# Confronting sraffian and marxian categories 

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#### Abstract

Starting from a sraffian physical commodities system, categories as circulating and variable capital, are calculated using as aggregator the labour values of the system. Using, again, the sraffian system where the physical commodities have already been expressed in labour units, the common - or equalized - rate of profit and the production prices are jointly calculated in the same joint way as Sraffa calculates the prices and the rate of profit, instead of obtaining such production prices from the direct redistribution of plus value among sectors used by Marx. In doing so, the two equalities (output and profit in labour units and production prices) do not hold, as it is known, but the modifications introduced by the newly calculated production prices are studied allowing to draw consequences of the comparison among these two different systems, namely: the distortion of the real labour value system introduced by the price of production system and its degree of dependence. One basic distortion is that the use of production prices - which following the marxian procedure, are calculated with data expressed in labour units - produce an increase - in most cases - of profits over existing plus value at the expenses of the circulating and variable capital if the total output in terms of prices and in terms of labour values are equated.


## Introduction

## Creating subsystems of wage and circulating capital production

The concept of sraffian sub-system is used to identify in the general, unit-producing system, the parts of the system that create the wage goods and the circulating capital needed. More specifically, the subsystems are used to calculate the quantity of every good and service needed to produce the actual system's wage and similarly, the quantities of every commodity that produce the circulating capital goods and services ${ }^{1}$. Once we have done it, we are able to substitute the original Leontief matrix by a $2 \times 2$ matrix, where the rows and columns are the circulating and variable capital inputs, required to produce the two sectors' output: the one producing the wage and the other producing the capital used. In order to express the inputs and outputs no longer in physical quantities but in labour values, prior to any aggregation to $2 \times 2$ matrix we must calculate and have the original labour values with which we are able to aggregate different commodities, independently of any price system. From this point, it is possible to obtain conjointly the common rate of profit and the corresponding marxian production prices - newly calculated and not based in an weighted average rate of profit of the plus value as Marx did -. They are, afterwards, confronted with the labour values. The mechanisms by which they differ are analyzed. Due to the introduction of production prices, redistribution of the generated plus value between both sectors does not match exactly sraffian profits if the general output have also to match

[^0]simultaneously the general output in terms of value ${ }^{2}$ as it is known, regardless of the input coefficients matrix and output being expressed in terms of labour. If the total output in prices and labour units are equated, the consequence of the production prices is that the real capital is reduced - in most cases - in favour of the profits. The rate of profit obtained in terms of prices would normally be not lower than the one in terms of labour values calculated with the Marxian procedure. In addition, both systems are not independent as far as they share the input matrix and the existence of a wage. If it does not consumes all the surplus then there is place for profits and plus value. One particular limitation is signalled, the real rate of profit in terms of prices should not exceed the one obtained with the existent plus value. There is only equilibrium in the standard system situation.

## Development

We use ${ }^{3}$, a Leontief matrix and the wage is expressed as a basket of goods. as follows:

$$
\left(\left[\begin{array}{lll}
a_{11} & a_{12} \cdots & a_{1 n} \\
a_{21} & a_{22} \cdots & a_{2 n} \\
a_{n 1} & a_{n 2} \cdots & a_{n n}
\end{array}\right]+\left[\begin{array}{ccc}
b_{1} l_{1} & b_{1} l_{2} \ldots & b_{1} l_{n} \\
b_{2} l_{1} & b_{2} l_{2} \cdots & b_{2} l_{n} \\
b_{n} l 1 & b_{n} l_{2} & b_{n} l_{n}
\end{array}\right]\right)=\mathbf{A}=\left[A+B L^{\prime}\right]
$$

$\mathrm{a}_{\mathrm{i}}, \ldots . \mathrm{a}_{\mathrm{ni}}$ (column of the matrix), are the necessary inputs to produce a unit of product i B is the column vector that corresponds to the basket of goods where components: $b_{1}, \ldots$ bn, are the quantities of good, 1 to n , that need to be consumed to reproduce a unit of work
$L^{\prime}$ is a row vector where components $l_{1}, \ldots, ., \ln$, are the quantities of work needed to produce a unit of product of branches, 1 to n

Assumptions are, $\mathbf{A}$ is a non-decomposable and productive matrix; a case of simple production is considered and only circulating capital exists. Every good or service is a basic commodity. With the known matrix $A$, vector $B$ and manpower $L$ in our example we can calculate relative prices in terms of wage and the labour values - see example below -.

