having approximately twice the spending power as "P." Budget lines are drawn for each consumer in each of the two regimes, with the uncontrolled price of X shown as twice the controlled price. The budget lines are subscripted for the two consumers R and P in the controlled (C) and uncontrolled (U) situations, in an obvious notation.

As drawn, the indifference curves reveal that quantity-and-price control turn out to be beneficial to P, whose loss from the quantity constraint is more than replaced by the advantage of purchasing X at the controlled price. Compare the indifference curves attained at the two equilibria for P, E_{PC} and E_{PU} . By contrast, the

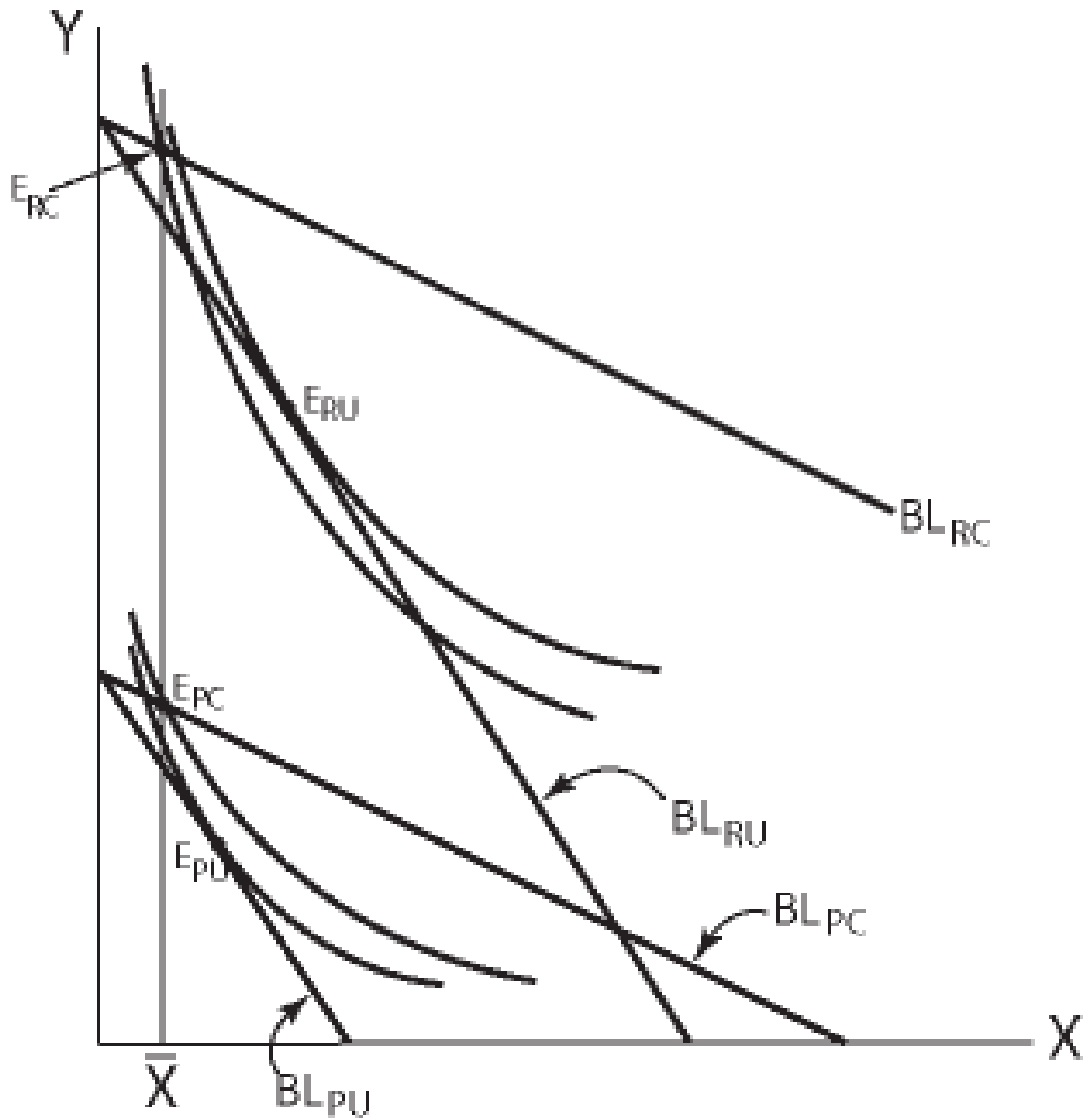


Figure 1: Comparison of Welfare Effects of Rationing and Price Control on Two Individuals, Rich and Poor.

