Heteroskedasticity – A Quick Guide

1 Definition

Multiple regression model: $y_i = X_i\beta + u_i$

Homoskedasticity (constant variance of error term): $\mathrm{Var}(u_i) = \sigma^2, \forall i$

Heteroskedasticity (non-constant variance of error): $\mathrm{Var}(u_i) = \sigma_i^2$

2 Consequences

Characteristics of OLS under heteroskedasticity:

- Parameter estimates ($\hat{\beta}$) are still unbiased, but are not most efficient.
- Regular standard errors are invalid, and hence hypothesis tests are unreliable.

3 Diagnosis

The most common test for heteroskedasticity is White's test. The null hypothesis is that the error term is homoskedastic. The test proceeds as follows:

1. Estimate the model of interest using OLS and save the squared residuals.

2. Run an *auxiliary regression*: the dependent variable is the squared residual series from above; on the right-hand side are the independent variables from the original model plus their squares and (possibly) cross-products.

3. The test statistic is nR^2 from the auxiliary regression. Under the null hypothesis of ho-moskedasticity this is distributed as χ^2, with degrees of freedom equal to the number of regressors in the auxiliary regression.

4. As usual, reject the null if the p-value of the test statistic is smaller than the chosen α or significance level.

4 Correction

- Sometimes the problem can be eliminated by *respecifying* the original model, e.g. adopting a formulation in logs rather than levels, or perhaps adding a quadratic term for one or more of the independent variables. It's also possible that removal of outliers solves the problem.

- Failing that, valid standard errors can be obtained via a heteroskedasticity-consistent ("ro-bust") procedure. In gretl this is just a matter of appending the `--robust` option to the `ols` command. Note that this does not change the $\hat{\beta}$ estimates at all and so does not "re-move" the heteroskedasticity; it's just a matter of calculating different standard errors that "work" in the presence of heteroskedasticity.

- The same principle that stands behind White's test can be used to perform a suitable weighted least squares regression (gretl command `hsk`).