The price system is calculated as follows:
$P^{\prime}=\left(P^{\prime *} \mathbf{A}^{*}(1+r)\right.$
Where
$\mathrm{P}^{\prime}$, is the row vector of prices
$r$, is the rate of profit
If the unit taken is the current wage, the price systems can be expressed as follows:

$$
P_{w}{ }^{\prime}=\left(P_{w}{ }^{\prime *} A+l^{\prime}\right) *(1+r)
$$

The labour values are calculated this way:

$$
\mathbf{N}=L^{\prime}(I-A)^{-1}
$$

[^1]We proceed to calculate the marxian circulating and variable capital C and V which converts the original n industries system in a two sector system when aggregated by means of the known labour values or prices.
$C_{S}=[A] * q_{S}$
$V_{S}=\left[B L^{\prime}\right] * q_{S}$
$\mathrm{q}_{\mathrm{s}}$ represents the column vector of wage goods and services needed to produce one unit of products in the system and thus Cs and Vs are the resulting vector of quantities of goods needed as inputs to produce such vector.

Similarly, if $\mathrm{q}_{\mathrm{c}}$ is the vector of quantities of circulating capital goods that are needed to produce a unit of product in the system, the inputs that produce such vector are as follows:
$C_{c}=[A] * q_{c}$
$V_{c}=\left[B L^{\prime}\right] * q_{c}$
The aggregates are finally obtained multiplying these vectors by the corresponding row vector of commodity prices or labour values.
$P^{1 *} C_{S}=\left.P * * A\right|^{*} q_{S}$
$\Lambda * C_{S}=\Lambda * A * q_{S}$
and so on.
Using the former unit we can obtain the input matrix in terms of prices

$$
\mathrm{M}_{\mathrm{p}}=\left[\begin{array}{ll}
C p c & C p s \\
V p c & V p s
\end{array}\right]
$$

or its equivalent in terms of value using the labour values,
$\mathrm{M}_{\mathrm{v}}=\left[\begin{array}{ll}C_{v c} & C_{v s} \\ V_{v c} & V_{v s}\end{array}\right]$
The resulting product obtained with these inputs in terms of prices is: $\mathrm{P} * \mathrm{q}_{\mathrm{c}}$ and $\mathrm{P} * \mathrm{q}_{\mathrm{s}}$ and similarly in terms of labour values.

## Example

The existing prices -in terms of current wage units - and labour values are: $\mathrm{pl}=2,11$, $\mathrm{P} 2=1,23$ and $\mathrm{P} 3=2,66$ and $\lambda_{1}=1,95 \lambda_{2}=1,1 \quad \lambda_{3}=2,51$ respectively. The rate of profit of A is 0,043 corresponding to an eigenvalue of 0,959 .
$\mathrm{A} * \mathrm{I}=\left[\begin{array}{ccc}0,3 & 0,2 & 0,2 \\ 0,1 & 0,1 & 0 \\ 0,1 & 0,2 & 0,05\end{array}\right] *\left[\begin{array}{l}1 \\ 1 \\ 1\end{array}\right]=\left[\begin{array}{c}0,7 \\ 0,2 \\ 0,35\end{array}\right]=\mathrm{q}_{\mathrm{c}}$
$\mathrm{BL}^{*} * \mathrm{I}=\left[\begin{array}{ccc}0,1 & 0,01 & 0,2 \\ 0,1 & 0,01 & 0,2 \\ 0,25 & 0,025 & 0,5\end{array}\right] *\left[\begin{array}{l}1 \\ 1 \\ 1\end{array}\right]=\left[\begin{array}{c}0,31 \\ 0,31 \\ 0,775\end{array}\right]=\mathrm{q}_{\mathrm{s}}$
Where $B^{\prime}$ is $[0,10,0,100,25]$ and $L$ is [ 10,102 ]
$\mathrm{M}_{\mathrm{p}}=\left[\begin{array}{ll}C_{p c} & C_{p s} \\ V_{p c} & V_{p s}\end{array}\right]=\left[\begin{array}{ll}1,125 & 1,080 \\ 1,418 & 1,888\end{array}\right] 4$
$\mathbf{I}^{\prime} *\left[\begin{array}{ll}C_{p c} & C_{p s} \\ V p c & V_{p s}\end{array}\right] *\left[\begin{array}{cc}\left(1+r_{c}\right) & 0 \\ 0 & \left(1+r_{s}\right)\end{array}\right]=\left[\begin{array}{l}q^{\prime} c \\ q^{\prime} s\end{array}\right]$
Dividing each column by the corresponding total sectoral output: $\mathrm{P}^{*} * \mathrm{q}_{\mathrm{c}}=2,654$ and $\mathrm{P}{ }^{*} \mathrm{q}_{\mathrm{s}}=3,0969$ the coefficients to produce a unit of product in terms of prices are obtained:
$M_{p u}=\left[\begin{array}{ll}0,4239 & 0,3489 \\ 0,5345 & 0,6099\end{array}\right]$
The total output is 5,750378 which corresponds to the inputs needed to produce a unit of every product which is the sum of prices and amounts 6 .