controlled regime lowers the welfare of R ($E_{RC} < E_{RU}$). For R, \bar{X} is an onerous burden to bear, whose removal would more than offset the loss from the higher “free market” price in the uncontrolled regime. The minor lesson here is that rationing and price control *may* help the poor and discomfort the rich; at least, a consistent set of indifference curves can be drawn for this case. The major lesson SS this is in fact my real interest in presenting this exercise SS is that the tools of microeconomic theory are in themselves neutral with respect to analysis and policy conclusions; they are not, in particular, necessarily on the side opposing political intervention into the spontaneous workings of the market. The decision to control the market for a good that is, for whatever reason, momentarily scarce must be based on a complex reading of the short- and long-range consequences of so doing, including the all-important issue of public support for the measure (which plays a major part in determining the deadweight costs of administering the controlled regime, the possibility of illegal markets, and so on).

Figure 1, however, recalls the famous aphorism that “paper will tolerate anything that is written on it.” Can the range of possible outcomes of price and quantity control be studied in a more rigorous manner?

2. Modeling the Welfare Consequences of Market Control

At Different Income Levels

I borrow only the most commonly used and long-established tools from the

micro toolbox. Begin with a CobbSDouglas utility function

$$U = X^\alpha Y^\beta = X^{1-\beta} Y^\beta \quad (1)$$

The restriction $\alpha + \beta = 1$, with $\alpha, \beta \in (0,1)$, is a standard simplifying normalization, which can be shown to have no effect on outcomes, including in the present study. Utility is here treated as a methodological construct with no cardinality, and any function, such as (1), that is monotonic increasing in any other such function can be adopted without loss of generality (see Henderson and Quandt, 1980, Ch. 2, p. 16). The restriction to (apparent) linear homogeneity enables us to express the elasticities of utility with respect to X and Y using only one parameter, β .

Including the budget constraint, $M = p_x X + p_y Y$, where p_x and p_y are the prices of X and Y , respectively, and M is “money income” (the amount a consumer is able to allot to consumer goods purchases per period of time), we find the well-known equilibrium (utility-maximizing) levels of consumption, and the associated level of utility. For ease of later exposition, these are written without qualifying subscripts or diacritical marks:

$$X = \frac{(1-\beta)M}{p_x} \quad Y = \frac{\beta M}{p_y} \quad (2)$$

$$U = \left(\frac{1-\beta}{p_x} \right)^{1-\beta} \left(\frac{\beta}{p_y} \right)^{\beta} M. \quad (3)$$

This is the utility level achieved by a consumer in the uncontrolled regime. The corresponding expression for the controlled regime is easily found. Assume, for the moment, that the consumer avails her/himself of the full ration allowance, \bar{X} . The consumption bundle will then be \bar{X} and the quantity of Y that remains available is $(M - \bar{p}_x \bar{X}) / p_y$. (Note the notation \bar{p}_x for the *controlled* price of X associated with the rationing constraint.) Controlled utility is then:

$$\bar{U} = \bar{X}^{1-\beta} \left(\frac{M - \bar{p}_x \bar{X}}{p_y} \right)^{\beta} \quad (4)$$

Comparing (3) and (4) is the central task in evaluating the two regimes, so long as the sole basis for this evaluation is individual consumer welfare.

The properties of this comparison can best be revealed by using a diagram in which we plot utility levels against levels of money income (Figure 2).

(4) is represented by the curve labeled $\bar{U}(\bar{p}_x)$; it has positive first and nega-

