A Sraffian Critique of the Classical Notion of Center of Gravitation

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Abstract

In this paper we use insights from Sraffa's classic, PCMC, to argue that the classical notion of 'centre of gravitation' is not a sound concept. The market mechanics of labour allocation through price signals and quantity adjustments, given effectual demands, do not lead to a 'centre of gravitation'. We work out all such possible market mechanisms, including the specific classical case, and show that the 'centre of gravitation' is a non-attractive point in all the cases.

1 Introduction

The classical economists (e.g. Adam Smith, David Ricardo and Karl Marx)[1] argued that in a capitalist economy there exists a market mechanism that ensures that the division of a given social labour for any given system of production, i.e. the given techniques and total labour in use, is brought in conformity with given 'social needs' represented by what Adam Smith called the "effectual demands". The general argument works in two stages as follows: (1) For any given system of production at any given point of time there exists a set of combination of net outputs of commodities that the system could produce given the total labour and the techniques in use. From among this set, there is one combination of the net output that corresponds to the 'social need' or the effectual demands of the society.[2] If the combination of the net output produced at any given time is not equivalent to the combination specified by the effectual demands, then the relative prices of the commodities which has positive excess effectual demands would rise[3] and of the commodities which

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[1] See Adam Smith (1981, ch. VII, pp. 72-81), David Ricardo (1951, ch. IV, pp. 88-92) and Karl Marx (1981, ch. 10, pp. 273-301).

[2] "...the demand of those who are willing to pay the natural price of the commodity may be called the effectual demanders, and their demand the effectual demand; ... A very poor man may be said in some sense to have a demand for a coach and six; he might like to have it; but his demand is not an effectual demand, as the commodity can never be brought to market in order to satisfy it." (Smith 1981, p. 73).

[3] "When the quantity of any commodity which is brought to market falls short of the effectual demand.... A competition will immediately begin among them [the effectual demanders], and the market price will rise more or less above the natural price, ..." (Smith 1981, pp. 73-74).

has positive excess effectual supplies would fall. It was contended by the classical economists that such movements in prices would result in rise in the rates of profits of the commodities with excess effectual demands and fall in the rates of profits of the commodities with excess effectual supplies. Such movements in the rates of profits in turn would direct reallocation of capital and labour by moving capital and labour from sectors that have rates of profits lower than the "natural" rate (the rate of profits that would prevail when all rates of profits are equal) to the sectors that have rates higher than the natural rate of profits. Such rescaling of the sectors would reduce the amount of excess effectual demands and supplies of the system. It should be kept in mind that though classical economists did not have developed notion of returns to scale, they nevertheless implicitly assumed constant returns (CRS) during the adjustment mechanism as the input-output structure of the techniques in use in their examples do not change during the rescaling of the sectors. From here on the argument works as follows: (2) As the supplies in the sectors with higher than the natural rate of profits rises, it leads to fall in their relative prices which in turn leads to fall in their rates of profits and the converse happens for the sectors that contract. This tendency is supposed to prevail till the system's combination of the net output matches with the effectual demands. At this stage, it is contended that the rates of profits in all the sectors would converge or become uniform and outputs will have no reason to change and thus the prices prevailing when the rate of profits are equal will be the equilibrium or the "natural" prices.

In the light of Sraffa's (1960) work, several weaknesses in the above argument can immediately be noticed.[4] First of all, given a complex input-output structure of a large number of basic goods, it cannot be contended that the direction of the movements of prices and the corresponding rates of profits would always be the same (see Steedman 1984 and also Section 2.4.1). The classical proposition would always be true if only the prices of the commodity under consideration changed with all other prices remaining constant. However, if price of a commodity rises but at the same time the prices of its inputs that it uses extensively also rise greatly then it may end up with a fall in its rate of profit along with a rise in its price. Secondly, the classical mechanism truly begins to work only after prices have adjusted to their maximum and minimum as a result of the initial excess effectual demands and supplies. As a matter of fact, the first stage of the story is incoherent as the adjustment mechanism assumes a downward sloping (in the price-quantity plane) demand curves passing through the given corresponding effectual demands and the supply adjustments are supposed to slide down or climb up on the given demand curves. Thus there is no reason to assume that the initial supply points are off the given demand curves to begin with. In any case, however, the reasoning again shows the weakness of the partial equilibrium reasoning. As all the sectors rescale, the quantity demanded of any basic good would fluctuate according to the rescaling of all the sectors even though its effectual demand is held to be a fixed quantity. No smooth demand curve of any particular commodity could be drawn along which

[4] Elsewhere (Sinha and Dupertuis 2006) we have argued that Sraffa's prices do not need the classical gravitational mechanism. The uniformity of the rate of profits is a logical property of Sraffa's given system and it must hold irrespective of supplies being equal or not to the effectual demands.

supply could be expected to adjust till it reaches the effectual demand point. In a continuous dynamic framework one would expect prices to rise (fall) so long as there is excess effectual demand (supply) for the commodity. Thirdly, even if we accept the above described adjustment process of the classical system that during the quantity adjustments the rates of profits of the commodities with higher (lower) than the 'natural' rate fall (rise), there is no guarantee that they will all be equal when supplies are equal to their effectual demands. This proposition has never been proven but simply assumed. It is true that when supplies are equal to effectual demands then prices will have no tendency to move. But these prices may be a vector of prices that gives rise to unequal rates of profits that leads to further rescaling that in turn causes changes in prices and therefore, the system fails to converge at the points of effectual demands.

Unfortunately, the literature in the area of classical notion of centre of gravitation is not only modest in size but also most of it is not directly relevant to the central issue; as most of them deal with the question of convergence in the context of economic growth (e.g. Duménil and Lévy 1985, 1987; Flaschel and Semmler 1987; Boggio 1992; Franke 1998)[5]. The classical economists separated the problem of economic growth and the problem of allocation of labour. The gravitation mechanism was exclusively designed to deal with the problem of allocation of labour and not with the problem of growth. In the growth context the classical economists were quite clear that the techniques of production in use as well as the 'natural wages' or the 'natural rate of profits' etc. cannot be held constant. In Sraffa's system, the theoretical distinction between the problematic of allocation of labour and growth becomes much sharper. To any given system of production there exists a unique standard system, which is a particular reallocation of its given total labour. This standard system is associated with a set of all possible reallocation of the given total labour with the same techniques— that is, any such possible allocation can be taken as an equivalent of the given system. It is this set that defines the universe of the problem of allocation of labour pure and simple. On the other hand, whenever total labour is allowed to change (given the same techniques), which must be the case in the context of growth, the standard system must change, as the standard system is not only depended on the techniques of production but also the size of the total labour of the system. Thus even when we assume constant returns and balanced growth, we cannot maintain that the system remains the same. It must continuously be changing as its utilisation of total labour changes. Furthermore, many of these papers deal with only two-good models (e.g. Nikaido 1983; Duménil and Lévy 1987). If, however, the system describes chaotic dynamics, then such models would simply fail to capture it, as chaotic dynamics requires at least three degrees of freedom which is possible only in a system with at least three goods. In what follows, in Section 2 we provide a reminder of the basic properties of a Sraffian system and mathematical proofs of the above propositions.