The eigenvalue and eigenvectors of the $\mathrm{M}_{\mathrm{pu}}$ matrix are as follows:
Inverse of the Eigevalue $=1,0403079(1 / 0,9587)$
Eigenvectors:
$\mathrm{P} 1=0,7069$
$\mathrm{P} 2=0,7073$,
or relative prices of the new aggregates

[^2]q1 $=0,5465$
$\mathrm{q} 2=0,8375$,
or quantity coefficients that modify the sectoral output so as to allow a proportionality of outputs and inputs in each sector.

## Joint calculation of marxian production prices and rate of profit following the sraffian procedure

We proceed to detail the relations of the labour values and the marxian price of production. Instead of using the physical inputs matrix, we will use it expressed in labour units and then aggregated to the marxian two sectors. We will apply the same procedure of Sraffa to calculate a uniform rate of profit and prices - at the same time for the derived two sectors system created using the previous labour values as aggregators. The result is labour values converted to labour production prices that allow a uniform - common - rate of profit among sectors which is a different procedure that the standard marxian calculation. We can see that even if we use only labour quantities the production prices derived does not maintain the output and profit proportionalities with labour values.

Taking the labour inputs obtained for the two sectors:
$\mathrm{M}_{\mathrm{v}}=\left[\begin{array}{ll}C_{v c} & C_{v s} \\ V_{v c} & V_{v s}\end{array}\right]=\left[\begin{array}{ll}1,043 & 1,003 \\ 1,324 & 1,763\end{array}\right]$
And dividing by the output in terms of labour values in each sector, 2,4635and 2,8907, amounting 5,35435 in total, it is obtained the matrix with unit coefficients:
$M_{\mathrm{vu}}=\left[\begin{array}{ll}0,4233 & 0,3470 \\ 0,5375 & 0,6099\end{array}\right]$
The eigenvalue and eigenvectors of this matrix are as follows:
Inverse of the Eigenvalue $=1,040329(1 / 0,9585)$
Eigenvectors:
Relative production prices
P1=0,7087
P2 $=0,7056$
Multipliers to create the standard system
$\mathrm{q} 1=0,5441$
$\mathrm{q} 2=0,839$

Modifying the input coefficients through price system

Giving a closer look to what we have done, we observe that the original labour value system in which the units employed were equal in the whole system: i.e. the labour content, we have attributed different prices to each sector so as to allow for a unified rate of profit that produces, in an automatic way, the redistribution of plus value.
It is expressed in the following example and formulas, which begin with the labour system with its sectoral rates of profit and continue with the application of the above relative production prices to obtain a new system with redistribution of plus vale although not exactly the one produced as we will see -.

