From Adam Smith to the Dynamics of the Profit Rate

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The fundamental structure of a scientific theory may be quite old; to take an example, last year was the 150th anniversary of Darwin's greatest work. But though it is 150 years old Darwin's theory of evolution remains very *fertile*. Basing themselves on its premises biologists are able to come up with new non-obvious but testable propositions about the biological world.

We contend that Adam Smith's work on economics, even older than that of Darwin, remains in this sense *fertile*. We can use it to make new predictions about the economy that are both testable and non-obvious. In particular we will show that starting from certain premises of Smith we can derive a formula for the rate of profit that is not at all obvious, but which when tested gives excellent results.

1 Smithian Premises

From Smith's work we draw the following premises:

1. That labour is the source of value.
2. That capital or 'stock' is the accumulated result of past labour.
3. That unproductive expenditure impedes the accumulation of capital.
4. That the development of the productivity of labour drives down the values of commodities.

Other classical economists, including Ricardo and Marx, shared these premises and were also concerned with the trajectory of the rate of profit. We make some references to Marx's version of the theory below.

2 Development of the premises

That labour is the source of value

Smith argued that the reality of exchangeable value was labour. Labour was the 'original currency' by means of which we purchase all our wants and necessities from nature, and things are only really valuable in terms of labour—so that when gold purchases commodities what it is really doing is purchasing the work that went into these commodities.

> What is bought with money or with goods is purchased by labour, as much as what we acquire by the toil of our own body. That money or those goods indeed save us this toil. They contain the value of a certain quantity of labour which we exchange for what is supposed at the time to contain the value of an equal quantity. Labour was the first price, the original

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> purchase-money that was paid for all things. It was not by gold or by silver, but by labour, that all the wealth of the world was originally purchased; and its value, to those who possess it, and who want to exchange it for some new productions, is precisely equal to the quantity of labour which it can enable them to purchase or command. (Smith, 1974, p. 133)

If an ounce of gold at a particular time and place has a definite value in terms of labour, that implies that any bundle of goods its owner chooses to purchase with the ounce will contain roughly the same amount of labour. Goods with a high money price will contain much labour. Those that are cheap in terms of gold will contain less labour.

> At all times and places, that is dear which it is difficult to come at, or which it costs much labour to acquire; and that cheap which is to be had easily, or with very little labour. Labour alone, therefore, never varying in its own value, is alone the ultimate and real standard by which the value of all commodities can at all times and places be estimated and compared. It is their real price; money is their nominal price only. (Smith, 1974, p. 136)

Profits may be measured in money, but for Smith this is only a 'nominal measure'. The real measure of profit, or any other sum of money, is the amount of embodied labour that it will exchange against or command.

> Wealth, as Mr Hobbes says, is power. But the person who either acquires, or succeeds to a great fortune, does not necessarily acquire or succeed to any political power, either civil or military. His fortune may, perhaps, afford him the means of acquiring both; but the mere possession of that fortune does not necessarily convey to him either. The power which that possession immediately and directly conveys to him, is the power of purchasing; a certain command over all the labour, or over all the produce of labour which is then in the market. His fortune is greater or less, precisely in proportion to the extent of this power, or to the quantity either of other men's labour, or, what is the same thing, of the produce of other men's labour, which it enables him to purchase or command. (Smith, 1974, p. 134)

The process of acquiring profit is thus a process of acquiring social power, the power to command labour.

If labour is the source of value—and the real measure of the value of a commodity is the labour that it will command—it follows that the value created in an economy will be proportional to the number of workers and how long they work. If we take a certain number of working hours per person per year to be the norm, we can express the labouring capacity, and hence the value creating capacity, of an economy in terms of the number of full-time equivalent workers it has. Note that when we do this we are abstracting from the problem of the differential valuation of the various nations' labours on the world market. This is legitimate if we are looking at the internal rate of profit of the economy.

From these assumptions we obtain our first equation. Let P denote the total profit in the economy, N the number of full-time equivalent workers, and w the wage expressed as the fraction of a full-time equivalent working year required to produce the goods consumed by the average labourer. We than have

$$P = (1 - w)N \tag{1}$$

Note that the dimension of P is millions of full-time person years per annum, which reduces to millions of full-time equivalent persons. We can view this as the population which produces those goods purchased out of profits. In Smithian terms, the reality of profit is the millions of people whom it commands.

That stock is accumulated labour

If we follow Smith and view stock as the accumulation of past labour, then it can be valued in terms of working years. The total stock of the nation will be expressible as so many million working years, accumulated over previous years. The rate of profit per annum, r, is then given by

$$r = \frac{P}{K} \tag{2}$$

where K is the stock measured in millions of person years. The units of r are persons/person-years which gives dimension t^{-1}, as we would expect for a rate of profit.

It is clear that r will fall if the rate of growth of K exceeds the rate of growth of P, and will rise if the converse holds. But it is also clear from equation (1) that the main determinant of the rate of growth of P will be the growth of the working population. A secondary influence will be any tendency for the wage share, w, to change over time. Specifically, we might expect w to fall if the working time required to produce the workers' means of subsistence falls as technical progress enhances the productivity of labour.

Why is movement of w secondary? Suppose the working population grows by 5% a year. If the wage share remains constant then by equation (1) total profit will also grow by 5%. But consider the effect of a reduction in the wage share: if w is initially 60% then a 5% reduction in the labour content of the real wage will produce a 3% increase in the profit rate; if the initial w is 40%, the same 5% reduction in the labour content of the wage will raise the rate of profit by 2%. In general, the further w is depressed the less significant is the growth in profit for a given proportional reduction in the labour time going as the real wage.

Besides, Smith points out that in countries with more productive labour, the real wage tends to rise. The great virtue of the division of labour is that it raises the living standards of even the meanest labourer. Contemplating the many and various articles used or consumed by even "the most common artificer or day-labourer" in the society of his age, Smith says

> [I]f we examine, I say, all these things, and consider what a variety of labour is employed about each of them, we shall be sensible that, without the assistance and co-operation of many thousands, the very meanest person in a civilized country could not be provided, even according to, what we very falsely imagine, the easy and simple manner in which he is commonly accommodated. Compared, indeed, with the more extravagant luxury of the great, his accommodation must no doubt appear extremely simple and easy; and yet it may be true, perhaps, that the accommodation of an European prince does not always so much exceed that of an industrious and frugal peasant, as the accommodation of the latter exceeds that of many an African king.... (Smith, 1974, p. 117)

Thus we would argue that it is in the spirit of Smith to assume w is relatively stable. In that case the proportional rate of change of profit is given by the rate of growth of the population, n,[1]

$$\frac{\dot{P}}{P} = n \tag{3}$$

Population, on the other hand, is not stable. In the early stages of capitalist development it grows very rapidly. Later, with the elevation of the social status of women and with education becoming more important, family sizes shrink. In highly developed capitalist countries the population stabilizes or even starts to decline, in the phenomenon known as demographic transition.