In Section 3 we first present a Sraffian model of only basic goods with at least three or more goods with no fixed capital or joint-production. We take a given system of production with a vector of net output. We describe all the possible

[5]See Ganguli (1997) for a comprehensive bibliography on this subject.

economically sustainable set of net outputs given the total labour of the system and the techniques in use. Out of this set, we arbitrarily pick a combination of net output other then the given combination of the net output as the vector of effectual demands. We postulate that the initial vector of relative prices is such that the rates of profits in all the sectors are equal. However, since the given supplies are not equal to the effectual demands, the relative prices of the commodities with positive excess effectual demands (supplies) rise (fall). This tendency persists as long as excess 'effectual' demands and supplies exist. As prices rise and fall, they simultaneously affect the rates of profits in all the sectors. We, in the first case, postulate that the sectors that show greater than the 'natural', or what we call the 'standard', rate of profit expand their supplies and the sectors that show lower than the standard rate of profit contract (on the assumption of CRS). Throughout this process of expansion and contraction we maintain that the total labour of the system remains fixed at the level of the initial period. Thus the model is of pure allocation and not growth. We find that in a continuous adjustment process of this kind, the system is non-convergent and quickly moves to negative territory implying that the system breaks down. In the second case, instead of taking the standard rate of profit as the bench mark for deciding which sector expands and which contracts, we use the average rate of profit of the system as a whole (the global rate of profit) as the bench mark— a bench mark that itself continuously changes with changes in the rates of profits of individual sectors. We find that in this case as well the results are qualitatively the same.

In Section 3.5 we try to mimic the classical adjustment process. We start off with a vector of prices such that the relative prices of all the commodities with positive excess effectual demands are higher than what they would be in the case of all the rates of profits being equal and conversely for the commodities with positive excess effectual supplies. We postulate that the sectors that show higher (lower) rates of profit than the standard rate expands (contracts) its supplies and every rise (fall) in supply is associated with a fall (rise) in its relative price. Again we find that the system is non-convergent. We repeat the exercise by taking the global, in place of the standard, rate of profit as the benchmark. The result in this case as well remains qualitatively the same. In Section 3.6 we try a special case where investment flows only to the sector that has the highest local rate of profit. Again the results remain qualitatively the same.

It should be pointed out though that all these results do not suggest that the system can *never* converge. All it says is that possibility of that happening is most unlikely— in mathematical terms the probability of convergence is zero.

In Section 3.7 we try only quantity adjustment with fixed prices. Of course, if we take a vector of prices that gives unequal individual rates of profits and try to rescale the system on the basis of the difference between the individual or local rates of profits and the standard rate or the global rate with prices held constant, then in this case the system can never converge. The reason for it is simple. Since prices are held constant, the individual rates of profits are independent of rescaling, thus some sectors would continuously expand and some would continuously contract till the system breaks down. Thus we only try a pure quantity adjustment mechanism with the rate of profits and prices held at the standard rate (which means that all the individual rates of profits are uniform) and the sectors adjusting only their quantities on the basis of inventory signals. In this case only, we find that the system converges. Thus, it seems, it is the 'Keynesian type' of fix-price quantity adjustment mechanism that is more conducive to the idea of centre of gravitation than the price induced adjustment mechanism of the classical type.

In Section 4 we briefly discuss Steedman (1984), which in our opinion comes closest to identifying the problems with the classical mechanism and was, in some sense, an inspiration for our own attempt here. We argue that though Steedman made a splendid beginning, he did not succeed in proving his case. Furthermore, though he does not cast his model in a growth context, it is not clear whether he succeeds in maintaining that the total labour remains constant throughout the adjustment process.

2 Some properties of Sraffa's systems

In this section, we work out some properties of Sraffa's systems which will be used to describe the gravitational mechanisms.

2.1 Basic properties

In part I of his book (1960), Sraffa introduces systems which are instantaneous descriptions of closed economies.[6] Such systems have exactly the same number of techniques and commodities and each technique produces only one commodity; for example:

$$\left\{\begin{array}{llllllllllll} 90 & \text{t.iron} & \oplus & 120 & \text{t.coal} & \oplus & 60 & \text{qr.wheat} & \oplus & 3/16 & \text{labour} & \Rightarrow & 180 & \text{t.iron} \\ 50 & \text{t.iron} & \oplus & 125 & \text{t.coal} & \oplus & 150 & \text{qr.wheat} & \oplus & 5/16 & \text{labour} & \Rightarrow & 450 & \text{t.coal} \\ 40 & \text{t.iron} & \oplus & 40 & \text{t.coal} & \oplus & 200 & \text{qr.wheat} & \oplus & 8/16 & \text{labour} & \Rightarrow & 480 & \text{qr.wheat} \end{array}\right. \quad (1)$$

Each line represents a technique (or a sector). This means that when the observation was made, it took 90 tons of iron, 120 tons of coal, 60 quarter of wheat and 3/16 of the total labour used to produce 180 tons of iron. The other lines are interpreted in the same way.

It should be noted that these notations may hide the fact that units of production are complex (in the sense "elabourate"). Therefore, since one does not have a more detailed view of the techniques involved, it could happen that, counter intuitively, these units of production are not linear. For example, if one doubles all the inputs in one sector, the output quantity may not double. Therefore, the assumption that these techniques are linear, i.e. it obeys CRS, should be treated carefully. Nevertheless, CRS may lead to a good approximation and the "gravitational mechanism", which this paper is about , assumes CRS.

Let us describe a system such as (1) involving n different commodities, numbered 1 to n. To describe the system one needs a square matrix $A = (a_{i,j})$ of size $n \times n$

[6]These descriptions involve only observables.

called the “input matrix”, a vector $L = (\ell_i)$ of size n which represents labour[7] and a vector $Q = (q_i)$ of size n called the “output vector”. The system denoted $A|L \Rightarrow Q$ is the one for which the unit of production i takes goods 1 to n by the quantities given by $a_{i,1}, \ldots, a_{i,n}$ respectively, with an amount of labour given by ℓ_i and an amount of output of good i by q_i.

Below we define some notations that would be needed for the analysis of the system:

- Let e_i be the column vector whose all entries are equal to zero except the i^{th} one, which is equal to 1. The set of these vectors forms what is called the canonical basis.
- When it will come to rescaling, we will need a notation: if v is a vector, matrix $\tilde{v}$ will be the diagonal matrix whose entries are the ones of v.
- If v and w are vectors, then $\langle v|w\rangle$ is their (euclidean) scalar product.
- The vector e is the column vector whose entries are all equal to 1.

