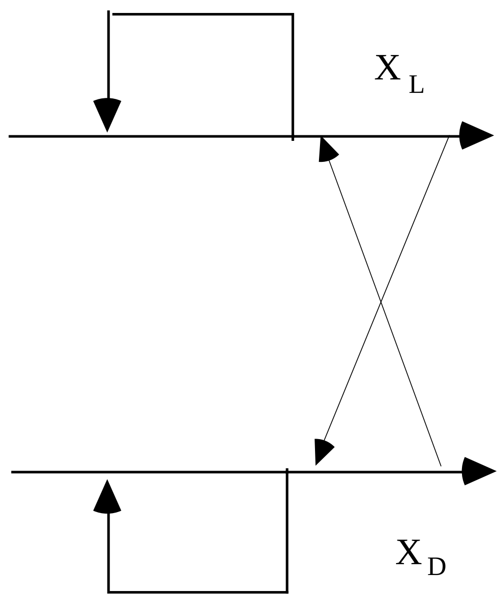
# Theory of Spontaneous Chiral-Symmetry Breaking

Spontaneous chiral symmetry breaking can generally be realized in systems that basically have the following features

• *AUTOCATALYSIS* • *CROSS-INHIBITION*




**The following simple model can be used to illustrate most general features.**

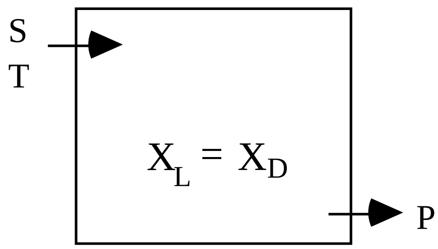
## A simple model (extension of Frank model)

$$S + T === X_L \qquad S + T + X_L === 2X_L$$

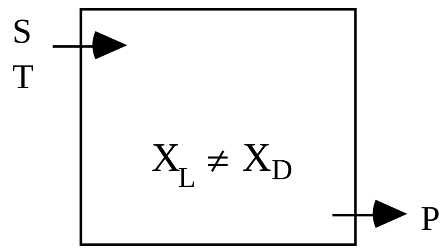
$$S + T === X_D \qquad S + T + X_D === 2X_D$$

$$X_L + X_D \rightarrow P$$

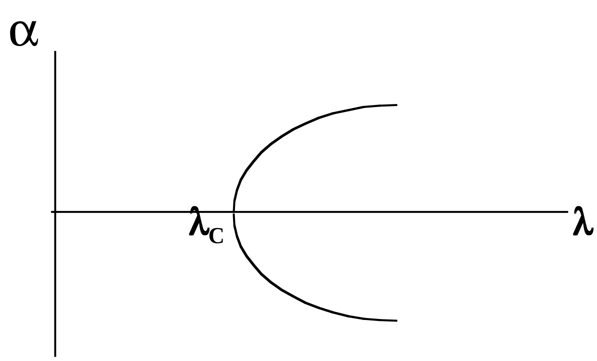
$$\boldsymbol{\lambda = [S][T] \qquad \alpha = ([X_L]-[X_D])/2}$$




$\boldsymbol{\lambda < \lambda_c}$




$\boldsymbol{\lambda > \lambda_c}$




$$\frac{d\alpha}{dt} = -A\alpha^3 + B(\lambda - \lambda_c)\alpha + \xi(t)$$

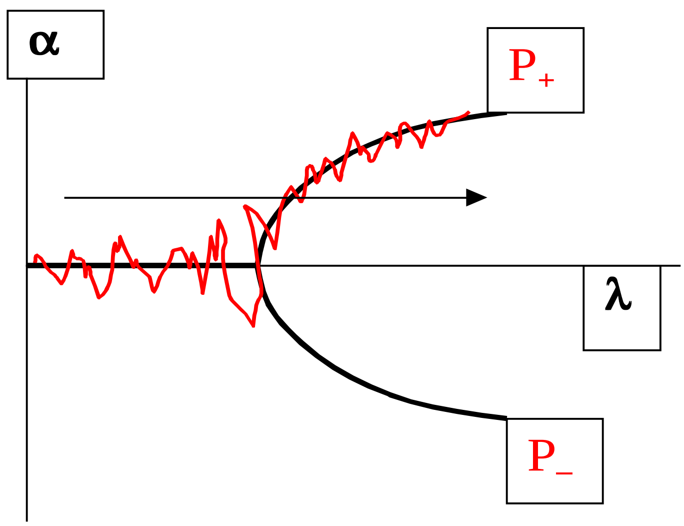
A, B and λ depend on kinetic rates constants