What is the implication of this? If population stabilizes, then by equation (2) any ongoing increase in the stock of capital must tend to depress the rate of profit. Marx (1971) argued that the rate of profit would tend to decline on various grounds. Among these grounds he recognized this population effect (under the title "absolute over-accumulation of capital") but gave it relatively little emphasis, presumably because demographic transition had not yet occurred when he was writing *Capital*.

What then influences the stock of capital?

[1] The dot notation denotes a time-derivative: $\dot{x} \equiv dx/dt$.

That unproductive expenditure impedes the accumulation of capital

One of Smith's great concerns was the promotion of productive industry. Chapter III of Book II of *The Wealth of Nations* is titled "Of the Accumulation of Capital, or of Productive and Unproductive Labour". Here he distinguishes between the labour of the manufacturer which

> fixes and realizes itself in some particular subject or vendible commodity, which lasts for some time at least after that labour is past. It is, as it were, a certain quantity of labour stocked and stored up to be employed, if necessary, upon some other occasion (Smith, 1974, p. 430)

and the labour of the menial servant which

> on the contrary, does not fix or realize itself in any particular subject or vendible commodity. His services generally perish in the very instant of their performance, and seldom leave any trace or value. . . . (Smith, 1974, p. 430)

This distinction is relevant to accumulation because

> [t]hat part of the annual produce of the land and labour of any country which replaces a capital never is immediately employed to maintain any but productive hands. It pays the wages of productive labour only. That which is immediately destined for constituting a revenue, either as profit or as rent, may maintain indifferently either productive or unproductive hands. (Smith, 1974, p. 432)

Consider the situation in which the entire profit is directed to the employment of productive labour and thus fixes itself in a material product that lasts beyond the execution of the labour. If the profit were 5% per annum, the annual growth of stock would likewise be 5%. If, on the other hand, 40% of the profit were spent unproductively then the growth of stock would be only 3% per annum. This principle of Smith's thus provides us with a first approximation to the proportional growth rate of stock.[2] Let u denote the portion of profit that is spent unproductively. We then have

$$\frac{\dot{K}}{K} \approx (1-u)\,r \tag{4}$$

But this is not the only factor that affects the capital stock. We have to take into account the part of the produce that is "destined for replacing a capital, or for renewing the provisions, materials, and finished work, which had been withdrawn from a capital", or what we would now call depreciation. In addition there is the technical devaluation of stock. That is, capital goods in place lose value as the replacement items become cheaper due to technical advance; this is most obvious for computer equipment. Although Smith does not emphasize this point, it can be deduced from other parts of his theory.

That the development of the productivity of labour drives down the values of commodities

> Labour, therefore, it appears evidently, is the only universal, as well as the only accurate measure of value, or the only standard by which we can compare the values of different commodities at all times, and at all places. (Smith, 1974, pp. 139–40)

It follows that as time passes and as the productivity of labour improves, the real price of commodities falls. This will in turn affect the existing stock of capital in a country. If the productivity of labour grows

[2] Strictly speaking, the current argument requires a slightly modified version of Smith's criterion for productive labour, as found in Cockshott and Zachariah (2006).

rapidly then that nation's stock will be subject to an equally rapid devaluation. Let g denote the growth rate of labour productivity, and δ the rate of depreciation. We then arrive at a second estimate of the rate of change in stock over time.

$$\frac{\dot{K}}{K} = (1 - u)\, r - g - \delta \tag{5}$$

3 Equilibrium rate of profit

If a nation's profit grows at the same proportional rate as its stock ($\dot{K} = \dot{P}$), then the rate of profit will be stable, or in equilibrium. Is there some particular rate of profit that will bring about this equilibrium?

Combining equations (5) and (3) we get, under the assumption of a constant wage-share,

$$\frac{\dot{K}}{K} = \frac{\dot{P}}{P} = (1 - u)\, r - g - \delta = n \tag{6}$$

and from this we can readily derive an expression for the equilibrium or steady-state rate of profit:

$$r^{\star} = \frac{n + g + \delta}{1 - u} \tag{7}$$

What does an equilibrium rate of profit, $r^{\star}$, mean in this context? It is the rate of profit which is compatible with the current growth rates of technology and the labour force, and the current split in the usage of profit between productive and unproductive uses. The equilibrium rate itself will change given changes in the growth of the labour force, the growth of technology, or the share of unproductive expenditure. The important points are as follows.

- If more (respectively, less) of profit is productively reinvested the equilibrium rate will fall (respectively, rise). This may seem paradoxical, but the point is that faster accumulation increases the denominator of the profit rate, while unproductive consumption of the economic surplus limits that increase.
- If the pace of technological improvement increases, the rate of profit will rise.

The steady-state rate of profit is the rate towards which the actual rate of profit will tend. If $r_t > r_t^{\star}$ then we would expect to see $r_{t+1} < r_t$, whereas if $r_t < r_t^{\star}$ then we would expect that $r_{t+1} > r_t$.

It may be argued that the formula we have derived is essentially the same as the expression for the steady-state rate of profit that emerges from the standard Solow growth model (Solow, 1963), with Cobb–Douglas production function. That is, $r^{\star} = \alpha(n + g + \delta)/s$, where α is the Cobb–Douglas exponent on capital—which in this theory equals capital's share in output—s is the saving rate, and α/s plays the same role as $1 - u$ in our version of the argument. However, it is of interest to note that this relationship can also be derived from a classical starting point, as we have shown. Moreover, Shaikh (1974) has shown that the Cobb–Douglas function appears to fit the data well, as an algebraic artifact, whenever the data display a roughly constant wage share—regardless of the "true" underlying production function. (This is because the Cobb–Douglas equation with constant returns to scale is close to the accounting identity which states that total income equals capital income plus labour income, if the wage share is relatively steady.)

We would argue that the Smithian view gives a deeper insight into the underlying relations; in addition, a derivation of the steady-state rate of profit that does not carry the baggage of additional assumptions regarding the production function (e.g. smooth substitutability of factors and payment according to marginal product) is to be preferred on grounds of parsimony.