There are matrices and vectors that are not adequate for economically viable systems. Furthermore, we want to deal only with indecomposable systems, i.e. we want to deal only with basic goods. If $A|L \Rightarrow Q$ is a Sraffian system, then the following assumptions must be satisfied:

Assumption 1 *All the entries of matrix A are positive or null and A is invertible.*

Assumption 2 *All the entries of the vectors L and Q are strictly positive.*

Assumption 3 *The system can assure it's own subsistence. This means that for every j, one has:*

$$\sum_i a_{i,j} \leq q_j. \tag{2}$$

Assumption 4 *The total amount of labour is equal to 1. This means that:*

$$\bar{L} = \sum_i \ell_i = 1. \tag{3}$$

Assumption 5 *All goods are basic goods. This means that, for every i and j, one has:*

$$\langle A^n e_i|e_j\rangle \neq 0. \tag{4}$$

[7] In fact, ℓ_i represents the proportion of total work the unit of production uses in the closed economy described by the system. This proportion of labour is an observable: when it comes to the description of an existing system, it can be worked out by the wages as Sraffa homogenizes the labour units by assuming “any differences in quality to have been previously reduced to equivalent differences in quantity so that each unit of labour receives the same wage.” (Sraffa 1960, p.10).

The standard system:

Definition 1 *A system $A|L \Rightarrow Q$ is said to be* **standard** *if there exists a constant ρ such that, for all j, one has:*

$$\sum_i a_{i,j} = \rho q_j. \tag{5}$$

A rescaling is a vector $\Lambda = (\lambda_i)$ with strictly positive entries such that $\langle \Lambda | L \rangle = 1$. The rescaled system, which means the system whose i sector is rescaled by λ_i for all i, is $\tilde{\Lambda} A | \tilde{\Lambda} L \Rightarrow \tilde{\Lambda} Q$. The set of all admissible rescalings is defined by:

$$\mathcal{R}_L = \{ \Lambda \mid \langle \Lambda | L \rangle = 1 \}. \tag{6}$$

One has $\mathcal{R}_{\tilde{\Lambda} L} = \tilde{\Lambda}^{-1} \mathcal{R}_L$.

Definition 2 *Two systems $A|L \Rightarrow Q$ and $A'|L' \Rightarrow Q'$ are said to be* **equivalent**[8] *if there exists an admissible rescaling $\Lambda \in \mathcal{R}_L$ such that $\tilde{\Lambda} A = A'$, $\tilde{\Lambda} L = L'$ and $\tilde{\Lambda} Q = Q'$.*

Lemma 1 *For every system, there is a unique standard system which is equivalent to it. It's called the* **standard system***.*

Therefore:

Lemma 2 *A class of equivalent systems (which consists of all systems that can be transformed from one to another by CRS) is completely described by the standard system in that class and the admissible rescalings.*

If one gives some prices to each good, then, one can compute the relations between wages, the local rates of profit (i.e. the rates of profit in each sector) and the global rate of profit of the whole system. The Sraffa price-wages-profit relations can be translated in the matrices language by:

$$(\mathrm{id} + \tilde{R}) AP + wL = \tilde{Q} P \tag{7}$$

where id is the identity matrix, $P = (P_i)$ is the vector of prices, w represents wages and $R = (R_i)$ is the vector of local rates of profit.

Lemma 3 *Equation (7) is invariant under rescaling. This means that, given some prices P, the corresponding wages and local rates do not depend on rescaling. In particular, the maximal rates of profits, which are the local rates of profit when $w = 0$, are completely determined by the standard system and the prices. The relation between prices and maximal rates of profit is then:*

$$\tilde{R} AP = (\tilde{Q} A^{-1} - \mathrm{id}) AP. \tag{8}$$

[8] We do not mean to say that any mathematical system is also a real system. For this one needs to assume constant returns to scale. Therefore, it is better to think of this equivalence as an equivalence between the given real system and imaginary systems. To assume CRS means nothing more than that all of these systems could be real.

The matrix $\tilde{Q}A^{-1} - \mathrm{id}$ has only one positive eigenvalue which admits an eigenvector with strictly positive entries. This eigenvalue is called the **standard rate** and is denoted by R^*. The prices which lead to all local rates equal to R^* are called the **standard prices**. There are infinitely many such prices. But there is only one choice of standard prices whose sum is equal to 1, this choice of prices will be denoted by P^*. From here on we will assume $w = 0$ for simplicity sake. A positive w expressed in Standard commodity will not make any difference to the analysis.

The global rate of profit: The global rate of profit R_G is the rate of profit of the system as a whole:

$$R_G = \frac{\langle \tilde{R}AP|e\rangle}{\langle AP|e\rangle} = \frac{\langle AP|R\rangle}{\langle AP|e\rangle}. \tag{9}$$

It is evident that the global rate of profit may not be invariant under rescaling (for the proof, see Sinha-Dupertuis 2007). We prove the following:

Proposition 1 *The maximal global rate of profit is invariant under all rescalings if and only if the maximal local rates of profit are equal two by two. In which case, the maximal global rate and the maximal local rates are all equal to the standard rate.*

Proof. Let us suppose first that the local rates are all equal to the same constant r, then from lemma 3, one has:

$$(\tilde{Q}A^{-1} - \mathrm{id} - r\,\mathrm{id})AP = 0. \tag{10}$$

Therefore, AP is an eigenvector with strictly positive entries of the matrix $\tilde{Q}A^{-1} - \mathrm{id}$ for the eigenvalue r. Since the only eigenvalue of this type is R^*, all local rates of profit are equal to the standard rate and so is the global rate. Since the local rates are invariant under rescaling, any choice of prices that give equal local rates give also an invariant global rate.

Let us suppose that the global rate of profit is invariant under rescaling, then, from (9), for any rescaling Λ, one has:

$$R_G = \frac{\langle \tilde{R}\tilde{\Lambda}AP|e\rangle}{\langle \tilde{\Lambda}AP|e\rangle}. \tag{11}$$

Since RHS of equation (11) is a ratio of scalar products, which are homogeneous of degree 1 in their first variable, R_G is homogeneous of degree 0. This implies that (11) is true not only for rescalings, but for any diagonal matrix. So, let v and w be any vectors with strictly positive entries. Then, since diagonal matrices commute between themselves, one obtains:

$$R_G = \frac{\langle \tilde{v}\tilde{w}\tilde{R}AP|e\rangle}{\langle \tilde{v}\tilde{w}AP|e\rangle} = \frac{\langle \tilde{w}\tilde{R}AP|\tilde{v}e\rangle}{\langle \tilde{w}AP|\tilde{v}e\rangle}. \tag{12}$$

Thus:

$$\Big\langle (\tilde{R} - R_G\,\mathrm{id})\tilde{w}AP|v \Big\rangle = 0. \tag{13}$$

Therefore, $\tilde{R} = R_G\,\mathrm{id}$, which leads to the conclusion. ■

2.2 Rescaling: Why total labour must be kept equal to one

What we want to show here is how rescalings really work with Sraffa's systems. We will see that the constraint that total labour must be kept to 1 is not a free assumption: it is necessary. Indeed, if total labour is not kept equal to 1, we are in a growth context.

