December 10, 2019 Name ________________

Solutions to Final Exam PHY 310/610

This test consists of three parts. In parts II and III, PHY 310 students can skip one question of those offered, while PHY 610 students must answer all questions.

Part I: Multiple Choice (mixed new and review questions) [40 points] (2 points each)
PHY 310/610: For each question, choose the best answer

1. Our best guess is that in the future, the scale factor of the universe *a* will
   A) Increase exponentially forever
   B) Always increase, but at an ever slowing rate
   C) Asymptotically approach a finite value in the future
   D) Reach a maximum and then recollapse to $a = 0$
   E) Reach infinity in a finite time

2. Hubble's constant is probably closest to which of the following?
   A) 60 km/s/Mpc **B) 70 km/s/Mpc** C) 80 km/s/Mpc D) 90 km/s/Mpc E) 100 km/s/Mpc

3. Which of the following is <u>not</u> considered a force carrying particle?
   A) Neutrino B) W boson C) Z boson D) Gluon E) Photon

4. The hottest stars are of spectral type
   A) A B) B C) F D) G **E) O**

5. Homogenous means the same everywhere, isotropic means the same in all directions, and static means unchanging over time. Which of these three terms describes the universe on large scales?
   A) Homogenous (only)
   B) Isotropic (only)
   C) Static (only)
   D) Homogenous and isotropic, but not static
   E) Homogenous, isotropic, and static

6. According to modern particle physics, how many types (flavors) of quarks are there, total?
   A) Three B) Four C) Five **D) Six** E) Eight

7. At some point, the temperature dropped to the point where various quarks collided with anti-quarks and were annihilated. Why are there any quarks left over today, if this is the case?
   A) The quarks were generated after this annihilation through lepton decay
   B) There must have been a tiny excess of quarks over anti-quarks
   C) The density got so low that typical quarks couldn't find anti-quarks to annihilate with
   D) Various processes collected quarks in some parts of the universe and anti-quarks in others
   E) They did all annihilate; this is why there are no free quarks today.

8. The name of our galaxy supercluster, according to this class, is
   A) Milky Way B) Andromeda C) Local Group **D) Laniakea** E) Coma

9. A distant galaxy, is at a distance *d* and is moving away from us at a velocity *v*. Why can't we simply use the formula $t = d/v$ to deduce when it left us and find the age of the Universe?
   A) Galaxies were always moving <u>slower</u> in the past than they are now.
   B) Galaxies were always moving <u>faster</u> in the past than they are now
   C) Galaxies were moving <u>slower</u> at some times and <u>faster</u> at others in the past
   D) Peculiar velocities cause errors in this, which are particularly large for distant galaxies
   E) There is nothing wrong with this; this is how we get the age of the Universe

10. Which of the following coincidences was *not* mentioned as being crucial to the development of life in the Universe?
    A) The value of Ω at very early times was ridiculously close to 1
    B) Neutrino decoupling came almost exactly at the same time as neutron/proton freezeout
    C) There is a resonance in ^{12}C that makes it especially easy to synthesize
    D) The proton/neutron mass difference is remarkably small
    E) The cosmological constant is ridiculously tiny compared to the Planck scale prediction

11. Which of the following events is expected to be farthest in the future?
    A) Stars die B) Galaxies evaporate **C) Matter decays** D) Isolated Universe E) Sun dies

12. Most of the mass of our galaxy is ___________ and is located in the _________ .
    A) Dark matter, disk
    B) Dark matter, halo
    C) Dark matter, bulge
    D) Ordinary matter, disk
    E) Ordinary matter, halo

13. According to general relativity, when do objects follow geodesics, the longest proper time path between two events in space-time?
    A) Whenever there are no forces other than gravity acting on them
    B) Whenever there is no gravity acting on them, though there can be other forces
    C) Whenever there are no forces of any kind acting on them
    D) Only when there are no forces on them and spacetime has no curvature
    E) Always

14. What type of galaxy do we probably live in?
    A) Elliptical B) Spiral **C) Barred Spiral** D) Irregular E) None of these

15. When we look at the cosmic microwave background radiation (CMBR) we are seeing the universe at the era of
    A) Nucleosynthesis B) Inflation **C) Recombination** D) GUT scale E) Planck era