In relation to classical thinking, the novelty of our approach lies in the dynamic nature of the analysis of profit: classical models typically take a static approach (the Sraffian school being the pre-eminent example). Marx talks of a long-term tendency of the rate of profit to fall, but his analysis contains no dynamic equations that could be used to make actual predictions; we show below that the steady-state rate of profit can be used to predict the dynamics of the actual rate.

In addition, our approach leads to different conclusions regarding the effect of unproductive expenditures. In most Marxian approaches, unproductive expenditure appears as a deduction from profit, hence a negative entry in the numerator of the profit rate (Gillman, 1957; Shaikh and Tonak, 1996; Cockshott *et al.*, 1995; Cockshott and Zachariah, 2006). In our equation the key point is that profit consumed unproductively represents a deduction from *accumulation*, hence limiting the increase in capital stock. Note, however, that our Smithian definition of unproductive expenditure is not identical to the standard Marxian definition.

Illustration

Suppose that in 2000 the wage share of value added was $\frac{2}{3}$ in the UK and $\frac{1}{2}$ in China, and that the capital to labour ratio was $3\frac{1}{3}$:1 in the UK and 1:1 in China.[3] Using equations (2) and (1), the profit rate would then be $(1 - \frac{1}{3})/3\frac{1}{3} = 10$ percent in the UK, and $(1 - \frac{1}{2})/1 = 50$ percent in China. We now make some simplifying assumptions as follows.

1. The working population is fixed in both nations ($n = 0$).
2. In both nations 50% of profit is reinvested ($u = 0.5$).
3. The depreciation rate, δ is the same in the two countries, at 10%; and the growth rate of labour productivity, g, is 2.5%.
4. The respective wage shares of value-added do not change over time.

Equation (7) gives an equilibrium profit rate of $\frac{0.025+0.1}{0.5} = 25\%$. At this profit rate the additional investment is just sufficient to compensate for the physical and technical depreciation of the capital stock. In the UK the capital stock falls (and the profit rate rises) until the equilibrium profit rate is reached, because at the original 10% profit rate reinvestment was taking place at only 5% a year, not enough to compensate for total depreciation of 12.5% a year. For China the initial profit rate lies above the equilibrium rate: the capital stock rises and the rate of profit falls until the equilibrium is attained.

The evolution of the rate of profit in the two countries is plotted in Figure 1. It can be seen that the rate of return in both countries tends to the same limit.

4 Testing the Theory

The formula for the equilibrium profit rate derived above is non-obvious, and it allows us to make empirical predictions—two criteria for a fertile theory. But are the predictions confirmed? The short answer is Yes. We support this claim below using the Extended Penn World Tables. This data set has certain limitations; in particular the capital stock and wage share figures refer to the entire economy and not just the capitalist sector, as we would wish. Nonetheless it allows for comparison between countries and will suffice for the trends of the variables. Total capital stock was taken as the net fixed standardized capital stock, estimated using the Perpetual Inventory Method. See Marquetti (2009) for details of the EPWT data, and see the Appendix to this paper for more on our use of the data.

[3] By capital to labour ratio here we mean the ratio of labour time embodied in the capital stock to the total labour performed per year.

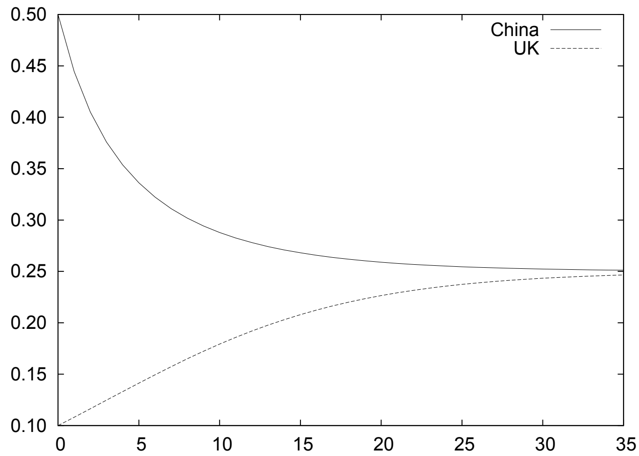

Figure 1: Simulation of convergence of the actual rate of profit to equilibrium

First Pattern

The graphs have several characteristic forms. The first is shown in Figure 2, which gives the trends for Japan and Switzerland. The solid line in the figures is the equilibrium rate of profit and the dotted line the actual rate. Note how the actual rate lags the equilibrium rate by a few years.

So our first result is that the equilibrium rate does act as an attractor for the real rate. This is validation of the basic theory set out above. Since the theory of the equilibrium rate consists of simple mathematical deductions from Smithian premises, it confirms the basic premises of Smith. This is striking since equation (7) is far from obvious. It contains none of the variables one might initially expect to determine the rate of profit, yet it predicts it with great accuracy.

The next striking point about the graphs for Japan and Switzerland is the way they seem to confirm the Marxian theory of the tendency of the rate of profit to fall. In a sense they do, but we will present graphs for other countries where the rate of profit rises. So why was the equilibrium rate on a declining trend for Japan and Switzerland?

Look at female fertility in these countries and you have the answer. In Switzerland the fertility rate fell from 2.5 children per woman in 1965 to 1.5 babies per woman in 1995. A similar if less pronounced decline can be seen in Japan, from 2 babies per woman to 1.5 in 1995. This decreases the rate of growth of the population and, by equation (7), brings down the equilibrium rate of profit.

China is not included in the Penn World Tables, but when the corresponding graph is computed from the China National Statistical Yearbook, the trend from 1994 to 2003 is remarkably similar to that shown by Japan 30 years earlier (see Figure 3). In 2004, however, there occurs an up-spike in both the actual and equilibrium rates of profit; this is the result of a sharp rise in the profit share (affecting current profits) combined with a fall in share of accumulation out of profit (affecting the equilibrium rate). There may be scope for doubt regarding the correctness of the 2004 China data.