The problem with growth is that if one tries to rescale a Sraffian system over or under the given total labour, then it turns out that the new system is not equivalent to the former system. In other words, the two systems do not have the same standard system. For example, take any given system and multiply all its equations by 2 and try to construct the standard systems for both the systems. Since in a standard system the labour inputs for each sector is taken as a proportion of the total labour used in the system, we will have twice of all commodity inputs and outputs for each row of the second system with the labour entries remaining the same. Clearly, the two standard systems will not be the same. Below, we formally prove this point.

Let us assume CRS. Suppose that all labour can be counted in a physical unit such as labour hours. Then a technique is a way to transform some goods into another one, all of them measured in quantities per labour hours. A real economic system is described by a complete set of basic techniques and the quantity of labour hours used in each sector. Let us denote such a description of a system by $T \to U||W$ where T is the square matrix of inputs of the techniques, U is the vector of the outputs and W is the quantity of work. The Sraffian system $A|L \Rightarrow Q$ associated with this system is given by: $a_{i,j} = t_{i,j}w_i$, $q_i = u_i w_i$ and $\ell_i = w_i/\overline{W}$ where $\overline{W}$ is the sum of all the w_i's, i.e. total quantity of labour.

Consider any rescaling Λ. If one tries to adapt the different sectors according to the rescaling, then the new system will be $T \to U||\tilde{\Lambda}W$. Denote the associated Sraffian system by $A'|L' \Rightarrow Q'$. On the basis of the discussion above, this Sraffian system satisfies the following relations with the original Sraffian system:

$$a'_{i,j} = t_{i,j}\lambda_i w_i,\ q'_i = u_i\lambda_i w_i,\ \ell'_i = \lambda_i w_i \left(\overline{\tilde{\Lambda}W}\right)^{-1} = \lambda \frac{\overline{W}}{\overline{\tilde{\Lambda}W}} \ell_i. \tag{14}$$

Therefore, the proportions between $a_{i,j}, q_i, \ell_i$ and $a'_{i,j}, q'_i, \ell'_i$ are the same (and, thus, these systems have the same standard system) if and only if:

$$\langle W|e\rangle = \overline{W} = \overline{\tilde{\Lambda}W} = \left\langle \tilde{\Lambda}W|e \right\rangle = \langle W|\Lambda\rangle\,. \tag{15}$$

Since $L = W/\overline{W}$, (15) is equivalent to:

$$\langle\Lambda|L\rangle = \langle\Lambda|W\rangle\,/\overline{W} = 1 \tag{16}$$

which proves our claim.[9]

[9] One should note that since unbalanced growth can be decomposed into a rescaling which maintains total labour equal to 1 followed by a balanced growth, the problems induced by balanced or unbalanced growth are of the same nature.

2.3 Affordable demands

Let us think of demand as a vector of quantities B. Assuming CRS, this demand may or may not be fitted by rescaling the system. A demand is said to be **affordable** if it can be fitted by rescaling.

Should a demand B be affordable, it would mean that there exist a rescaling Λ such that $\tilde{\Lambda}Q = B$. The criterion here is that total labour needs to remain equal to 1. Therefore, a demand is affordable if:

$$\left\langle \tilde{Q}^{-1}B|L \right\rangle = \left\langle B|\tilde{Q}^{-1}L \right\rangle = 1. \tag{17}$$

The other condition on demand to be affordable is that the system should be capable of self reproducing. Therefore, for all j, one should have:

$$\sum_i \frac{b_i}{q_i} a_{i,j} \leq b_j. \tag{18}$$

In terms of matrices, this means that, for all j:

$$(\tilde{Q}^{-1}\tilde{B}A)^t e_j \leq \tilde{B}e_j \tag{19}$$

which can also be written as:

$$A^t \tilde{Q}^{-1}\tilde{B}e_j \leq \tilde{B}e_j. \tag{20}$$

This is not ensured for all vectors B (even under assumption 3). But if the system is in a surplus producing state, there are infinitely many such B's. In fact, the set of these vectors is of positive measure.

2.4 Number of goods: Why two is not enough

The behavior of the two goods case can not be taken as a general one because of the problems with the movements of rates of profit and prices and some general property of chaotic dynamics.

2.4.1 Rates of profit

Steedman (1984) shows that prices and the rates of profit are not always moving in the same direction. Here we prove it with our own argument.

Let us choose a system $A|L \to Q$. The rate of profit of sector i is given by:

$$R_i = \frac{q_i p_i}{\sum_j a_{i,j} p_j} - 1 \tag{21}$$

The partial derivative of R_i with respect to p_k is then:

$$\frac{\partial R_i}{\partial p_k} = \left(\sum_j a_{i,j} p_j \right)^{-2} \begin{cases} -q_i a_{i,k} p_i & \text{if } i \neq k, \\ q_i \sum_{j \neq i} a_{i,j} p_j & \text{if } i = k. \end{cases} \tag{22}$$

Therefore, the derivative is never a constant and no partial derivative is zero if all prices are strictly positive (and it satisfies $\langle \nabla R_i | P \rangle = 0$.) Therefore, the locus of local rates of profit is not a plane and, locally, there is a one to one correspondence between the vector of prices and the vector of local rates of profit.

Let us take a system with strictly more than two goods and suppose that the prices are such that first good has a high rate of profit, the second good has a low rate of profit and rates of all other goods are equal to the standard rate. One can change the prices for the first two goods and let all other prices remain the same. In general, all the rates of profit will change. Indeed, the rate of profit of the i^{th} good will remain constant if and only if $a_{i,1}$ and $a_{i,2}$ are equal, which is a very restrictive case.

2.4.2 Chaotic behaviour

Gravitational mechanisms act on prices and rescalings this means that, in an n goods system, they act on $2n$ variables. Since total labour is kept equal to 1 and since the prices are normalized, these variables have a $2n - 2$ degree of freedom.

It is a general principle of continuous dynamics that chaotic behaviours can only arise for conservative systems (i.e. solutions are unique) if the degree of freedom is greater or equal to 3. Indeed, since the solutions of the system can not cross, with only two degrees of freedom, each solution divides the plane into two parts and each other solution which belongs to one part can not switch to the other one. This do not happen with three degrees of freedom since the solutions do not divide the space into parts.

For discrete dynamics, chaos may arise even in the two good case. But if the discrete system is the discretization of a conservative continuous system (which is the case of most of the discrete mechanisms), such a behaviour is obstructed.