16. The luminosity distance is defined as $d_L^2 = L/(4\pi b)$, where L is the power it is putting out and b is the power per unit area we receive. This corresponds to the physical distance in an unchanging, flat universe. Why are things more complicated in the real universe?
A) The distances are changing with time, since the Universe is expanding (only)
B) The objects are moving away, causing them to be red-shifted to lower energy (only)
C) Space might be curved, meaning the formula is basically wrong (only)
D) All of the above are correct
E) None of the above are correct

17. Which of the following ingredients contributes approximately 26% of the total density of the Universe?
A) Baryons B) Cosmological constant C) Neutrinos D) Photons **E) Dark matter**

18. Which of the following isotopes is believed to not have been produced primordially, even in small quantities
A) ^{2}H B) ^{4}He C) ^{3}He D) ^{7}Li **E) ^{12}C**

19. The velocity of a distant object towards or away from us is normally measured by
A) Doppler shift
B) Measuring how the object's apparent size changes with time
C) Using radar distancing to measure its distance over time
D) Noting how the parallax angle changes with time
E) Seeing how the apparent brightness changes over time

20. If particles are colliding with a cross section σ, have a number density n, and are moving relative to each other at a speed Δv, which formula gives us the rate for collisions Γ?
A) $\Gamma = \frac{n\sigma}{\Delta v}$ B) $\Gamma = \frac{n\Delta v}{\sigma}$ C) $\Gamma = \frac{\sigma\Delta v}{n}$ D) $\Gamma = \frac{\Delta v}{n\sigma}$ **E)** $\Gamma = n\sigma\Delta v$

Part II Short Answer [40 points/50 points] (10 points each)
PHY 310: Choose **four** of the following five questions and give a short answer (1-3 sentences)
PHY 610: Answer **all five** questions

21. Explain qualitatively how one can use Cepheid variable starts to measure distances.

Cepheid variable stars have periodic variations in their luminosity. They have a simple relationship between their period and their luminosity. Hence by measuring their period, we know their luminosity. By also measuring the brightness, one can then deduce the distance.

22. Explain how our observations of white dwarf supernovae at high redshift radically changed our understanding of the Universe.

White dwarf supernovae can be used as a standard candle, because they are consistently the same luminosity. Because they are so bright, they can be seen so far away that we can see how fast the universe was expanding in the past. Observations indicated the expansion is currently accelerating, which then allowed us to deduce the existence of dark energy, which is the presumed cause of this acceleration.

23. What problems of the standard ΛCDM cosmological model are inflation supposed to fix?

The ΛCDM cosmological model cannot, in itself, explain why the universe is almost perfectly flat, why it is the same in all directions and locations, and where the small fluctuations that cause temperature variations in the cosmic microwave background as well as large scale structure in the universe. Inflation gives a plausible, if unproven, explanation for these.

24. List the five events below in order from earliest to latest:

e^+/e^- annihilation, first structure, inflation starts, quarks confined, recombination

Inflation starts, quarks confined, e^+/e^- annihilation, recombination, first structure.

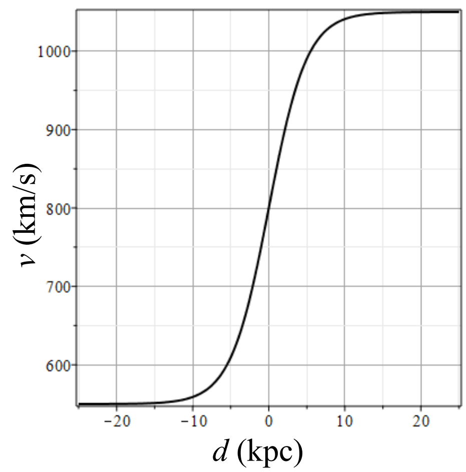
25. Give at least two reasons there might effectively be multiple universes. Would the low-energy physics in these multiple universes necessarily be the same?

There might be multiple universes because: (i) in some models of the universe, different areas of the universe come out at different times, effectively producing multiple universes; (ii) if the universe appears spontaneously out of nothing, then you could spontaneously created multiple universes; and (iii) in the many worlds interpretation of quantum mechanics, the description of the universe contains many "pieces" that effectively represent different possible universes. In many versions of these theories, the low energy physics could look very different in different universes, possibly even having different numbers of dimensions.