Typical European Union Pattern

The next typical pattern we see is illustrated by the Netherlands. In the first period the shape of the curve is like that of Japan or Switzerland—a steady decline in the equilibrium rate driven by falling population growth. But from the start of the 1980s we see a recovery of the profit rate. This has two underlying causes: first, increased immigration compensates for a low birthrate, and second, rising consumption of

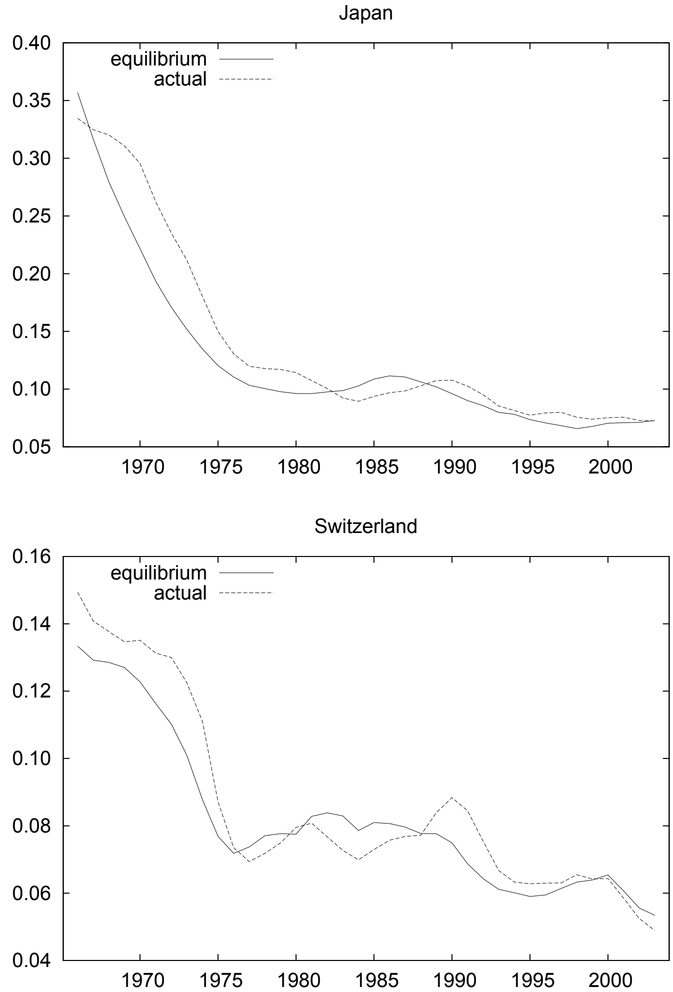

Figure 2: The first characteristic pattern, Japan and Switzerland. Note how the actual rate lags the equilibrium rate by several years, as theory would predict.

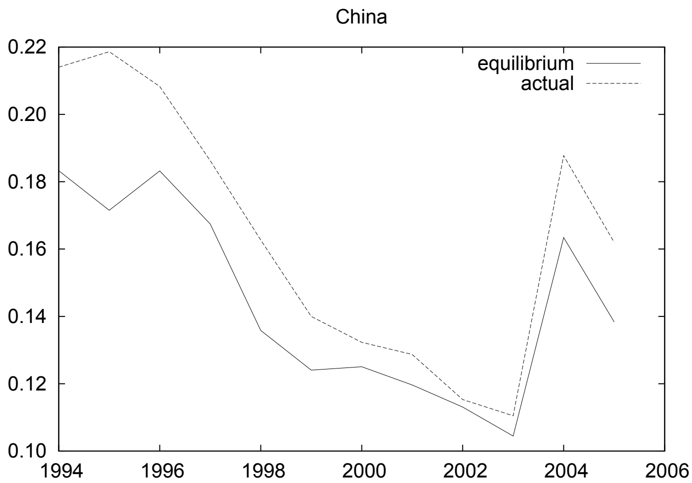

Figure 3: China: the trend from 1994 to 2003 is like that in Japan 30 years earlier.

the surplus by unproductive services (finance, advertising, etc.) slows the growth of capital stock. As equation (7) shows, a rise in unproductive expenditure has the paradoxical effect of raising the equilibrium rate of profit.

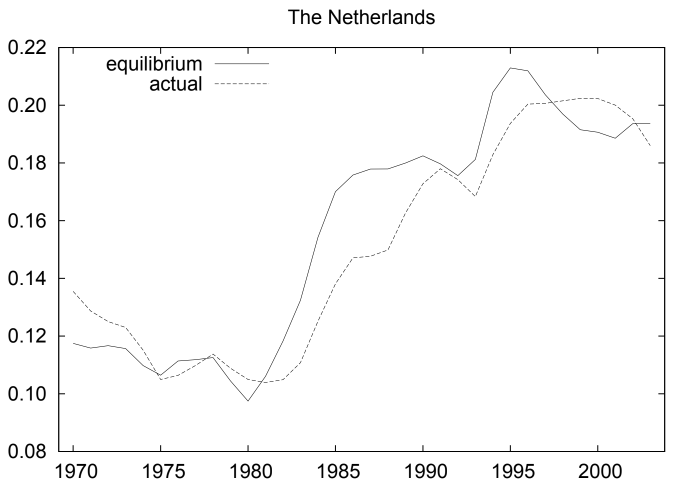

Figure 4: Netherlands, which shows the typical pattern for European Union countries

As Figure 4 shows, the equilibrium rate rises first, the actual rate following upward with a 2 or 3 year lag.

African Pattern

Africa shows a quite different pattern, illustrated by Egypt in Figure 5. Unfortunately data on the wage share are not available from 1981 to 1995; we resorted to interpolating values for this period (linearly) based on the data for 1980 and 1996. This plot is therefore less reliable than the others, but with that caveat we see that the equilibrium rate once again predicts what the actual rate of profit will be in subsequent years. But unlike the other two patterns, the equilibrium rate is rising. We would explain this as the result of a rapidly growing workforce (from 29 million in 1965 to 63 million in 2000), along with a falling capital–output ratio. The share of the surplus being reinvested is relatively low (u is near to 1). The consequence is a rising tendency of the rate of profit.

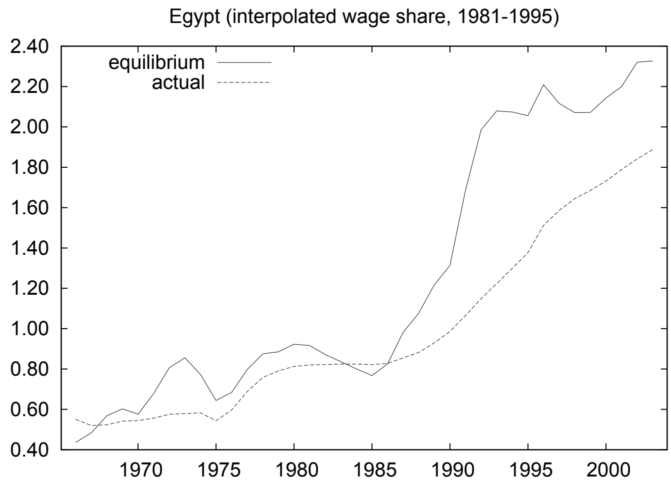

Figure 5: Egypt, which shows a quite different pattern

5 Predictive power

We now examine in a more systematic way the effectiveness of the steady-state formula for predicting the actual rate of profit a few years ahead. In this section we use data from 16 countries—those with the highest data quality minus Hong Kong and Luxembourg, which we set aside on account of their exceptionally small size and high degree of openness.