Therefore, although it may diverge, considering only two goods cases (which give only two degrees of freedom) may be too restrictive and can miss some interesting behaviour.

3 Gravitational mechanisms

In this section, we will consider different processes based on gravitational mechanisms. What is shown is that the space of systems for which such processes converge is of codimension at least one. Furthermore, apart from these cases, the set of initial conditions for which the process converges is of measure zero. In plain words, this means that the probability for such a process to converge is zero.[10]

[10]Here, 'zero' does not mean it is impossible: 'probability equal to zero' occurs for example when there is a finite number of possibilities out of infinity.

3.1 Overview of gravitational mechanisms

Let us take a system, an affordable demand for this system and a set of prices. This system may be in a certain state such that demand is not fitted or rates of profits are not equal. Such a state is not in equilibrium because investments will move according to these indicators. Our purpose here is to describe mathematically and study the convergence of various possible 'gravitational mechanisms'.

3.1.1 Suitable variables for gravitational mechanisms

From now, we will assume CRS. Since we will deal with rescaling of equivalent systems, we introduce variables that can be easily handled. For example, we don't want to deal with all the system entries since there are multiple constraints on them. One way to achieve this is to look at all the systems we will encounter (even the given real system) as rescalings of their common standard system. This way the mathematical models of gravitational mechanisms are more easily described and the computations are made simpler. This in no sense privileges the standard system. As laws for the solar system are completely independent of the choice of a referential; thus to mimic what an observer on Earth can see one may be tempted to set the Earth as the center of the solar system, however the equations are simpler and more understandable if one sets the Sun as the center of the system.

Given any Sraffian system $A_r|L_r \Rightarrow Q_r$ (where r stands for *real*), its standard system will be denoted by $A|L \Rightarrow Q$ and the rescaling that brings this standard system to the real system will be denoted by $\Lambda_0 = (\lambda_{0,i})$. This means that one has $A_r = \tilde{\Lambda}_0 A$, $L_r = \tilde{\Lambda}_0 L$ and $Q_r = \tilde{\Lambda}_0 Q$. The vector Λ_0 is nothing more than an initial condition.

The variables are Λ (rescaling) and P (prices). The equations for the gravitational mechanisms will only deal with these variables. Since total labour must be kept equal to 1, at each stage Λ should satisfy $\langle \Lambda | L \rangle = 1$ (this is more or less like the ecliptic plane). Furthermore, one should normalize the prices: the local and global rates of profit do not really depend on prices, but on the proportions between them. We will maintain the sum of prices equal to 1. This way, there is a one-to-one correspondence between prices and rates of profit. The manner in which the prices are normalized is of no importance so long as we keep normalizing them in the same way.

3.1.2 General principles of gravitational mechanisms

The different processes we will encounter should reflect how capital and labour move according to the market situation. The indicators which will be used for investments are either just the sign or sign and the distance between local rates of profit and a benchmark of profit, which could be either the global rate or the standard rate. If the rate of profit of one sector is above (below) the benchmark, then it will expand (contract). This, however, is only a trend: if one sector has a huge expansion, it could happen that other expanding sectors might shrink because of this 'highly efficient' sector. The condition is that the rescalings induced by the movements of capital should maintain total labour equal to 1.

In a second stage, one has to deal with demand. The prices should rise or fall according to the position of supply with respect to demand. Depending on the chosen processes, we will have to make it clear whether the information is only if supply is greater or smaller than the demand or whether there are more information available. We will see that the processes do not converge in both the cases.

To sum up, the process (continuous or discrete) follows the following scheme:

1. the prices move according to demand and supply;
2. rates of profit are worked out on the basis of market prices;
3. rescaling moves according to the distance between local rates of profit and some reference that could either be the standard rate (classical case) or the global rate;
4. these rescalings change supplies;
5. the process is repeated.

There are four ways to classify the gravitational processes:

- a gravitational process may be continuous or discrete;
- the information on demand and supply may be continuous (i.e. the difference between the quantities of supply and demand is given) or not (i.e. only the sign of this difference is given);
- the reference for the rates of profit may be the standard rate or the global rate;
- the change in prices can be dependent on previous prices (the non classical case) or not (the classical case).

Some combinations of these cases will never show up, for example: the classical case can not be used in a continuous process;[11] discrete information can not be used in a continuous process; discrete information for the classical case is meaningless;[12] etc. The possibilities are:

- non classical continuous process with standard rate and continuous information;
- non classical continuous process with global rate and continuous information;
- non classical discrete process with standard rate and continuous information;

[11]A continuous process uses a relation between demand, supply and the derivative of prices. Therefore, prices are determined by their former values.

[12]The classical case needs a demand curve to determine prices. Discrete information would not be sufficient for determining prices according to demand.

- non classical discrete process with global rate and continuous information;
- non classical discrete process with standard rate and discrete information;
- non classical discrete process with global rate and discrete information;
- classical discrete process with standard rate and continuous information;
- classical discrete process with global rate and continuous information.

All these processes lead to dynamical systems. These dynamical systems will all have the same fixed point: demand fitted and all rates of profit equal to the standard rate.[13] This means that $P = P^*$ and $\Lambda = \Lambda^*$ where, if B is the demand, $B = \tilde{\Lambda}^* Q$.

To study convergence, we will only be interested by what is happening in the neighborhood of the fixed point. This will allow us to linearize equations.

3.2 Continuous processes

Let us introduce effectual demand in terms of quantities as a vector B. Think of Λ and P as functions of a time variable. Let R_{ref} be the reference for the rates of profit (this means the standard rate or the global rate). In the classical case, R_{ref} is a constant and in the non classical case, it's a function of the prices. Since a constant can be seen as a function, in fact, in both cases we can think of R_{ref} as a function. Therefore, the gravitational equations can be stated once for all. Fromerly, for a small amount of time h, one should have:

$$\left\{ \begin{array}{rcl} \Lambda(t+h) - \Lambda(t) & = & hF(R - R_{ref}e) \\ P(t+h) - P(t) & = & hG(B - \tilde{\Lambda}Q) \end{array} \right. \tag{23}$$

where F and G are continuous derivable functions of the form $F = (F_1, \ldots, F_n)$ and $G = (G_1, \ldots, G_n)$, this means that the rescaling and the price of a good cares about its own indicators only.

