Economics 215, 2015 Allin Cottrell

OLS cheat sheet

Here are some basics that you should know about Ordinary Least Squares. Note that several of the points that are simply asserted here are proved and/or explained more fully in the notes titled "Regression Basics in Matrix Terms". Key assumptions are marked as, for example, "[**A1**]".

1. The linear multiple regression model can be written compactly in vector–matrix form as

$$y = X\beta + u \tag{1}$$

where the dependent variable y is $n \times 1$; the regressor matrix X is $n \times k$; the parameter vector β is $k \times 1$; and the error term u is $n \times 1$.

2. The OLS estimator of β, which we write as $\hat{\beta}$, is given by

$$\hat{\beta} = (X'X)^{-1}X'y \tag{2}$$

This exists provided that $X'X$ is non-singular, which requires that the X matrix is of full column rank (no exact collinearity among the columns of X, [**A1**]).

Assuming $\hat{\beta}$ exists, two useful additional vectors may be formed: fitted values, $\hat{y} = X\hat{\beta}$, and residuals, $\hat{u} = y - \hat{y} = y - X\hat{\beta}$.

3. If the data-generating process conforms to (1) [**A2**] then the *expectation* of $\hat{\beta}$ is given by

$$E(\hat{\beta}) = \beta + E\left[(X'X)^{-1}X'u\right] \tag{3}$$

On condition [**A3**] that $E(u|X) = 0$ the second term above disappears and we have $E(\hat{\beta}) = \beta$, or in other words the OLS estimator is unbiased.

4. The *variance* of $\hat{\beta}$ (a $k \times k$ matrix) is, from first principles,

$$\text{Var}(\hat{\beta}) = E\left[\left(\hat{\beta} - E(\hat{\beta})\right)\left(\hat{\beta} - E(\hat{\beta})\right)'\right]$$

If the condition for the estimator to be unbiased is met, then

$$\text{Var}(\hat{\beta}) = (X'X)^{-1}X'E(uu')X(X'X)^{-1} \tag{4}$$

If the error term has a constant variance, σ_u^2 [**A4**], and the drawings from the error distribution are independent, such that $E(u_i u_j) = 0$ for all $i \neq j$ [**A5**], then $E(uu') = \sigma_u^2 I_n$ and the OLS variance simplifies to the "classical" formula,

$$\text{Var}(\hat{\beta}) = \sigma_u^2 (X'X)^{-1} \tag{5}$$

which can be estimated by using

$$s_u^2 = \frac{\sum_{i=1}^{n} \hat{u}_i^2}{n-k}$$

in place of the unknown σ_u^2.

5. Note the role of the various assumptions: [**A1**] is required for $\hat{\beta}$ to exist; in addition, [**A2**] and [**A3**] are required for OLS to be unbiased; and in addition [**A4**] and [**A5**] are needed for "classical" standard errors to be valid. (The standard errors routinely reported alongside OLS estimates are just the square roots of the diagonal elements of $\text{Var}(\hat{\beta})$).