Country by country measures

Table 1 contains two sets of columns, one based on the raw or unfiltered data and one based on a filtered version of the data. The filtering has two aspects. First, the EPWT contains two variant measures of the growth of labour productivity: one unadjusted and one smoothed via a locally weighted regression of the 'loess' type. The latter is intended to purge fluctuations in labour productivity at business cycle frequencies. In our 'filtered' calculations we use the smoothed productivity series. Secondly, our filtered results employ a moving average for both the actual and the steady-state rate of profit. Specifically, the final filter is a three-year weighted moving average (see Appendix). As this filter involves no look-ahead, we take it to be suitable for the purpose of forecasting. The unfiltered columns offer results that have been subjected to no such filtering. Note that all of the per-country plots employ the filtered data.

The basic rationale for filtering is that the mechanism we describe in section 3 is, in the real world, overlaid by business cycle fluctuations and other noise, partially obscuring the relationship that we seek to highlight.

The columns numbered (1) and (2) in Table 1 give a measure of the closeness of fit between the actual profit rate and the lagged value of the steady-state rate. In effect we slide the two series against each other in search of the best fit for each country in turn. Here, 'best fit' is defined by maximization of the sample correlation coefficient. Column (1) reports the maximum correlation and column (2) reports the lag at which the maximum occurs. The maximum correlations range from 0.875 to 0.995 for the filtered data, and from 0.683 to 0.961 for the unfiltered version.[4] Results based on alternative criteria (minimization of the Mean Absolute Prediction Error, or of the Mean Square Error) can be viewed by accessing the website built by one of the authors (Tajaddinov, 2009).

The columns numbered (3) and (4) present an alternative measure, namely the 'attractor ratio', $\hat{A}$: this is the proportion of annual observations, indexed by t, for which the equilibrium rate $r^\star$ is an effective leading indicator for the actual rate, r—that is, the motion of the actual rate is towards the equilibrium rate. More formally, the criterion is the following.

$$\left(r_t^\star \geq r_t \wedge r_{t+1} \geq r_t\right) \vee \left(r_t^\star \leq r_t \wedge r_{t+1} \leq r_t\right)$$

The p-value associated with the attractor ratio is calculated from the binomial distribution: it is $\text{Prob}(x > \hat{A})$ for x distributed as binomial with per-trial probability 0.5 and number of trials equal to the relevant number of observations. The null hypothesis is that the actual profit rate is just as likely to move away from rather than towards the equilibrium rate. Using the filtered data, the attractor ratio exceeds 0.8 for most countries and the small p-values decisively reject the null for all countries. Using the raw data the performance is less impressive, but the ratio exceeds 0.5 in all cases and the null is rejected at the 5% significance level for 7 of the 16 countries.

While the correlations are of some interest, the attractor ratio is the more telling statistic. Correlation simply measures the linear association between two variables, but the attractor ratio measures the frequency with which the actual rate moves towards the steady state, as the theory predicts. Under some conditions r may be moving towards $r^\star$ yet the two variables are moving in opposite directions (e.g. $r^\star$ is rising, but r lies above $r^\star$ and is falling).

Speed of convergence

The 'attractor ratio' calculations discussed above give us an idea of the frequency with which the *direction* of movement of the rate of profit conforms to the predictions of the theory. We can also ask: how well does the *magnitude* of the changes in the rate of profit match the theory? To answer this, we need a dynamic equation for the rate of profit. This is derived in the Appendix; here we just state the result:

$$\dot{r} = (1 - w)(1 - u)\,\frac{N}{K}\left(r^\star - r\right) \tag{8}$$

Given suitable data, we can calculate the predicted change in the rate of profit for each country and year using (8) and compare this with the actual change. The comparison may be done via a simple regression of the actual changes on the predicted changes. The 'ideal' result for the theory would be an intercept that is statistically indistinguishable from zero and a slope statistically indistinguishable from one.

We face some data problems in this context. First there is the question of measuring N/K. The EPWT dataset contains a series `N` representing total employment (number of persons) and a series `K` representing the capital stock in constant dollars. But these are not commensurable, and we chose to proxy the monetary equivalent of N using value added (Net National Product) in constant dollars.[5] (The

[4]Note that lag zero of the steady-state rate never yields the maximum correlation with the actual rate when using the filtered data, but it gives the highest correlation for half of the countries in the sample when using the raw data. This reflects the common impact of high-frequency noise on both series and supports the rationale for filtering.

[5]Specifically, our N variable is defined as `X - D` (constant-price GDP minus depreciation) from EPWT. See the Appendix for details on the EPWT series.

alternative would be construct a measure of the capital stock in person-hours.) Then there is the issue of smoothing versus no smoothing. This is a dilemma, since the relationship between year-to-year changes that we're trying to examine may be masked by noise when using the raw data, but also might be distorted by any filter that is applied.

Table 2 reports three variant OLS regressions, using data from the same 16 countries as in Table 1. In the first, the raw data are used; in the second we use the EPWT's smoothed labour-productivity series when computing $r^\star$; and in the third we in addition apply the moving average filter mentioned above when computing both r and $r^\star$.

The results are quite encouraging: the 95 percent confidence interval for the constant includes zero for all the regressions, and the slope is close to one for the regression using the raw data. The main problem is that the slope coefficients in the second and third regressions are significantly greater than one, suggesting that the actual change in the rate of profit exceeds the theoretical prediction. This may be an artifact of filtering (which substantially improves the fit, as measured by R^2). Another possible explanation is endogenous change in the wage share, w. If the current rate of profit exceeds the steady-state rate, our theory predicts a period of relatively rapid accumulation, which then leads to an increase in the capital to labour ratio and a fall in the rate of profit. It is likely, however, that rapid accumulation tends to raise w (as was argued by Smith, Ricardo and Marx), at least temporarily, hence contributing an extra downward pressure on the rate of profit.