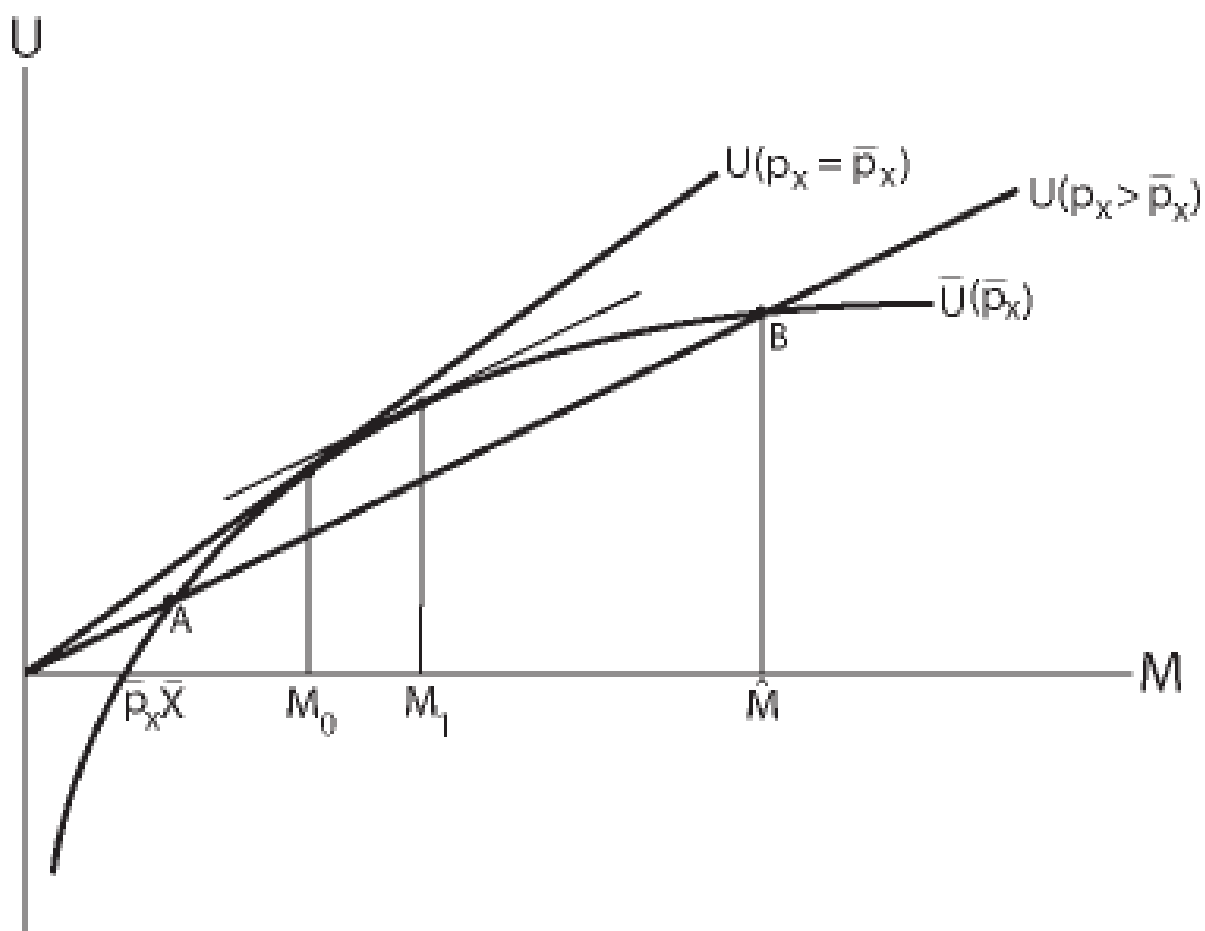


Figure 2: Levels of Utility in the Controlled and Uncontrolled Regimes, at Different Income Levels

tive second derivatives, and rises from an M -intercept of $\bar{p}_x \bar{X}$, the value of the rationed quantity of X . (Clearly, at this level of income attainable Y would be zero, and utility therefore also zero.) At income levels lower than $\bar{p}_x \bar{X}$, (4) does not produce real numbers for U ; thankfully, we do not need to venture there.

The utility curve for the uncontrolled regime, (3), plots as a linear ray from the origin. We may first find the ray that is tangent to \bar{U} , by setting

$$U = \bar{U}, \frac{\partial U}{\partial M} = \frac{\partial \bar{U}}{\partial M}, \text{ and solving for } M, \text{ from which we find}$$

$$M_0 = \left(\frac{1}{1-\beta} \right) \bar{p}_x \bar{X}. \quad (5)$$

Putting M_0 in turn into both U and \bar{U} and simplifying, we find that this tangency point occurs when $p_x = \bar{p}_x$. The U curve that is tangent to \bar{U} at M_0 , then, represents the limiting case in which the uncontrolled market price is the same as the controlled price: either price control is not applied, or (for some reason) the relative scarcity of X that made rationing a distinct possibility does not force the price up-

ward from \bar{p}_x in the absence of government intervention. This limiting case, in fact, provides us with the paradigmatic anti-intervention conclusion. With $p_x = \bar{p}_x$, U lies above \bar{U} at every level of income except M_0 . Rationing, therefore, has no effect on consumer welfare at that income level, and is harmful to consumer welfare at every other level.