Units and Constant	Physical Constants	Age of Universe	Temperature
$\text{pc} = 3.086\times10^{16}\ \text{m}$	$k_B = 8.617\times10^{-5}\ \text{eV/K}$	Matter	$T_0 = 2.725\ \text{K}$
$\text{eV} = 1.602\times10^{-19}\ \text{J}$	$k_B = 1.381\times10^{-23}\ \text{J/K}$	$t = \dfrac{17.3\ \text{Gyr}}{(z+1)^{3/2}}$	**Mass Density**
$M_\odot = 1.989\times10^{30}\ \text{kg}$	$\hbar = 6.582\times10^{-16}\ \text{eV}\cdot\text{s}$	Radiation	$\rho = g_{\text{eff}} \dfrac{\pi^2 (k_B T)^4}{30(\hbar c)^3 c^2}$
$\text{y} = 3.156\times10^{7}\ \text{s}$	$\hbar = 1.055\times10^{-34}\ \text{J}\cdot\text{s}$	$t = \dfrac{2.42\ \text{s}}{\sqrt{g_{\text{eff}}}}\left(\dfrac{\text{MeV}}{k_B T}\right)^2$	**Friedman Equation ($\Omega = 1$)**
$G = 6.674\times10^{-11}\ \text{m}^3/\text{kg/s}^2$	$\hbar c = 1.973\times10^{-7}\ \text{eV}\cdot\text{m}$		$H^2 = \frac{8}{3}\pi G\rho$

Part III: Calculation [100/120 points] (20 each)
PHY 310: Answer **five** of the following six problems
PHY 610: Answer **all six** of the following problems

26. Galaxy Carlsonii is edge on to us, and the velocity of the stars away from us is measured at various distances from the center, as shown at right.



(a) How fast, approximately, is this galaxy moving away from us? What are the rotational velocities of the stars at distances of 5 kpc and 20 kpc from the center, relative to the center of the galaxy?

The center of the galaxy is moving away from us at 800 km/s. At +5 kpc, the velocity appears to be about 990 km/s, and at –5 kpc, it is about 610 km/s; each of these is about 190 km/s compared to the center at these radii. At +20 kpc, it is about 1049 km/s, and at –20 kpc, it is about 551 km/s. Each of these differ from 800 km/s by about 249 km/s, so we have

$$v(5\text{ kpc}) = 190\text{ km/s}, \quad v(20\text{ kpc}) = 249\text{ km/s}.$$

(b) Find the total enclosed mass in $M_\odot$ within a radius of 5 kpc . Repeat at 20 kpc.

To find the mass, we equate the gravitational force $F = GMm/R^2$ to the centripetal force, $F = mv^2/R$, so we have

$$\frac{GMm}{R^2} = \frac{mv^2}{R}, \quad M = \frac{v^2R}{G}.$$

Substituting in our explicit values, we have

$$M(5\text{ kpc}) = \frac{v^2R}{G} = \frac{(190\times10^3\text{ m/s})^2(5.00\times10^3\text{ pc})}{6.674\times10^{-11}\text{ m}^3\text{kg}^{-1}\text{s}^{-2}}\cdot\frac{3.086\times10^{16}\text{ m}}{\text{pc}} = 8.35\times10^{40}\text{ kg}$$

$$= (8.35\times10^{40}\text{ kg})\cdot\frac{M_\odot}{1.989\times10^{30}\text{ kg}} = 4.20\times10^{10}\,M_\odot,$$

$$M(25\text{ kpc}) = \frac{v^2R}{G} = \frac{(249\times10^3\text{ m/s})^2(2o.00\times10^3\text{ pc})}{6.674\times10^{-11}\text{ m}^3\text{kg}^{-1}\text{s}^{-2}}\cdot\frac{3.086\times10^{16}\text{ m}}{\text{pc}} = 5.733\times10^{41}\text{ kg}$$

$$= (5.733\times10^{41}\text{ kg})\cdot\frac{M_\odot}{1.989\times10^{30}\text{ kg}} = 2.88\times10^{11}\,M_\odot,$$

(c) Does this galaxy show evidence of dark matter?