$$
\begin{align*}
& (1,043+1,324) *(1,0406)=2,463 \\
& (1,003+1,763) *(1,0449)=2,890 \\
& \left(C_{c}+V_{c}\right) *\left(1+r_{c}\right)=q_{c}  \tag{3}\\
& \left(C s+V_{s}\right) *\left(1+r_{s}\right)=q_{s} \\
& (0,739+0,934) *(1,04329)=1,745 \\
& (0,711+1,244) *(1,04329)=2,039 \\
& \left(P_{p p c} * C_{c}+P_{p p s} * V_{c}\right) *(1+r)=P_{p p c} * q_{c} \\
& \left(P_{p p c} * C_{s}+P_{p p s} * V_{s}\right) *(1+r)=P_{p p s} * q_{s}
\end{align*}
$$

If we now make the output in prices equal to the output in value and apply this ratio to the rest of quantities in the last two equations (which is only a matter of changing the prices 0,7087 and 0,7056 by 1,002 and 0,997 which have the same proportion) we go from the initial price production system:
(Example: matrix of inputs expressed in production prices)
)

| Sector | C | V | $\mathrm{C}+\mathrm{V}$ | Output |
| ---: | ---: | ---: | ---: | :--- |
| Circulating <br> capital | 0,739191818 | 0,93432024 | 1,673512 | 1,74597 |
| Variable <br> capital | 0,711015323 | 1,244136499 | 1,955152 | 2,039804 |
| Total sectors | 1,45020714 | 2,178456739 | 3,628664 | 3,785773 |

to a new system where the total output in prices equals the total output in labour values:
(Example: matrix of inputs expressed in production prices as above but maintaning total output equal to total output in labour values - 5,35 - )

| Sector | C | V | $\mathrm{C}+\mathrm{V}$ | Output |
| ---: | ---: | :--- | :--- | :--- |
| Circulating <br> capital | 1,045444956 | 1,32141666 | 2,366862 | 2,469339 |
| Variable <br> capital | 1,005594712 | 1,759592297 | 2,765187 | 2,884911 |
| Total sectors | 2,051039669 | 3,081008956 | 5,132049 | 5,35425 |

We can observe that in order to allow an identical rate of profit for each sector we have but finally modified the labour quantities converting the above matrix $\mathrm{M}_{\mathrm{v}}(1)$ and $\mathrm{M}_{\mathrm{vu}}$ (2) in the following way:
$\mathrm{M}_{\mathrm{v}}$ corresponds to:
$M_{p p}=\left[\begin{array}{ll}1,045 & 1,003 \\ 1,321 & 1,763\end{array}\right]$
and $\mathrm{M}_{\mathrm{vu}}$ corresponds to:
$M_{p p u}=\left[\begin{array}{ll}0,4233 & 0,3485 \\ 0,5351 & 0,6099\end{array}\right]$
The price system has, thus, acted as a modifier of the physical inputs matrix and, now, if we calculate with it a "new production prices and rate of profit" we can see that both prices are identical: 0,7071 and that the rate of profit is the former one.

New production prices using new unit coefficients matrix

| Sectors | C coefficients | V coefficients | (sum) | $\begin{array}{r} \text { Unit } \\ \text { prices } \end{array}$ | $\begin{array}{r} \hline \mathrm{C}+\mathrm{V} \text { in unit } \\ \text { prices } \\ \hline \end{array}$ | Rate of profit | $\begin{array}{r} \text { Unit } \\ \text { output } \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Circulating capital | 0,423370333 | 0,535129667 | 0,9585 | 0,7071 | 0,67775535 | 1,043297 | 0,7071 |
| Variable capital | 0,348570469 | 0,609929531 | 0,9585 | 0,7071 | 0,67775535 | 1,043297 | 0,7071 |

## Distortion of the value of circulating and variable capital and plus value in the same quantity

We are now able to compare $\mathrm{M}_{\mathrm{v}}$ (1) and $\mathrm{M}_{\mathrm{pp}}$ (4) this is to say, the original input matrix in labour values and the one after a redistribution of plus value have been produced:

Original labour value input matrix

| Sector | C | V | $\mathrm{C}+\mathrm{V}$ | Output | Plus value | Rate |
| ---: | ---: | ---: | :--- | ---: | ---: | ---: |
| Circulating <br> capital | 1,043025 | 1,32415 | 2,367175 | 2,4635 | 0,096325 | 0,040692 |
| Variable capital | 1,003267 | 1,763232 | 2,766499 | 2,89075 | 0,124251 | 0,044913 |
| Total sectors | 2,046292 | 3,087382 | 5,133674 | 5,35425 | 0,220576 | 0,042966 |

Production prices input matrix

| Sector | C | V | $\mathrm{C}+\mathrm{V}$ | Output | Plus value | Rate |
| ---: | ---: | ---: | :--- | :--- | :--- | :--- |
| Circulating <br> capital | 1,045444956 | 1,32141666 | 2,366862 | 2,469339 | 0,102477576 | 0,043297 |
| Variable capital | 1,005594712 | 1,759592297 | 2,765187 | 2,884911 | 0,119723799 | 0,043297 |
| Total sectors | 2,051039669 | 3,081008956 | 5,132049 | 5,35425 | 0,222201375 | 0,043297 |

Differences between both matrix
$\left.\begin{array}{|r|r|r|l|l|l|l|}\hline \text { Sector } & \text { C } & & \text { V } & \text { C+V } & \text { Output } & \text { Plus value }\end{array} \begin{array}{l}\text { (Variation of } \\ \text { rates) }\end{array}\right]$

It can be seen that the profits in terms of labour values calculated by substracting $\mathrm{C}+\mathrm{V}$ from the sum of sectoral values resulting, $0,2205=((5,3542-5,1336)=$ (Total value)$(\mathrm{C}+\mathrm{V})$ ), differ slightly from the profit obtained in the production prices system: 0,2222 .

In other words, valuing commodities and services according to the production prices allows a variation with respect to the exact value contents of every commodity allowing a common rate of profit but distorting the two equalities that would be obtained if a direct redistribution of plus value according to the capital employed in each sector, following the marxian standard procedure had been done. In this case total labour values in output and in profits are proportional to its price valuation. ${ }^{5}$

The problem with this transformation is that the prices system's profit is higher - in this case - from the real one expressed by the plus value, and that the sum of circulating and variable capital is lower in the same quantity, creating a disfunction with what is happening in reality as expressed by values.

It is significative observing that if we had lowered the wage goods in the example used we would have obtained the same result as the calculated prices of production and uniform rate of profit.

## Differences in rates of profit in terms of value and in terms of prices

An important observation is that the rate of profit in terms of value would be lower - or at least equal - than the one in prices in most cases as we discuss in the following paragraphs.
Expressing the two sectors with the marxian weighted average rate and afterwards introducing in them the production prices we have:

$$
\begin{aligned}
& \left(C_{c}+V_{c}\right) *\left(1+r_{\text {average }}\right)=q^{\prime}{ }_{c}>q_{c} \\
& \left(C_{S}+V_{S}\right)^{*}\left(1+r_{\text {average }}\right)=q_{s}^{\prime}<q_{S} \\
& \left(q^{\prime}{ }_{c}+q^{\prime}{ }_{S}=q_{c}+q_{S}\right) \\
& \left(C_{c}+\frac{P_{p p s}}{P_{p p c}} * V_{c}\right) *(1+r)=q_{c} \\
& \left(\frac{P_{p p c}}{P_{p p s}} * C_{s}+V_{S}\right) *(1+r)=q_{s}
\end{aligned}
$$

Provided that we come from values where there is only one common "price" or unit without profit rates equalization - and that in order to find a common rate of profit it is needed to vary the existing common unit giving a different price to each branch, the solution to compensate the different rates would be making the lower price sector the one corresponding to the higher rate in terms of labour - see (3), so as to decrease the rate in this branch and increase in the other, as follows:

[^3]\[

$$
\begin{aligned}
& \left(C_{c}+\frac{P_{p p s}}{P_{p p c}} * V_{c}\right) *\left(1+r_{c}\right)<q_{c} \\
& \left(\frac{P_{p p c}}{P_{p p s}} * C_{s}+V_{s}\right) *\left(1+r_{s}\right)>q_{s}
\end{aligned}
$$
\]