We have to satisfy the constraints on rescaling and prices. Since we want total labour equal to 1, scalar products between the vectors $\Lambda(t+h)$ and $\Lambda(t)$ with L should be equal to 1. Therefore, $\Lambda(t+h) - \Lambda(t)$ should be orthogonal to L. In the same way, to ensure that total prices are equal to 1, the vector $P(t+h) - P(t)$ should be orthogonal to e. Thus, let π_L and π_e be the projections over the planes orthogonal to vectors L and e respectively, then one should have:

$$\left\{ \begin{array}{rcl} \Lambda(t+h) - \Lambda(t) & = & h\pi_L(F(R - R_{ref}e)) \\ P(t+h) - P(t) & = & h\pi_e(G(B - \tilde{\Lambda}Q)) \end{array} \right. \tag{24}$$

Finally, by letting h tend to zero, the systems of differential equations for continuous gravitational processes turn out to be of the form:

$$\left\{ \begin{array}{rcl} \dot{\Lambda} & = & \pi_L(F(R - R_{ref}e)) \\ \dot{P} & = & \pi_e(G(B - \tilde{\Lambda}Q)) \end{array} \right. \tag{25}$$

[13] When all local rates of profit are equal to the same constant, this constant is the standard rate and is also the global rate.

To ensure that (Λ^*, P^*) is the fixed point, we assume, without loss of generality, that the functions $F_1, \ldots, F_n$ and $G_1, \ldots, G_n$ take the value zero at zero. It should also be noted that these functions have to be strictly increasing for gravitational mechanism to make sense.

Finally, assume that the functions $F_1, \ldots, F_n$ are the same functions and functions $G_1, \ldots, G_n$ are the same functions as well. This last assumption may not really be needed, but it makes sense to suppose that investors will react to rates of profit regardless of the commodity and, in the same way, the 'prices-supply-demand' equation should not be dependent on the nature of the commodity. With these assumptions, (Λ^*, P^*) is the only fixed point.

Since we are only interested in the behaviour of the system near the fixed point, we can assume that the F_i's and G_i's are constant multiplications.[14] Therefore, we want to study the behaviour of the system:

$$\left\{ \begin{array}{lcl} \dot{\Lambda} & = & \alpha \pi_L (R - R_{ref} e) \\ \dot{P} & = & \beta \pi_e (B - \tilde{\Lambda} Q) \end{array} \right. \tag{26}$$

where α and β are strictly positive constants.

The projections can be described by non invertible matrices. Let M_L and M_e be the matrices of the projections π_L and π_e respectively. Thus, (26) becomes:

$$\left\{ \begin{array}{lcl} \dot{\Lambda} & = & \alpha M_L (R - R_{ref} e) \\ \dot{P} & = & \beta M_e (B - \tilde{\Lambda} Q) \end{array} \right. \tag{27}$$

3.2.1 With the standard rate

The system becomes:

$$\left\{ \begin{array}{lcl} \dot{\Lambda} & = & M_L \alpha R - C \\ \dot{P} & = & B' - M_e \beta \tilde{\Lambda} Q \end{array} \right. \tag{28}$$

where $C = R^* M_L \alpha e$ and $B' = M_e \beta B$ are constants.

Since (Λ^*, P^*) is the only fixed point, the behaviour of the process is dependent on the sign of $Cr(\Lambda, P) = \langle \dot{\Lambda} | \Lambda^* - \Lambda \rangle + \langle \dot{P} | P^* - P \rangle$. Notice that P is the vector of standard prices if and only if $\dot{\Lambda} = 0$. Therefore, for any rescaling Λ, one has $Cr(\Lambda, P^*) = 0$.

Suppose that demand is fitted. Here $\dot{P} = 0$, then Cr becomes:

$$Cr(\Lambda, P) = \langle M_L \alpha R | \Lambda^* - \Lambda \rangle - \langle C | \Lambda^* - \Lambda \rangle \tag{29}$$

If $\Lambda \neq \Lambda^*$, then the space of rates of profit is a $n-1$ unbounded surface which is not a plane and the image of it by the projection π_L covers the plane orthogonal to e. Since $n \geq 3$, there is a choice of prices such that Cr is strictly negative. Since Cr is linear in $P^* - P$, there is an open half-line starting at P^* such that Cr is negative.

[14] If this simple process diverges, then all continuous processes are diverging.

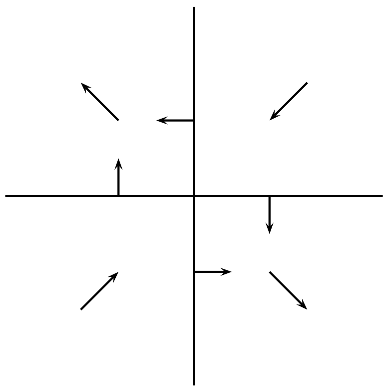
This implies that (Λ^*, P^*) is not an attractive point. There is only one possibility left for such a process to be more or less 'stable'. It is the one where a repulsed trajectory turns around the fixed point and comes back from the rear side (this is similar to the magnetic flow lines). To rule out this case one could compute the second order derivatives of the local rates of profit. We have done these computations and have found that this does not turn out to be the case apart from maybe a very small number of systems.

In any case, there is a way to avoid these computations— It is by describing the flow of the system. If one fixes some prices, then $\dot{\Lambda}$ and $P^* - P$ are constants. Therefore, Cr is of the form:

$$Cr(\Lambda, P) = d + f(\Lambda^* - \Lambda) \tag{30}$$

where $d = \langle B' - M_e \beta \tilde{\Lambda}^* Q | P^* - P \rangle$ is a constant and where f is a linear form. This means that for any prices, there is a half-plane of Λ's for which Cr is negative. Furthermore, for any demand B, there is a half-plane of prices such that there is a half-plane of Λ's containing Λ^* for which Cr is negative. It should be noted that changing both sings of $\Lambda^* - \Lambda$ and $P^* - P$ does not change Cr.

If one takes any slice of the space of the flow by a plane containing the fixed point, one obtains the following configuration in the neighborhood of the fixed point:



Such a configuration is not convergent in the way that the space of solutions converging to the fixed point is of dimension at most $2n - 3$, whereas the space of solutions is of dimension $2n - 2$.

Therefore, we obtain the following:

Theorem 1 *Let $n \geq 3$. For any continuous system with standard rate, the space of solutions that are converging is of codimension at least 1 in the space of solutions.*

This means that convergent solutions are very rare. And, in fact, if one allows small perturbations into the process, then no solutions will converge.

3.2.2 With the global rate

This case is very similar to the former one. In fact, every argument applies here as well. Therefore, we obtain:

Theorem 2 *Let $n \geq 3$. For any continuous system with global rate, the space of solutions that are converging is of codimension at least 1 in the space of solutions.*

It should be noticed that if $B = Q$, then the global rate of profit has a null derivative at $P = P^*$. This means that when demand is equal to the supply of the standard system, there is no qualitative difference in the behaviour of the process with standard and global rate. Therefore, since it is not converging in this case for the global rate, it will not converge for other demand vectors as well.

3.3 Non classical discrete processes with continuous information

This case is in fact the discrete analogue of the former one. The equations for the gravitational mechanism are:

$$\begin{cases} \Lambda_{t+1} &= \Lambda_t + \alpha M_L(R(P_t) - R_{ref}e) \\ P_{t+1} &= P_t + \beta M_e(B - \tilde{\Lambda}_t Q) \end{cases} \quad (31)$$

where the notations are as before.