Incremental predictive power

A standard question asked of forecasting models is: does the model beat a simple autoregression, in which the variable of interest is predicted purely on the basis of its own past values? In the present context the issue is whether the steady-state profit rate, $r^\star$, really gives additional explanatory power over the actual rate, after taking into account the history of the actual rate itself. We tackled this by means of a panel-data regression, using data from the 16 countries represented in Table 1. In this context we used the smoothed series for growth of labour productivity from the EPWT but performed no further filtering of the data.

Table 3 shows the results of pooled OLS estimates of the current aggregate rate, r_t, using three yearly lags of both r and $r^\star$ as regressors, along with a constant and, for good measure, a linear time trend. The model is therefore

$$r_{i,t} = \alpha + \beta t + \sum_{j=1}^{3} \gamma_j r_{i,t-j} + \sum_{k=1}^{3} \delta_k r^\star_{i,t-k} + u_{i,t}$$

where i indexes the countries and t indexes the years.

The minimum time-series length of the country samples was 31 years and the maximum 37. The standard errors reported are of the heteroskedasticity- and autocorrelation-robust (HAC) type, as proposed for panel data by Arellano (2003).

Pooled OLS imposes the assumption of a common constant or intercept, α. This can be tested via a fixed-effects regression (not shown in the Table), in which the common α is replaced by a set of country-specific α_is. The null hypothesis—that there is no difference in α across countries—is not rejected: the test statistic is $F(15, 555) = 0.660$ with a p-value of 0.824. Furthermore, the R^2 value of 0.973 suggests that we are not doing too much violence to the data in imposing a common specification across the 16 countries in the sample.

It can be seen from Table 3 that $r^\star$ is strongly significant even in the presence of the lags of r_t. The individual p-value for $r^\star_{t-1}$ is 3.96×10^{-17} and the Wald test statistic for all lags of $r^\star$ is $\chi^2(3) = 436.5$, with p-value 2.77×10^{-94}, convincingly rejecting the hypothesis that $r^\star$ has no effect.[6] The

[6] The Wald χ^2 statistic tests the restriction that the true coefficients are zero for all lags of $r^\star$. It is calculated from the robust estimate of the covariance matrix of the coefficients.

panel regression therefore amply confirms the impression given by the per-country plots, namely that the steady-state rate of profit has significant predictive value for the actual rate.

6 Conclusion

We have shown that, starting from Smithian premises about the economy, one can construct a set of equations that make non-obvious predictions about the trends of profitability across the world economy. We have presented a small sample of our results here, but the predictive power is there quite generally.

But what does equation (7) tell us about the long term future of the world economy? We believe it shows that the epoch of capitalist profit is transitory. It shows that as population growth slows, and eventually declines, profitability will fall. Indeed with a declining population the equilibrium profit rate could be negative.

The steady state condition is one in which profit is merely sufficient to maintain the value of capital stock against the devaluation it continually experiences due to technical advances. In Japan and Korea today we see this state being approached. Europe postponed it by being much more open to immigration than the far eastern countries. Japan, lacking sufficient domestic labour, has long been a big capital exporter. China, with its one-child families, will soon be in the same position as Japan in the 1980s. Already it is becoming a major capital exporter. But, as compared to European capital export a century ago, there is great disproportion in scale. In the early 20th century Europe was much smaller in population than Asia and Africa, so it was unable to export sufficient capital to saturate the supplies of labour on these continents. Today's industrialized capitalist Asia has only Africa available to it as a relatively untapped source of new labour. In the face of the huge flow of capital out of China, even the abundant labour reserves of Africa are likely to be rapidly exhausted.

Then civilization will enter a new phase, as across the world labour becomes scarce relative to capital. The economic and moral order of commercial civilization—the order of the *Wealth of Nations*—will come to an end. Perhaps then the principles of Smith's *Theory of Moral Sentiments* can come to the fore.

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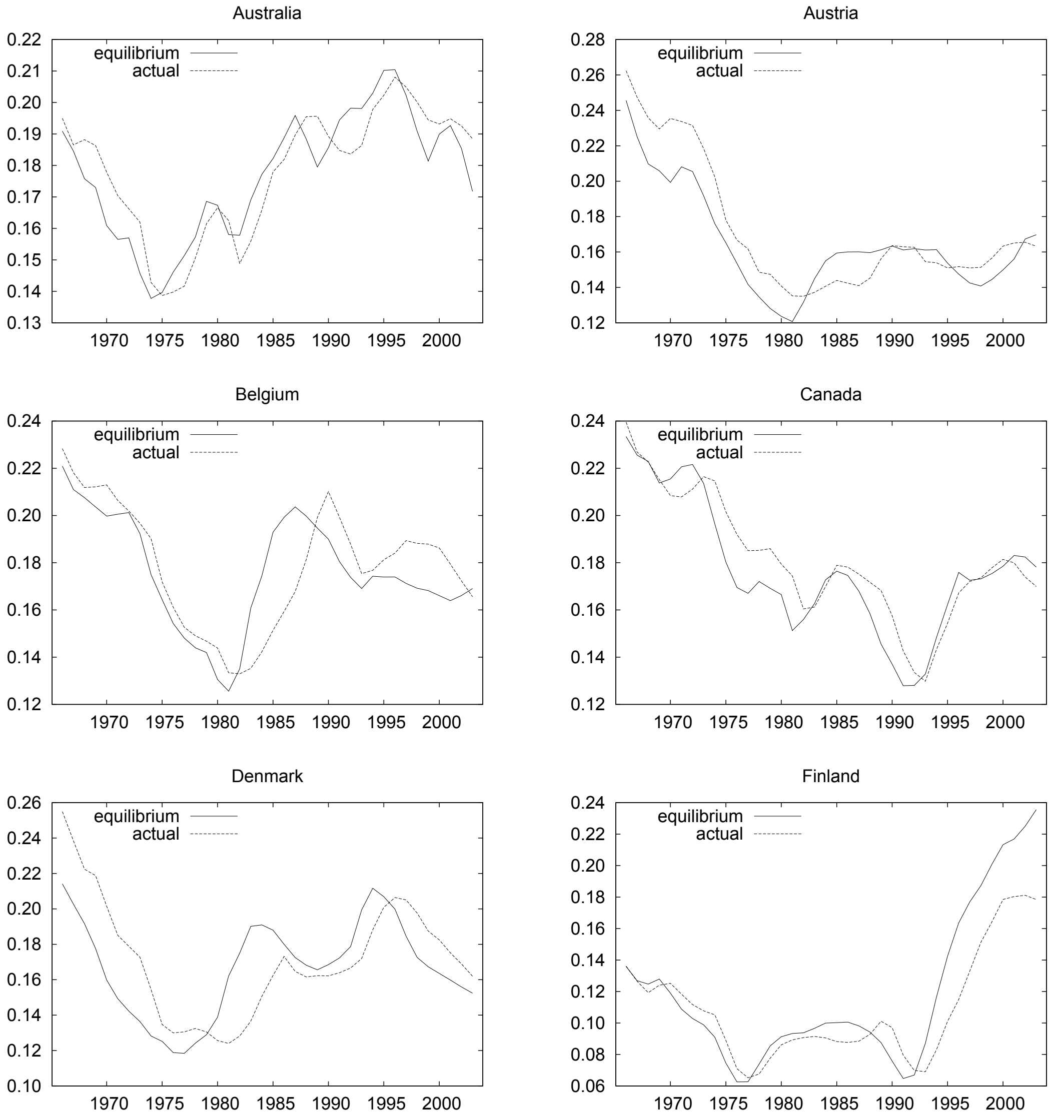