Thus, the lower price of the sector with higher rate of profit reduces the inputs needed in the other branch and conversely, the correspondingly higher price of the second sector increases the price of the inputs in the sector with higher rate, as it has happen in the example.
It seems to be only one intermediate solution situated between both rates ${ }^{6}$ being higher in most cases or at the limit equal, to the average one in labour values. That specific case would require that the relative increase in the second price be compensated by the decrease in the first one, similarly to when the average rate of profit is calculated in the marxian way. It is difficult to think in a reduction of the first price that would provoke an increase in the second - higher - price so as to make the total inputs greater than the previous ones; and bearing in mind that the output remains equal it can be expressed that way:

$$
\left(C_{c}+\frac{P_{p p s}}{P_{p p c}} * V_{c}\right)+\left(\frac{P_{p p c}}{P_{p p s}} * C_{s}+V_{s}\right)<\left(C_{c}+V_{c}\right)+\left(C_{s}+V_{s}\right)
$$

A simplified condition for the rate to be higher than the average is as follows:

$$
\begin{equation*}
\frac{C_{s}}{V_{c}}<\frac{P_{p p s}}{P_{p p c}} \tag{6}
\end{equation*}
$$

If we compare the sectors rates - where the sector producing C has more inputs per output and hence, less rate of profit - and move quotients from both sides as follows:
$\frac{C_{c}}{q_{c}}-\frac{C_{s}}{q_{s}}>\frac{V_{s}}{q_{s}}-\frac{V_{c}}{q_{c}}$
,it can be seen that the sector that gives more input in relation to one sector's output, to the other sector, is the one with more profitability - it keeps proportionally less of his own product as input and, moreover, it receives in return `proportionally less from the other sector - and this is the case of sector producing the wage goods, V in our example. This makes more probable that in the left quotient of condition (6) the denominator numerator $\mathrm{V}_{\mathrm{c}}$ be higher than the numerator $\mathrm{C}_{\mathrm{s}}$ and hence lower than unity. Also the sector with more profitability is the one with the "assigned" lower relative price, as we have said, and the second part of the inequality in condition (6) is also lower than unity. Being so, it not expected a price in sector C which, in order to decrease the rate in sector V , increases so much its price making the common rate lower than the average one in labour terms.

[^4]
## Profit and plus value

Be the prices - taken in wage units - and labour values represented by these two expressions:

$$
\begin{aligned}
& P^{\prime}{ }_{w}=(1+r) L^{\prime *}\left(I+(1+r) * A+(1+r)^{2} * A^{2}+\ldots+(1+r)^{n} A^{n}+\ldots\right) \\
& \Lambda^{\prime}=L^{\prime *}\left(I+A+A^{2}+\ldots+A^{n}+\ldots\right)
\end{aligned}
$$

or similarly:

$$
\begin{aligned}
& P^{\prime}{ }_{w}=L^{\prime *}(I-(1+r) A)^{-1} \\
& \Lambda^{\prime}=L^{\prime *}(I-A)^{-1}
\end{aligned}
$$

in which, as long as there is a positive profit rate, it always hold as it is already known that:

$$
P_{w}^{\prime} \mid \ \Lambda^{\prime}
$$

Substracting both expressions, this is to say, the sum of prices and the sum of values and knowing also that prices are equal to values when there is no profit - , we obtain the expression of profit:

$$
P^{\prime}{ }_{w}-\boldsymbol{\Lambda}^{\prime}=\text { profits }
$$

There is a need for plus value to exist to have real profit and there should be a precise relation between profits in the price system with the plus value in the labour values system so as to profits to have real content. Both price and labour systems share the coefficient matrix and the existence of a wage that if does not consume all the surplus enables the existence of both profit and plus value.