Looking at (3), we can see that a rise in p_x will rotate the U curve downward, into the position represented by $U(p_x > \bar{p}_x)$ in Figure 2. We may take this to be the normal case. The U and \bar{U} curves intersect twice, at points A and B. A occurs at a low level of M (not labeled in the Figure) and B at \hat{M} . These two values appear to define a range of income levels at which the controlled regime results in a higher level of utility than the uncontrolled regime, with the opposite holding at the extreme ranges of income, above B and below A. This lower level, however, is deceptive. The \bar{U} curve traces utility levels on the assumption that the consumer in fact purchases the entire allowable ration, \bar{X} . The ration, however, is an inequality constraint. Putting \bar{X} and \bar{p}_x into (2) and solving for M , we find $M = M_0$, as determined by (5). This, then, is the income level at which the consumer would purchase the ration quantity, \bar{X} , in the absence of the ration constraint. At any income level

below M_0 , the consumer would select a quantity of X less than \bar{X} , having chosen freely, at the controlled price, along the ray $U(p_x = \bar{p}_x)$. The complete potential utility frontier, then, follows this ray out to the tangency point, and \bar{U} thereafter, and A is dominated by this frontier. Our attention may therefore turn to B, which defines \hat{M} . This is the critical income level, below which the consumer is helped, and above which s/he is harmed, by quantity and price controls.

Before examining \hat{M} , it will be useful to derive one more property of the model, as shown in Figure 2. Given $p_x > \bar{p}_x$, we find M_1 , at which the slopes of U

and \bar{U} are equal. Setting $\frac{\partial U}{\partial M} = \frac{\partial \bar{U}}{\partial M}$ and solving, we find

$$M_1 = \left(1 + \frac{\beta}{1 - \beta} \frac{p_x}{\bar{p}_x}\right) \bar{p}_x \bar{X} = \left(1 + \frac{a\beta}{1 - \beta}\right) \bar{p}_x \bar{X} \quad (6)$$

In the second term of this equality, we define $a \equiv \frac{p_x}{\bar{p}_x}$, the ratio of the uncontrolled

to the controlled price; this, and β , are the two parameters that will shape our con-

clusions regarding \hat{M} , which, like M_0 and M_1 , will emerge as a multiple of $\bar{p}_x \bar{X}$, the controlled-price value of the ration quantity. Note, finally, from (6), that when $a = 1$ (the limiting case), M_1 reduces to M_0 .

We come now to the heart of the matter: determination of \hat{M} . Equating (3) and (4), and simplifying, we obtain

$$\psi M^{1/\beta} - M + \bar{p}_x \bar{X} = 0 \quad (7)$$

where $\psi = \beta \left(\frac{1-\beta}{a \bar{p}_x \bar{X}} \right)^{\frac{1-\beta}{\beta}}$. Note that ψ contains the uncontrolled price, p_x , in the

form of the ratio a . The expression (7) is unfortunately, and obviously, non-algebraic in M ; we will have to approach its properties, first, by means of two special cases; second, using a numerical method, which will be developed later in this section.

Case: $\bar{p}_x = 0$. The government imposes a ration quantity, \bar{X} , but also requisitions or purchases the good from producers and distributes it to the population without charge. (7) now has a readily attainable solution:

$$\hat{M}(\bar{p}_x = 0) = \left(\frac{1}{\psi} \right)^{\frac{\beta}{1-\beta}} = \left[\frac{1}{\beta^{\frac{\beta}{1-\beta}} (1-\beta)} \right] p_x \bar{X} \quad (8)$$

In this case, of course, the multiplier must be applied to $p_x \bar{X}$, the value of the ration at the *uncontrolled* price. This multiplier contains only one parameter, β (the other parameter, a , is clearly not applicable). I postpone full discussion of possible parameter values until section 3, but for $\beta = 0.9$, the expression in square brackets ≈ 25.8 . To give a pre-indication of the sort of calculation one can do with this, choose a monetary value for $p_x \bar{X}$ of \$200 per month (a month's worth of the rationed good would cost \$200, at market prices). The monthly \hat{M} is then 200×25.8 , or \$5160/month, which translates, at a tax rate of 0.3, into an annual pre-tax income of about \$88,500. On this account, people whose $\beta = 0.9$ and who earn less than \$88,500 per year before taxes would benefit from rationing, while those with incomes above that amount would be worse off.