The rotation curves are not falling off with distance. As we can see from our calculations, the mass in 20 kpc is nearly seven times larger than the mass within 5 kpc. So this indicates the presence of dark matter.

27. A muon is a particle with a mass of $mc^2 = 105.7\text{ MeV}$. One of the processes by which it can disappear in the early universe is decay $\mu^- \to e^- \nu\bar{\nu}$, which occurs at a rate $\Gamma = 4.55\times10^5\text{s}^{-1}$. This process can also occur in reverse.

(a) At approximately what temperature $k_B T$ would we expect this particle to disappear, if it stays in thermal equilibrium? Is this in the radiation or matter-dominated era?

An average particle energy is around $3k_B T$, so particles disappear roughly when $3k_B T = mc^2$, or $k_B T = \frac{1}{3}mc^2 = 35.2\text{ MeV}$. The matter/radiation equality is around 1 eV, and since this is way hotter, it is clearly in the radiation dominated era.

(b) Not counting the muons, what is g_{eff} at this time? Include photons (boson with 2 spin states), electrons (fermions with 4 spin states) and neutrinos (fermions with 6 spin states).

We have $g_B = 2$ and $g_F = 4+6 = 10$, so

$$g_{eff} = 2 + \tfrac{7}{8}\cdot 10 = 10.75 .$$

(c) What is the approximate time at this temperature? Use the g_{eff} you computed in part (b).

We simply use the appropriate time formula to obtain

$$t = \frac{2.42\text{ s}}{\sqrt{g_{\text{eff}}}}\left(\frac{\text{MeV}}{k_B T}\right)^2 = \frac{2.42\text{ s}}{\sqrt{10.75}}\left(\frac{\text{MeV}}{35.2\text{ MeV}}\right)^2 = 5.96\times10^{-4}\text{ s}.$$

(d) Is this process fast enough to keep it in thermal equilibrium with everything else?

To determine if the process is fast enough, we simply look at the product Γt, which is given by

$$\Gamma t = \left(4.55\times10^5\text{s}^{-1}\right)\left(5.96\times10^{-4}\text{s}\right) = 271.$$

In other words, it has easily enough time to decay at the time the particle should be disappearing.

28. The boiling point of liquid nitrogen is 77.36 K, and the density is 808 kg/m³.

(a) What was the red-shift *z* at the time when the universe was at the temperature of the boiling point of liquid nitrogen? Was the universe matter or radiation dominated? Calculate the age of the universe at this time.

The red-shift factor is simply given by

$$1+z=\frac{T}{T_0}=\frac{77.36\text{ K}}{2.7255\text{ K}}=28.38,\quad z=27.38.$$

The universe is generally matter dominated for $1<z<3000$, so this is clearly still in the matter-dominated era. We therefore use the matter-dominated formula for the age of the universe, namely

$$t=\frac{17.3\text{ Gyr}}{(z+1)^{3/2}}=\frac{17.3\text{ Gyr}}{(28.38)^{3/2}}=0.114\text{ Gyr}=114\text{ Myr}.$$

(b) What was the temperature k_BT in eV at the time the density of the radiation matched that of liquid nitrogen? Assume $g_{\text{eff}}=3.36$ at this time. Was the universe matter or radiation dominated? Calculate the age of the universe at this time.

We equate the density to the formula for the density in terms of temperature, which yields

$$\rho=g_{\text{eff}}\frac{\pi^2(k_BT)^4}{30(\hbar c)^3c^2},$$

$$(k_BT)^4=\frac{30(\hbar c)^3c^2\rho}{\pi^2 g_{\text{eff}}}=\frac{30(1.973\times10^{-7}\text{ eV}\cdot\text{m})^3(2.998\times10^8\text{ m/s})^2(808\text{ kg/m}^3)}{\pi^2(3.36)}$$

$$=0.505\text{ eV}^3\cdot\text{kg}\cdot\text{m}^2/\text{s}^2=\frac{0.505\text{ eV}^3\cdot\text{J}}{1.602\times10^{-19}\text{ J/eV}}=3.149\times10^{18}\text{ eV}^4,$$

$$k_BT=(3.149\times10^{18}\text{ eV}^4)^{1/4}=4.21\times10^4\text{ eV}=42.1\text{ keV}.$$