$\xi(t)$ = Random fluctuations

**GENERAL EQUATION NEAR TRANSITION POINT FOR ALL SYSTEMS**

**• Two-fold symmetry breaking leads to the following general equation near the transition point**

$$\frac{d\alpha}{dt} = -A\alpha^3 + B(\lambda - \lambda_c)\alpha + \sqrt{\varepsilon} f(t)$$



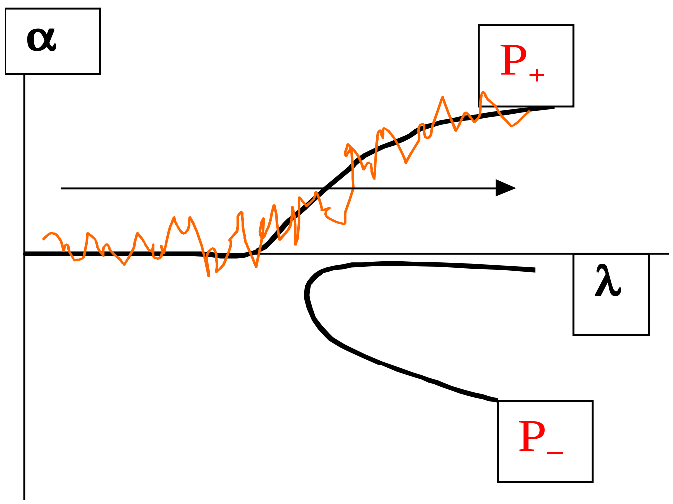

**As the systems sweeps through the critical point, it makes a transition randomly to one of the two branches. In the absence of any asymmetric influence:**

$$P_{+} = P_{-} = 0.5$$

# SENSITIVITY TO A SMALL ASYMMETRIC INFLUENCE "Cg"

**In the presence of a small asymmetric influence "Cg" the above equation is modified to:**

$$\frac{d\alpha}{dt} = -A\alpha^3 + B(\lambda - \lambda_c)\alpha + Cg + \sqrt{\varepsilon}f(t)$$

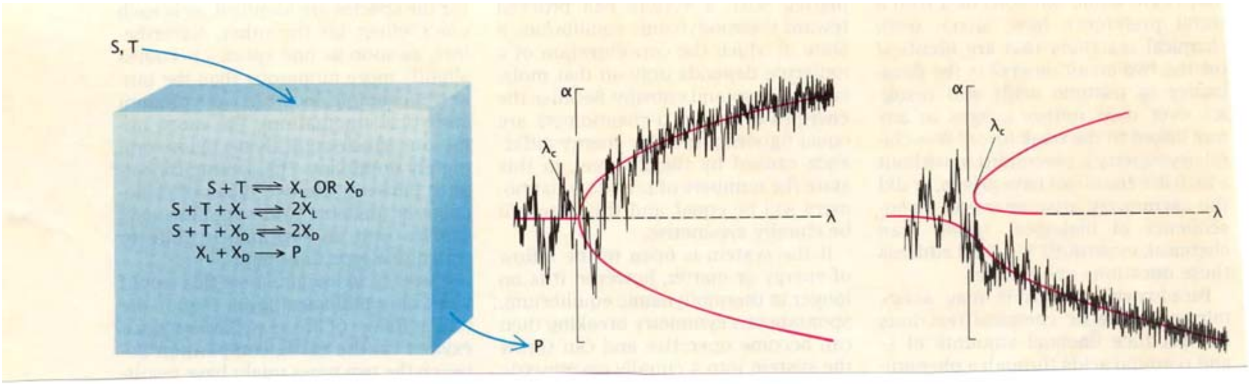



$$\lambda = \lambda_0 + \gamma t$$

$$P_+ = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{N} e^{-x^2/2}dx$$

$$N = \frac{Cg}{\sqrt{\varepsilon/2}}\left(\frac{\pi}{B\gamma}\right)^{1/4}$$

**This formalism is general – like that of phase transitions**




$$K_L = K_D\,(1 + g) \qquad\qquad g = \Delta E/kT$$

Probability $P_+ > 98\%$

| g | Volume/L | Run Time |
|---|---|---|
| $10^{-11}$ | 10 | 30 sec |
| $10^{-13}$ | 100 | 1 hr |
| $10^{-15}$ | 100 | 300 hrs |
| $10^{-17}$ | 1km x 1km x 1m | 10,000 yrs |