Case: $\beta = 1/2$. For this special value, (7) becomes quadratic. $\psi = \frac{1}{4a\bar{p}_x\bar{X}}$.

Using the + root (since we are looking for the higher of the two roots), we obtain, after simplification:

$$\hat{M}(\beta = 1/2) = 2a \left[1 + \sqrt{\frac{a-1}{a}} \right] \bar{p}_x \bar{X} \quad (9)$$

Again, postponing full discussion of parameter values and implications, for $a = 2$ the multiplier in (9) is ≈ 6.828 . For $\bar{p}_x \bar{X} = 100$, this gives an \hat{M} of \$682.80/month, or \$11,700/year, suggesting a very low cutoff income level, and a significant majority of the population that would be worse off in the controlled regime than in the uncontrolled regime.

Since we need more general results, for $\bar{p}_x > 0$ and for a variety of values for β and a , we return to (7). To explain the numerical estimation strategy, I first analyze the LHS of this expression as

$$\begin{aligned} F(M) &= \psi M^{1/\beta} - M + \bar{p}_x \bar{X} \\ F'(M) &= \frac{\psi}{\beta} M^{\frac{1-\beta}{\beta}} - 1 = 0 \quad \Rightarrow \quad M_2 = a \left(\frac{1}{1-\beta} \right) \bar{p}_x \bar{X} = a M_0 \quad (10) \\ F''(M) &= \frac{\psi}{\beta} \left(\frac{1-\beta}{\beta} \right) M^{\frac{1-2\beta}{\beta}} > 0 \end{aligned}$$

$F(M)$ has a unique minimum at $M_2 (> M_1)$. We can also evaluate $F(\cdot)$ at M_2 ; after simplifying, we find $F(M_2) = (1 - a) \bar{p}_x \bar{X}$, which is < 0 for $a > 1$. Similar processing reveals

$$F(M_0) = \left(\frac{\beta}{1 - \beta} \right) \left(\frac{1}{a^{\frac{1-\beta}{\beta}}} - 1 \right) \bar{p}_x \bar{X},$$

and

$$F(M_1) = \left(\frac{a\beta}{1 - \beta} \right) \left[\left(\frac{1 - \beta + a\beta}{a} \right)^{1/\beta} - 1 \right] \bar{p}_x \bar{X},$$

both of which can be shown to be < 0 , for $a > 0$ and $\beta \in (0,1)$. The LHS of (7) is therefore negative for all relevant values of M less than \hat{M} , and this provides a simple iteration procedure to find \hat{M} for a variety of values of a and β . Starting at M_2 , with $\bar{p}_x \bar{X} = 100$, M is incremented by 0.01 until $F(M) > 0$. The resulting M is our estimate of \hat{M} for every set of values for

(a, β) .¹

Before presenting the results, I return briefly to the question of the generality of (7) with respect to specification of the utility function, a point that is perhaps not completely obvious. The general CobbSDouglas function is

$$U = X^m Y^n, \text{ where } m + n = \gamma \neq 1. \quad (11)$$

In this case, the two maximized utility functions, corresponding to (3) and (4), become

$$U = \left(\frac{m}{\gamma p_x} \right)^m \left(\frac{n}{\gamma p_y} \right)^n M^\gamma \text{ and } \bar{U} = \bar{X}^m \left(\frac{M - \bar{p}_x \bar{X}}{p_y} \right)^n \quad (12)$$

The normalization to (1) uses $\alpha \equiv \frac{m}{\gamma}$ and $\beta \equiv \frac{n}{\gamma}$, with, of course, $\alpha = 1 - \beta$. Equat-

1 The calculations were carried out on the City University of New York mainframe system, using the now-archaic programming language PL-1. I will be glad to send a copy of the source text upon request.

ing the two utility expressions in (12) and simplifying, and notably taking the $1/\gamma$ -th power of the resulting expression, yields

$$\beta \left(\frac{1-\beta}{p_x \bar{X}} \right)^{\frac{1-\beta}{\beta}} M^{1/\beta} = M - \bar{p}_x \bar{X},$$

which is, of course, identical to (7).