Since this case is the discretization of a non converging continuous process, it can not converge. To see this, one can set $\Lambda_{t+1} = F(\Lambda_t, P_t)$ and $P_{t+1} = G(\Lambda_t, P_t)$. If we compute the derivative of these functions at the fixed point, we see that the norms[15] of these derivatives are always greater than 1. Therefore, the fixed point is not attractive and this for both cases (with global and standard rate).

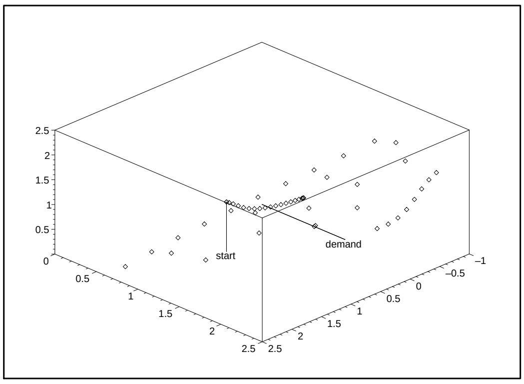
3.3.1 Example

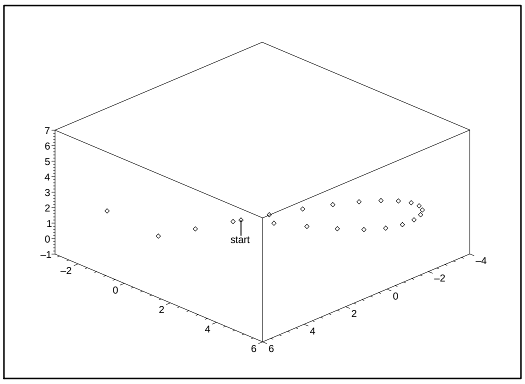
Here is an example of the first steps of this process.[16] The given system is (1). We are starting from the standard system with the standard prices and demand B is equal to the output vector of the given real system (therefore B is affordable). The constants α and β are set equal to $1/100$.[17] The reference rate is the global rate. The orbit of rescalings is on the left and the orbit of prices is on the right:

[15] The derivatives of multidimensional valued functions of several variables are operators. The norm of an operator is the maximum of its values upon vectors of norm equal to 1.

[16] The animations of this process and the following examples can be asked directly from the authors.

[17] The choice of these numbers does not modify the general behavior of the process, it only changes its speed.





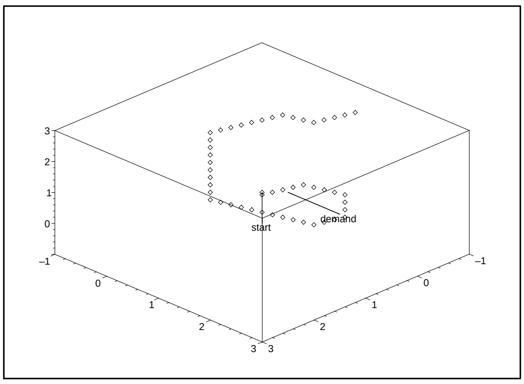
Although it may not appears so, both orbits are contained in planes which is a direct consequence of normalization of prices and constancy of total labour. One sees that the system is not only divergent, it also breaks down since it moves to negative territory.

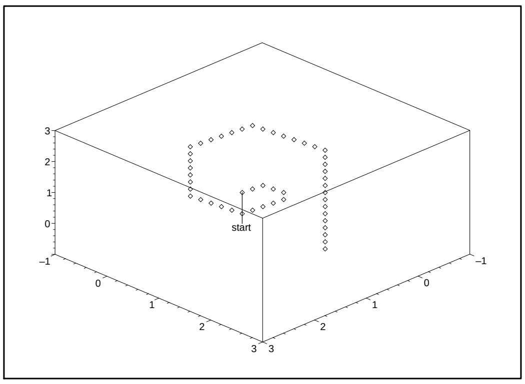
3.4 Non classical discrete processes with discrete information

This case is the simplest of all. “Discrete information” means that the distance from demand to supply is not known, what is only known here is whether the demand is larger than supply or not. This case will not converge since, even if we are very close to the stable point, the next prices can go very far form the standard ones. In fact, to converge, this process needs to attain the fixed point in a finite number of steps. Therefore, the only converging trajectories are the ones whose initial conditions are obtained by iterating the inverse of the process on the fixed point. The set of all these initial conditions may not be countable, but it will be of measure zero.

3.4.1 Example

Here is an example of the first steps of this process. The system and the initial conditions are as before except that α and β are chosen to be equal to $1/10$. The reference rate is the standard rate. The orbit of rescalings is on the left and the orbit of prices is on the right:





3.5 Classical processes

In the classical case, each good has a demand curve and prices are determined only by demand and supply. As before, since we are only concerned by the neighborhood of the fixed point, we can suppose that the process is linear. The equations of the gravitational mechanism are:

$$\left\{ \begin{array}{lcl} P_{t+1} & = & P^* + M_e D(B - \tilde{\Lambda}_t Q) \\ \Lambda_{t+1} & = & \Lambda_t + \alpha M_L (R(P_{t+1}) - R_{ref} e) \end{array} \right. \tag{32}$$

where D is a diagonal matrix which represents the demand curve.

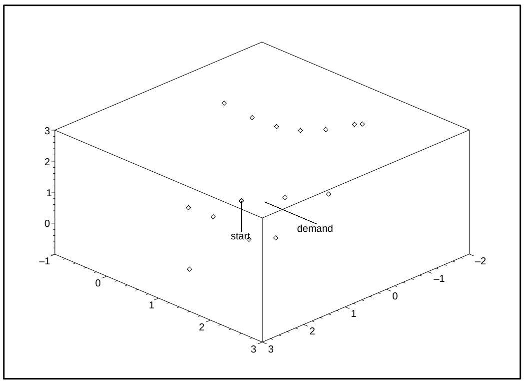
Here, we can replace prices into the equation for rescaling:

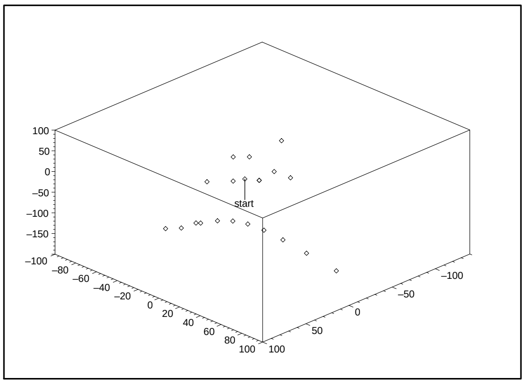
$$\Lambda_{t+1} = \Lambda_t + \alpha M_L (R(P^* + M_e D(B - \tilde{\Lambda}_t Q)) - R_{ref} e). \tag{33}$$

Then, we can do exactly the same thing we did in 3.3. Computing the derivative, we see that there is a direction in which this derivative is strictly greater than 1. Therefore, the fixed point is not attractive. The case where the trajectories could come back from the rear can be ruled out by computing the second order derivative (this is not easy and requires many pages of rough computations). If one does so, one will see that there is enough degrees of freedom so that this case represents only few particular systems. Therefore, this process does not converge either.