Figure 6: Trajectories of the steady-state and the actual rate of profit

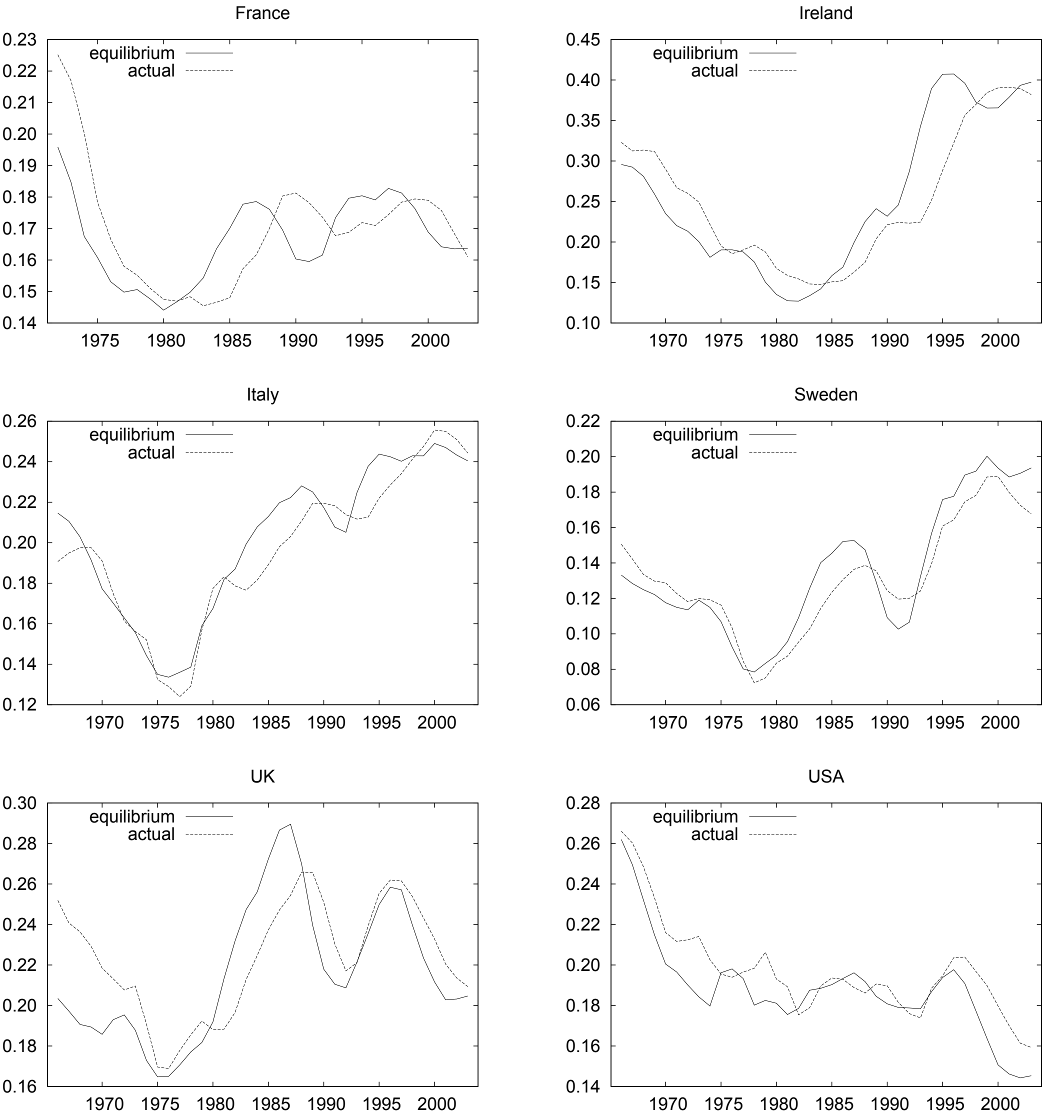

Figure 7: Trajectories of the steady-state and the actual rate of profit

	Filtered data				Unfiltered data			
	Max $\hat{\rho}$	Lag	$\hat{A}$	p-value	Max $\hat{\rho}$	Lag	$\hat{A}$	p-value
Country	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Australia	0.959	1	0.84	0.0000	0.779	0	0.56	0.1684
Austria	0.935	1	0.68	0.0100	0.849	0	0.59	0.0998
Belgium	0.898	2	0.81	0.0000	0.738	1	0.64	0.0266
Canada	0.970	1	0.76	0.0004	0.841	1	0.59	0.0998
Denmark	0.922	3	0.84	0.0000	0.710	1	0.67	0.0119
Finland	0.982	1	0.84	0.0000	0.888	1	0.54	0.2612
France	0.920	2	0.87	0.0000	0.683	1	0.58	0.1481
Ireland	0.981	3	0.81	0.0000	0.881	1	0.64	0.0266
Italy	0.961	1	0.84	0.0000	0.849	0	0.62	0.0541
Japan	0.995	3	0.81	0.0000	0.961	0	0.69	0.0047
Netherlands	0.972	2	0.91	0.0000	0.908	1	0.66	0.0205
Norway	0.974	1	0.73	0.0023	0.921	0	0.54	0.2498
Sweden	0.963	1	0.81	0.0000	0.889	0	0.51	0.3746
Switzerland	0.975	1	0.65	0.0235	0.848	0	0.64	0.0266
UK	0.875	2	0.78	0.0001	0.767	1	0.77	0.0001
USA	0.928	1	0.68	0.0100	0.807	0	0.62	0.0541

Table 1: Columns (1) and (2) show the maximum sample correlation, $\hat{\rho}$, between the actual and equilibrium rates of profit and the lag (in years) at which the maximum occurs. Columns (3) and (4) show the attractor ratio, $\hat{A}$ (the proportion of years for which the steady-state rate of profit correctly predicts the direction of movement of the actual rate), and the p-value for the null hypothesis that $A = 0.5$ (no better than chance).