Be $\Lambda^{\prime} B$ the expression of the unit wage and $1-\Lambda^{\prime} B$ the plus value rate,
the total system plus value is:

$$
\Lambda^{\prime}-\Lambda^{\prime *} \Lambda^{\prime} B=\Lambda_{\text {plusvalue }}{ }^{\prime}
$$

which is also equal to the following expression:

$$
\left.\mathbf{\Lambda}_{\text {plusvalue }}=\left(1-\mathbf{\Lambda}^{\prime} B\right) * L^{\prime} * I+\left(1-\mathbf{\Lambda}^{\prime} B\right) * L^{\prime} A+\left(1-\mathbf{\Lambda}^{\prime} B\right) * L^{\prime} A^{2}+\ldots+\left(1-\mathbf{\Lambda}^{\prime} B\right) * L^{\prime} A^{n}+\ldots\right)
$$

, representing the unpaid o retained part of the labour force L employed.
Conversely to the sraffian rate of profit, the rate of plus value is additive and does not follow a power as with r .

Connecting prices and values means that the profit should be based on an existing plus value as follows:

$$
P_{w}^{\prime}-\Lambda^{\prime}=L^{\prime *}(I-(1+r) A)^{-1}-L^{\prime *}(I-A)^{-1}=\left(1-\Lambda^{\prime} B\right) * L^{\prime *}(I-A)^{-1}
$$

$$
\begin{align*}
& L^{\prime}=\left(2-\Lambda^{\prime} B\right)^{*} L^{\prime *}(I-A)^{-1} * I-\left(2-\Lambda^{\prime} B\right)^{*} L^{\prime *}(I-A)^{-1} * A^{*}(1+r) \\
& L^{\prime}-\left(2-\Lambda^{\prime} B\right)^{*} L^{\prime *}(I-A)^{-1} * I=-\left(2-\Lambda^{\prime} B\right)^{*} L^{\prime *}(I-A)^{-1 *} A \\
& -\left(2-\Lambda^{\prime} B\right) * L^{\prime *}(I-A)^{-1} * A^{* r} \\
& \frac{L^{\prime}}{\left(2-\Lambda^{\prime} B\right)}-L^{\prime *}(I-A)^{-1}+L^{\prime *}(I-A)^{-1 * A=-L^{\prime *}(I-A)^{-1} * A^{*} r} \\
& r=\frac{\left(L^{\prime *}(I-A)^{-1}-L^{\prime *}(I-A)^{-1} * A-\frac{L^{\prime}}{2-\Lambda^{\prime} B}\right) * I}{\left(L^{\prime *}(I-A)^{-1} * A\right) * I}=\frac{\left(\Lambda^{\prime}-\Lambda^{\prime *} A-\frac{L^{\prime}}{2-\Lambda^{\prime} B}\right) * I}{\left(\Lambda^{\prime *} A\right)^{*} I} \tag{7}
\end{align*}
$$

## Concluding remarks

Questions about the relation of a sraffian and a marxian system have been dealt. Our starting point is labour values represent better than any other system the real costs and profits of the economy no matter how the sraffian prices system present it. The deviation from it must have therefore consequences: the profits, rate of profit, output should not differ from its labour counterpart, since equilibrium among these elements only happens in the situation of the sraffian standard system. Despite being true that the prices and values share the physical conditions of production, as Steedman indicates, with the LTV the conditions of the economy are better expressed than with the price system and it is so because the latter is an alteration of the former system so as to reach an equalized rate of profit.
As a consequence, when confronting labour values and prices the former express better than prices the real rate of profit which is not dependent on a set of prices necessary for the equalization of the rate between sectors.
Secondly the prices system can not be independent of the labour system. Steedman shows that both labour values and prices systems share the physical conditions of production, and is right signalling that the rate of profit (and prices) determination are based on the physical conditions - including labour - and the real wage; it have to be said, however, that the last condition, the real wage, is not an explicit element in the sraffian system where the wage is the percentage of the surplus which is not profit and it is empty of a real wage concept. Instead, the labour theory of value (LTV) introduces the necessary labour force as a physical element and particularly the real wage in form of the cost of the unit labour force. This is a key element for the determination of the rate of profit as Steedman acknowledges. It is worth noting, as well, that the surplus less the plus value is the real wage.
This is not a nominal discussion ${ }^{7}$, because the inclusion of the real wage in the prices and rate of profit determination make it very close to the value system from which there