3.5.1 Example

The initial conditions are as before. The reference rate is the standard rate.





3.6 A very special case: maximal local rate

Here we consider what happens if the investments always go to the sector with maximal rate of profit. Since this mechanism is not smooth, we have to consider it as a discrete one. To work out the equations, let us denote by Ω the function defined, for any vector $V = (v_i)$, by $\Omega(V) = (w_i)$ where w_i is:

$$w_i = \frac{1}{\#\{i \mid v_i = \max_j v_j\}} \left\{ \begin{array}{ll} 1 & \text{if } v_i = \max_j v_j, \\ 0 & \text{otherwise} \end{array} \right. \tag{34}$$

where # stands for the number of elements of the set.

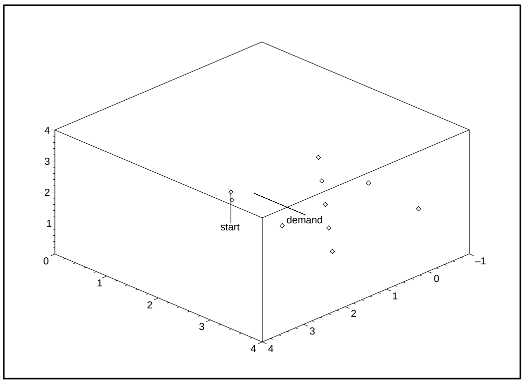
The equations for the gravitational mechanism are:

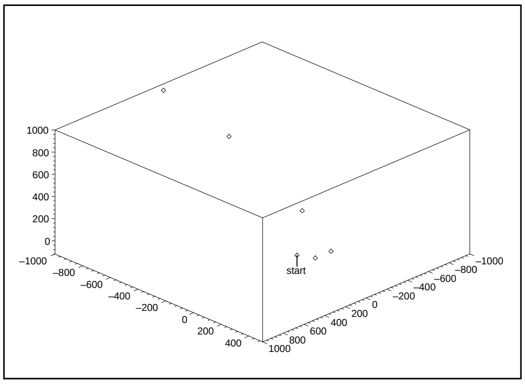
$$\left\{ \begin{array}{lcl} \Lambda_{t+1} & = & \Lambda_t + \alpha M_L(\Omega(R(P_t) - R_{ref} e)) \\ P_{t+1} & = & P_t + \beta M_e(B - \tilde{\Lambda}_t Q) \end{array} \right. \tag{35}$$

This process is not well defined at the fixed point, therefore, we extend the definition by putting $\Omega(0) = 0$. Still the function defining this process is not derivable at the fixed point. Giving a rigorous proof of the divergence would be too long and does not give much information, therefore, we will only do a sketch: to show that this process will not converge, we can compute the derivative at some points in the vicinity of the fixed point. One sees that in each neighborhood of the fixed point there are some points where the derivative exists and is greater than one. Thus the fixed point is not attractive. Should, by any chance, the process converge for a particular system, since the function defining the mechanism is not smooth, the process would not converge for other close systems.

3.6.1 Example

The constants α and β are set equal to 1. The reference rate is the global rate. The orbit of rescalings is on the left and the orbit of prices is on the right:





3.7 A very special process: fixed prices

In the former we have dealt with moving prices and we saw that none of these gravitational processes converge. One can ask the (not so stupid) question: "What if prices are fixed?".

Of course, if we keep looking at the former processes without changing prices, this will mean that we are dealing with equation $\dot{\Lambda} = \alpha M_L(R - R^*)$. So either the process will not be converging (if $P \neq P^*$) or it will not move at all (if $P = P^*$).

So, in fact, in this kind of process, we mean that the rescaling depends on inventory maintenance which in turn depends on demand. The equation would be:

$$\dot{\Lambda} = B - \tilde{\Lambda} Q = B - \tilde{Q} \Lambda. \tag{36}$$

This process is converging if and only if the scalar product:

$$\begin{aligned}\left\langle \dot{\Lambda}|\Lambda^* - \Lambda \right\rangle &= \left\langle B - \tilde{Q}\Lambda|\tilde{Q}^{-1}B - \Lambda \right\rangle = \left\langle B - \tilde{Q}\Lambda|\tilde{Q}^{-1}(B - \tilde{Q}\Lambda) \right\rangle \\ &= \sum_i q_i^{-1}(b_i - q_i\lambda_i)^2 \end{aligned} \tag{37}$$

is positive. Which is always the case.

4 A comment on Steedman (1984)

In the first part of his article, Ian Steedman (1984) shows that changes in prices do not necessarily lead to changes in the rates of profit in the same direction. Although it is not sufficient, this is a clue that suggest that the gravitational mechanisms may not be convergent.[18] In the last part of his article, he goes further and tries to show that these mechanisms are divergent.

Steedman's main argument seems to be that even if one ignores the non linear terms in his price adjustment equations, the convergence of the system could only be proved under "certain conditions", and those conditions cannot be defended on economic grounds that they would hold in all or most plausible cases. However, as no one knows what these conditions are, it seems too hasty to rule them out without consideration.

Furthermore, Steedman uses matrices D and S as weights for his demand and supply equations respectively. Without such weights his gravitational mechanisms would not be completely described. In fact, if one compares his equations to ours, one sees that these matrices should be the linear parts of the functions F and G together with the projections π_L and π_e. However, each of Steedman's matrices S and D has "positive diagonal (and perhaps a dominant diagonal)" (p.135). This leads to an alternative for which in each case a problem arises:

- if these matrices have dominant positive diagonals, they would be invertible. Therefore, there is no projections in these matrices. This means that Steedman is not keeping total labour equal to 1.[19]
- if they do not have a dominant diagonal, then they are too general to be meaningful for the gravitational mechanisms. And they still could be invertible so that the total labour is not kept to 1.

It seems that this is the reason why Steedman could not conclude: his matrices are either the wrong ones or too general to draw a conclusion.

[18]Indeed, these gravitational mechanisms where thought first to be convergent due to partial equilibrium reasoning.

[19]We have seen above, 2.2, that the total amount of labour must be kept to 1 for a proper description of Sraffa's systems. Suppose for a moment that the total amount of labour could be changed during the adjustment process then it is possible that during the process the system as a whole could shrink to zero or keep rising toward infinity.

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