Dependent variable: Δr for country i, year t
Independent: $(1-w)(1-u)\,\frac{N}{K}(r^\star - r)$ for country i, year $t-1$

		coeff.	std. error	t-ratio	p-value
(1)	const	−0.0009	0.0005	−1.755	0.0797
	slope	0.9239	0.0936	9.876	1.96e-21
		$n = 610$, $R^2 = 0.138237$			
(2)	const	−0.0002	0.0004	−0.5531	0.5804
	slope	1.9969	0.0914	21.84	1.68e-78
		$n = 610$, $R^2 = 0.439624$			
(3)	const	−0.0001	0.0002	−0.3517	0.7252
	slope	1.8220	0.0623	29.23	7.70e-116
		$n = 578$, $R^2 = 0.597239$			

Table 2: Regressions of actual year-to-year changes in the rate of profit on the changes predicted by the theory. Panel (1) uses the raw data; in panel (2) we use smoothed productivity growth data; in panel (3) a moving average filter is employed.

Pooled OLS: dependent variable r_t
Robust (HAC) standard errors

	Coefficient	Std. Error	t-ratio	p-value
const	−0.001076	0.00046264	−2.314	0.0210
t	3.9702e–05	2.1096e–05	1.882	0.0604
r_{t-1}	0.69237	0.044327	15.620	4.83e–46
r_{t-2}	−0.12339	0.047770	−2.583	0.0100
r_{t-3}	0.084726	0.028862	2.936	0.0035
$r^\star_{t-1}$	0.43344	0.049901	8.686	3.96e–17
$r^\star_{t-2}$	−0.12430	0.055806	−2.227	0.0263
$r^\star_{t-3}$	0.034550	0.051676	0.669	0.5040

$n = 578$, $R^2 = 0.973094$

H_0: parameters are zero for $r^\star_{t-1}$, $r^\star_{t-2}$, $r^\star_{t-3}$
Asymptotic test statistic: $\chi^2(3) = 436.481$, p-value 2.76856e–94

Table 3: Panel-data regression of the current aggregate profit rate on three lags of the current rate and three lags of the steady-state rate, plus a time trend

Appendix

Our use of the EPWT data

From the Extended Penn World Tables, version 3.0, we use the variables listed below. (For clarity we have renamed the original variables `Delta(%)` and `x(fc)` as `Delta_pc` and `x_fc` respectively.)

`Quality`	Quality rating of the country data, used for screening
`D`	Estimated depreciation, from net fixed standardized capital stock
`Delta_pc`	Depreciation rate, percent
`i`	Investment per worker-year
`K`	Estimated net fixed standardized capital stock
`N`	Number of employed workers
`X`	Real GDP in 2000 purchasing power parity (PPP)
`ws`	Wage share in the GDP
`x`	Labor productivity: real GDP in 2000 PPP per worker
`x_fc`	As `x`, but adjusted for the business cycle

On the basis of the above data we construct the following series:

`P = (1-ws)*X - D`	Aggregate profit
`r = P/K`	Actual economy-wide rate of profit
`n = ldiff(N)`	Growth rate of employment
`g = ldiff(x)`	Growth rate of labour productivity
`fg = ldiff(x_fc)`	As g, but filtered
`delta = Delta_pc/100`	Depreciation rate as decimal fraction
`I = i*N`	Aggregate investment
`u = 1-I/P`	Unproductive expenditure ratio
`rstar = (n+g+delta)/(1-u)`	Equilibrium profit rate
`rstar_f = (n+fg+delta)/(1-u)`	Equilibrium profit rate, filtered

In the expressions above, `ldiff()` denotes the first difference of the natural log: $\log x_t - \log x_{t-1}$. The series `r` and `rstar` (or the smoothed version, `rstar_f`) form the main focus in the text.

In the profit-rate plots we applied a further filter to the profit rate series (both actual and equilibrium), namely a trailing weighted moving average:

$$f(x_t) = 0.5x_t + 0.3x_{t-1} + 0.2x_{t-2}$$

Dynamic equation for the rate of profit

From equations (1) and (2) in the text, the rate of profit is given by

$$r = \frac{P}{K} = \frac{(1-w)N}{K}$$

If w is assumed to be constant, then the time-derivative of the rate of profit is

$$\dot{r} = (1-w)\,\frac{d(N/K)}{dt} \tag{9}$$

But, via basic calculus,

$$\frac{d(N/K)}{dt} = \frac{N}{K}\left(\frac{\dot{N}}{N} - \frac{\dot{K}}{K}\right)$$

Again from the text, we have $\dot{N}/N = n$ and equation (5) for $\dot{K}/K$:

$$\frac{\dot{K}}{K} = (1-u)\,r - g - \delta$$

and so

$$\frac{d(N/K)}{dt} = \frac{N}{K}\,(n + g + \delta - (1-u)\,r) \tag{10}$$

We can now use (10) in (9) to get

$$\dot{r} = (1-w)\frac{N}{K}\,(n + g + \delta - (1-u)\,r) \tag{11}$$

The solution of (11) for $\dot{r} = 0$ is

$$r^{\star} = \frac{n + g + \delta}{1-u}$$

and using the above expression to substitute for $n + g + \delta$ in (11) gives

$$\dot{r} = (1-w)(1-u)\frac{N}{K}\left(r^{\star} - r\right) \tag{12}$$

Equation (12) gives us an expression for the time-derivative of the actual rate of profit as a function of the gap between the steady-state and actual rates.

The actual rate of profit converges towards the steady state on condition that $(1-w)(1-u)N/K > 0$. The wage share, w, must be a positive fraction, and both N and K are strictly positive. What about $1-u$? The maximum value of u is 1 (meaning that the entire profit is consumed unproductively). For any value of u short of 1 the convergence condition is surely satisfied. If $u = 1$, on the other hand, the model breaks down: the rate of convergence goes to zero, but the steady-state rate of the profit, $r^{\star}$, is itself undefined.