[^5]is only the difference of the equalisation - particularly when the matrix coefficients have been expressed in terms of labour-.
Else, the rate of profit would appear as if it was a tax over the labour values with no relation with them. Therefore, only the consideration of a real wage makes it possible to determine the plus value and gives content of the sraffian wage and profits. The effect of prices of production can be compared to a change in physical coefficients - reducing some coefficients matrix and increasing others to produce a common rate of profit -. This change may have been produced also by means of other economic procedures as a change in the real wage, an appreciation or depreciation of constant capital, without reference to a prices system.
Therefore, for theses two reasons- representing the costs and profits of the system and incorporating the real wage -, there is no sraffian profit and wage categories autonomy from labour value. The profits have to rely on the difference between labour values and the workforce labour value. The rate of profit equation above (7), where it is expressed in terms of labour values, show that in order the profit to have full meaning it has to adapt to this fixed rate ${ }^{8}$. So, we do not agree with the lack of importance of the LTV declared by Steedman or with Vegara when he states that "..."it lacks theoretical significance to relate plus value with profits" pp 141.
Besides that, the use of aggregating concepts as variable capital and circulating capital are important for the analysis, instead of having to deal with different individual commodities. In this sense, we disagree with Steedman when he says (pp63) that aggregation in terms of $\mathrm{C}, \mathrm{V}$, etc., is only valid if all elements had the same price-value coefficient. The value magnitudes are not based in price considerations only in physical conditions - as Steedman himself says - and are absolute. That is a difference with prices - which are relative - which in turn make it possible the equalisation of profit rate.
This previous discussion introduces a new perspective to the lack of proportionality of global output and profits in terms of both the new production prices and labour values, because this, far from being a problem, is an opportunity to extract consequences from it, as has been shown. The contradiction between the maintenance of a real profit and the maintenance of a global output have to be solved in a standard system, otherwise the system falls in a contradiction between its core values and its price expression.

[^6]
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[^0]:    ${ }^{1}$ Instead of [the subsystem concept] being used to produce a unit of a given good while the rest of the goods or services are in a self-replacing state, as Sraffa does

[^1]:    ${ }^{2}$ With the known key exception of the sraffian standard system, where inputs and outputs are themselves proportional. In this situation there is no waste of outputs because they are produced in the exact proportion in which they are used and needed.
    ${ }^{3}$ Terminology and examples of Josep Maria Vegara i Carrió, Economía Política y Modelos Multisectoriales, Biblioteca Tecnos de Ciencias Económicas, Madrid: Editorial Tecnos, 1979, 190 pp. 84-309-0795-5.

[^2]:    ${ }^{4}$ These are the inputs necessary to produce the wage and circulating capital goods used in the unit producing system. This output amount 5,75 price units and not to the total 6 price units obtained by adding the three unit prices of the system indicated above: 2,11 $1,232,66$. The price amount of the total inputs is 5,5134 . This is due to the fact that the global output produced is not the unit but the sum of the two subsystems.

[^3]:    ${ }^{5}$ This procedure is also valid when inputs and outputs are present in the same proportion in the system.

[^4]:    ${ }^{6}$ And that the price in the higher rate sector have to be always the lower one, otherwise with the production prices relation being inverse in the two equations if the proportion of prices follow an opposite direction they would inevitably diverge

[^5]:    ${ }^{7}$ Steedman would indeed be right criticizing that the existence of surplus value be a necessary and sufficient condition for profits when both come from the same conditions, i.e. the conditions of profitability of the systems. This is to say, of the system's capability to produce a physical net product, and hence, the surplus can not be a condition for profits to exist.

[^6]:    ${ }^{8}$ A set of prices can then be calculated according to that specific rate. In the example, the rate of profit is 0,0776 and prices associated to this rate are:2,0878,1,2419, 2,5895. Plus value is 0,3699 . We disagree with Steedman's affirmations: "The rate of profit is not other than by a fluke, equal to $S /(C+V)$ " and "No value magnitude plays any significant role in determination of the profit rate (or the prices of production) '. pp 65