3. Results and Interpretations

We proceed immediately to some calculated values for \hat{M} , presented in Table

1. These are, as always, based on an assumed level of 100 for $\bar{p}_x \bar{X}$, interpreted as the monthly cost of the rationed quantity, \bar{X} , at the controlled price \bar{p}_x . The figures in the table are therefore monthly (after-tax) income levels, below which consumers benefit from rationing and price control, and above which they are made worse off.

The price differential, a , has a fairly straightforward interpretation, although there is undoubtedly no simple way to determine the extent to which p_x would rise in the presence of significant scarcity, if it were left free of political interference. The positive relation between a and \hat{M} seems consistent with intuition: the greater the price differential, the higher one's income would have to be for the uncontrolled

		β				
		0.5	0.6	0.7	0.8	0.9
	2	683	902	1264	1985	4143
	3	1090	1450	2045	3229	6773
	4	1493	1992	2817	4459	9375
	5	1894	2532	3586	5685	11967
a	6	2295	3071	4354	6909	14555
	7	2696	3610	5122	8132	17141
	8	3097	4149	5889	9355	19726
	9	3497	4687	6656	10577	22310
	10	3897	5225	7423	11798	24893

Table 1: Calculated Values of \hat{M} , for Selected Values
of a and β

regime to be preferred to rationing and price control. The effect is still not as powerful as one might expect. At the lowest shown β of 1/2, for example, a differential of 10 yields an \hat{M} of \$3897, suggesting an annual pre-tax income of \$66,805, and a significant segment of the income distribution that would prefer the uncontrolled regime.

Increasing β alters the picture, however. β , it will be recalled, is the elasticity of utility with respect to Y , "all other goods," and as it rises, the elasticity of utility with respect to X falls. It is tempting, but would be misleading, to identify the elasticities with the shares of income devoted to X and Y . Gasoline, for example SS to take a good for which rationing has been applied in the past, and might be a distinct possibility in the future SS may form a small share of total expenditure, but its elasticity coefficient may be much higher, to the extent that automobile use has become a structural necessity for both work and recreation. This is why I have chosen a range for β that goes as low as 0.5. Now while the intuition is much less clear in the case of β than in the case of a , it appears that as the rationed good becomes *less* (marginally) important to total utility, the number and proportion of people for whom rationing is beneficial *rises*. The effect is quite pronounced. At $a = 2$, a β of 0.9 implies a cutoff income of \$4143 per month, or an annual pre-tax income of \$71,023 (again using a tax rate of 0.3). At the other extreme of $a = 10$, the corre-

sponding figures are \$24,893 and \$426,737. Here, rationing indeed helps all but the extremely wealthy. I have also run the model for values of β up to 0.99, and can report that at these high levels, the values of \hat{M} soar exponentially.

These results could undoubtedly be subjected to a much more precise analysis, using Gini coefficients for the income distribution, and sample data or historical evidence to estimate a and β . That will not be attempted in this brief note. I must, however, report my sense of the results in relation to my own preconceptions. I admit to having begun this investigation expecting to find that, for a wide and reasonable range of values of the parameters, it would turn out that a large majority of the population falls below \hat{M} , with a distinct minority above it. This in turn would provide a basis for widespread political support for the controlled regime. The general shape of the numbers, however, suggests a conclusion that the cutoff income level is more likely to divide the population into significant large groups, and that, in these narrow terms, political support for rationing and price control in situations where these measures might be contemplated is more problematic than my preconception implied. I will close by suggesting that both popular support and scientific justification for quantity and price control in a specific market must rely on a much broader set of criteria than individual hedonistic calculation alone. Even within the narrow frame of individual utility maximization, however, rigorous modeling points to a

range of outcomes, and certainly no automatic presumption that un-(politically)-constrained equilibria must of necessity be deemed to be superior to politically inspired intervention.

REFERENCES

Henderson, James M., and Richard E. Quandt. 1980. Microeconomic Theory: A Mathematical Approach. Second Edition. New York: McGraw-Hill.