This is well above the matter/radiation equality around 1 eV, so it is radiation dominated. It is also well below the electron/positron annihilation energy (around 170 keV), so the value $g_{\text{eff}}=3.36$ is appropriate. We therefore can use the radiation equation for the age, namely

$$t=\frac{2.42\text{ s}}{\sqrt{g_{\text{eff}}}}\left(\frac{\text{MeV}}{k_BT}\right)^2=\frac{2.42\text{ s}}{\sqrt{3.36}}\left(\frac{\text{MeV}}{0.0421\text{ MeV}}\right)^2=745\text{ s}\approx12.4\text{ minutes}.$$

This puts it a little after primordial nucleosynthesis.

29. A long time ago in a galaxy far away, Han Solo measured the distance and red-shift to several galaxies. The values he found can be found at right

(a) For all five galaxies, find the velocity in km/s.

Gal.	z	d (Mpc)	v (km/s)
A	0.0006	0.75	180 km/s
B	0.0096	24.1	2880 km/s
C	0.0147	36.4	4410 km/s
D	0.0187	46.4	5610 km/s
E	0.0213	52.8	6390 km/s

Since the values of z are small, we can use the approximation

$$v = zc = z\left(299{,}800 \text{ km/s}\right).$$

The values have been included in the table.

(b) Estimate the value of Hubble's constant using galaxies *A–D* in km/s/Mpc. One of the data points does not work very well; what probably went wrong for this galaxy?

Hubble's Law says that $v = H_0 d$, so $H_0 = v/d$. We then quickly compute Hubble's constant using each of the numbers we just computed

$$H_A = \frac{180 \text{ km/s}}{0.75 \text{ Mpc}} = 240 \text{ km/s/Mpc}, \quad H_B = \frac{2880 \text{ km/s}}{24.1 \text{ Mpc}} = 120 \text{ km/s/Mpc},$$

$$H_C = \frac{4410 \text{ km/s}}{36.4 \text{ Mpc}} = 121 \text{ km/s/Mpc}, \quad H_D = \frac{5610 \text{ km/s}}{46.4 \text{ Mpc}} = 121 \text{ km/s/Mpc}.$$

Three of these numbers are (unrealistically) similar, but galaxy *A* is clearly the outlier. What went wrong? The problem is peculiar velocities: this galaxy is so close that its peculiar velocity could easily be enough to throw this number off. The correct value is about $H_0 = 121 \text{ km/s/Mpc}$.

(c) Estimate the unknown distance to galaxy *E*.

Since the red shift is still smallish, but not so small that it will ruin Hubble's Law, we assume Hubble's law is accurate. We solve $v = H_0 d$ for d to get

$$d = \frac{v}{H_0} = \frac{6390 \text{ km/s}}{121 \text{ km/s/Mpc}} = 52.8 \text{ Mpc}.$$

30. In a certain fictitious universe, which is homogenous, expanding and flat like ours, astronomers measure the concentration of baryons, cosmological constant, and radiation as

$$\rho_{b0} = 9.60\times10^{-22}\ \text{kg/m}^3, \quad \rho_{\Lambda 0} = 1.50\times10^{-23}\ \text{kg/m}^3, \quad \rho_{r0} = 4.00\times10^{-24}\ \text{kg/m}^3$$

In this universe, there are no neutrinos and no dark matter.

(a) Write a formula for the density of these three concentrations ρ_b, ρ_Λ and ρ_r at all times in terms of the red-shift factor $z+1$.

For ordinary matter, the mass density drops as a^{-3}, so $\rho_b \propto (z+1)^3$. For radiation, the number density drops by the same factor, but the energy per particle drops also proportional to a^{-1}, so $\rho_r \propto (z+1)^4$. The cosmological constant does not scale. So in summary, we have

$$\rho_b = \rho_{b0}(1+z)^3, \quad \rho_\Lambda = \rho_{\Lambda 0}, \quad \rho_r = \rho_{r0}(1+z)^4.$$

(b) Find the value of *z* when (i) radiation equals baryons, (ii) baryons equals cosmological constant. In each case, did this occur in the past or in the future?

We simply equate the two factors and solve for z in each case. For radiation equals baryon, we have

$$\rho_b = \rho_{b0}(1+z)^3 = \rho_r = \rho_{r0}(1+z)^4,$$

$$1+z = \frac{\rho_{b0}}{\rho_{r0}} = \frac{9.60\times10^{-22}\ \text{kg/m}^3}{4.00\times10^{-24}\ \text{kg/m}^3} = 240, \quad z = 239.$$

Since z is positive, this is in the past. For baryons equals cosmological constant, we have

$$\rho_b = \rho_{b0}(1+z)^3 = \rho_\Lambda = \rho_{\Lambda 0},$$

$$(1+z)^3 = \frac{\rho_{\Lambda 0}}{\rho_{b0}} = \frac{1.50\times10^{-23}\ \text{kg/m}^3}{9.60\times10^{-22}\ \text{kg/m}^3} = 0.0156,$$

$$1+z = (0.0156)^{1/3} = 0.25, \quad z = -0.75.$$

Since this is negative, this is in the future.

(c) What is the Hubble constant H_0 and inverse Hubble time H_0^{-1} in $\mathbf{y^{-1}}$ and y?

Since the universe is flat, we can assume $\Omega = 1$ and therefore we can use the appropriate Friedman equation:

$$H_0^2 = \tfrac{8}{3}\pi G\rho = \tfrac{8}{3}\pi\left(6.626\times10^{-11}\ \text{m}^3/\text{kg/s}^2\right)(9.60+0.15+0.04)\times\left(10^{-22}\ \text{kg/m}^3\right) = 5.47\times10^{-31}\text{s}^{-2},$$

$$H_0 = \sqrt{5.47\times10^{-31}\text{s}^{-2}} = \left(7.40\times10^{-16}\text{s}^{-1}\right)\left(3.156\times10^7\ \text{s/y}\right) = 2.34\times10^{-8}\ \text{y}^{-1},$$

$$H_0^{-1} = 4.28\times10^7\ \text{y} = 42.8\ \text{My}.$$

	Mass (MeV/c^2)	spin	strange	type
p^+	938	½	0	baryon
K^+	494	0	+1	meson

31. One way to make the Ξ particle is to collide two protons head on with energy 1800 MeV each in the reaction $p^+ + p^+ \to \Xi^? + p^+ + K^+ + K^+$. This is a strong reaction, and all particles are made of combinations of up, down, and strange quarks or their anti-particles.

(a) What is the baryon number, charge and strangeness of the $\Xi^?$? Is it a fermion or boson?

Since all of these quantities are conserved, we can use conservation laws to determine the values of each. The p^+ is a baryon and hence has baryon number 1, but the kaon has baryon number 0. So we have

$$\begin{aligned} \text{baryon:} \quad & 1+1=b+1+0+0, \quad b=1, \\ \text{charge:} \quad & 1+1=q+1+1+1, \quad q=-1, \\ \text{strangeness:} \quad & 0+0=s+0+1+1, \quad s=-2. \end{aligned}$$

The protons are fermions and the kaons are not. Since there must be a total number of an even number of fermions (left plus right), and the three protons totals three, the $\Xi^?$ must be a fermion. Its real name is the Ξ^-.

(b) What is the maximum possible mass that the $\Xi^?$ particle could have in Mev/c^2?

Energy is conserved, so the energy on the left must equal the energy on the right. The minimum energy for each particle on the right is the mass, while the energy on the left is given, so we have

$$(1800\text{ MeV})+(1800\text{ MeV}) \ge mc^2 + (938\text{ MeV})+(494\text{ MeV})+(494\text{ MeV}),$$
$$mc^2 \le 2(1800\text{ MeV})-(938\text{ MeV})-2(494\text{ MeV})=1674\text{ MeV}.$$

The actual mass of the Ξ^- is about 1321 MeV/c^2.

(c) What is the quark content of the $\Xi^?$?

It is a baryon, so it has three quarks. The strange quark has strangeness –1, while the up and down quarks have strangeness 0, so to get strangeness –2, there must be two strange quarks. The strange quarks each of charge $-\frac{1}{3}$, so to combine two of these with a third quark to get total charge –1, we must also have another quark with charge $-\frac{1}{3}$, which makes it a down quark. So the quark content is [dss].