December 11, 2021 Name ________________

Solutions to Final Exam PHY 310/610

This test consists of five parts. In parts II and III, PHY 310 students can skip one question of those offered, while PHY 610 students must answer all questions.

Part I: Multiple Choice (mixed new and review questions) [40 points] (2 points each)
PHY 310/610: For each question, choose the best answer

1. The value of the Ω at very early times (at least after inflation) is closest to which number?
   A) 0.003 B) 0.999 **C) 1.000** D) 1.001 E) 1.953

2. The particle that carries the strong force between quarks is called a
   A) *W* boson B) *Z* boson **C) Gluon** D) Photon E) Graviton

3. When we look at the cosmic microwave background, we are seeing the universe at the time of what event?
   A) Nucleosynthesis B) Big bang C) Inflation D) First structure **E) Recombination**

4. Which of the following spectral classes of stars would be coolest?
   A) A B) B C) F D) O **E) M**

5. Galaxies that have undergone recent collisions are most likely to be
   A) Elliptical B) Spiral C) Barred Spiral **D) Irregular** E) None of these

6. The name of the galaxy we live in is
   A) Milky Way B) Andromeda C) Local group D) Laniakea E) Virgo

7. The largest contribution to the mass density today is _______ and second largest is _____ .
   A) Dark matter, baryons
   B**) Dark energy, dark matter**
   C) Dark energy, baryons
   D) Dark matter, dark energy
   E) Baryons, dark matter

8. A very reliable method to measure the distance to some of the nearest stars is
   A) Radar distancing
   B) Parallax
   C) Cepheid variable stars
   D) Type Ia supernovae
   E) Hubble's law

9. Active galaxies are believed to have which object generating the energy at the center?
   A) Supernova B) Star C) Globular cluster D) Hot nebula **E) Black Hole**

10. Which of the following is important for something to make an ideal standard candle?
    - A) It must be consistently the same luminosity (only)
    - B) It must be bright enough to see at long distances (only)
    - C) It must always be at the same distance from us (only)
    - **D) A and B are correct, but not C**
    - E) A, B, and C are all correct

11. From our perspective, almost all galaxies seem to be moving away from us with a speed proportional to their velocity. If we were looking at things sitting on a distant galaxy we would see almost all the galaxies moving

    A) Away from us B) Towards us C) Past us D) Not moving E) None of these

12. Gravity should become as strong as other forces at approximately the time of
    - **A) Planck scale**
    - B) Electroweak scale
    - C) GUT scale
    - D) Quark confinement
    - E) Recombination

13. The Sun, in addition to going around the galaxy in a circle, also does which of the following?
    - A) Moves alternately inwards/outwards from its current position (only)
    - B) Occasionally reverses its direction, going around the other way (only)
    - C) Bobs up and down compared to the galactic plane (only)
    - D) A and B are correct, but not C
    - **E) A and C are correct, but not B**

14. If we knew both the angular size α (in radians) and the physical size s of an object, how could we find the distance d?

    A) $d = \alpha s$ B) $d = \frac{\alpha}{s}$ **C)** $d = \frac{s}{\alpha}$ D) $d = \frac{1}{\alpha s}$ E) None of these

15. Which of the following is *not* explained by inflation?
    - A) Why the universe is almost perfectly flat
    - **B) Why there is more matter than anti-matter in the universe**
    - C) Why the universe is approximately the same in all directions (isotropic)
    - D) Why the universe is approximately the same at all locations (homogenous)
    - E) Where the small fluctuations in the density come from

16. Shortly after the time of quark confinement, the quarks and anti-quarks annihilated. How come there are any leftover quarks today?
    - A) The quarks and anti-quarks became segregated, so other places have anti-quarks
    - B) The annihilation was not equal, so more anti-quarks were destroyed than quarks
    - C) They *were* all annihilated, but later processes made more quarks
    - **D) There must have been a tiny excess of quarks over anti-quarks at the time**
    - E) Santa Clause used his magic bag to deliver more quarks for Christmas

17. About how old was the universe when electrons and positrons annihilated?
    A) 10^{-42} s B) 10^{-6} s **C) 20 s** D) 57,000 y E) 6×10^{8} y

18. What method can sometimes be used to determine the mass of a cluster of galaxies?
    A) Gravitational lensing; observing how it bends light from objects behind the cluster
    B) Estimating the mass of each galaxy from rotation curves and adding them up
    C) Calculating the orbital speed of galaxies circling the cluster
    D) Counting the number of black holes and multiplying by the mass of one black hole
    E) Measuring the gravitational red shift from galaxies near the center of the cluster

19. Which of the following was suggested as a possible reason to believe in multiple universes?
    A) Many Worlds interpretation of quantum mechanics (only)
    B) Spontaneous creation of the universe (only)
    C) Chaotic/Eternal inflation with "pockets" resulting in multiple universes (only)
    D) All of the above
    E) None of the above

20. Which force is *not* currently explained by particle physics
    A) Weak force
    B) Gravity
    C) Strong force
    D) Electromagnetism
    E) Actually, all of them are explained by particle physics

Part II Short Answer [40 points/50 points] (10 points each)
PHY 310: Choose **four** of the following five questions and give a short answer (1-3 sentences)
PHY 610: Answer **all five** questions

21. Explain how measuring rotational velocities of spiral galaxies can be used to indicate the existence of dark matter in galaxies. In particular, how would these rotational velocities look different if there were no dark matter?

The rotational velocity as a function of distance to the center is measured by Doppler shift, and it is observed that these velocities do *not* fall off with distance, instead approaching a constant. This indicates that the matter is *not* concentrated in the center, which is what you'd expect looking at where the stars were, because if it were the velocities should fall off as the square root of distance.

22. Explain at least two reasons why Hubble's Law $v = H_0 d$ does not work very well at very *high* red-shifts/velocities.

Hubble's Law does not apply at high red-shifts/velocities because: (i) You are looking so far in the past that you are seeing the universe as it was, not as it is; (ii) The "distance" is ambiguous, since it could be the distance that the object *was*, the distance it *is now*, or the distance the light traveled, or even the luminosity distance; (iii) On these scales, the curvature of the universe becomes relevant, distorting the apparent luminosity; (iv) The object is moving so fast that the relativistic headlight effect causes it to send most of its light away from us; and (v) The red-shift causes the luminosity to be diminished because the frequency (and hence energy) of each photon is reduced, and also the number of photons per second is similarly reduced.

23. There were lots of protons and neutrons present in the early universe. Why didn't they just immediately bond together to make heavier nuclei? Once they did bond together, what isotope did they mostly combine to make? Why was there any hydrogen left over at the end?

The temperatures were initially so high that though protons and neutrons could combine to make deuterium, it would have immediately been disintegrated back into its constituents. Once the temperature dropped enough, this process could get going, but by then the light proton was favored over the heavier neutron, and hence there was a preponderance of protons. The protons and neutrons mostly got bound together into ^{4}He, but there were leftover protons that became hydrogen.

24. List the five events below in order from earliest to latest:

electroweak scale, first structure, matter/radiation equality, nucleosynthesis, Planck era

Planck era, electroweak scale, nucleosynthesis, matter/radiation equality, first structure

25. Give a list of five significant events that are expected to happen in the distant future (more than ten billion years from now).

Galaxies become isolated, galaxies merge into single supergalaxies, last stars form, all stars die, galaxies evaporate, matter decays, black holes evaporate.

Units and Constants

$\mathrm{pc} = 3.086\times10^{16}\ \mathrm{m}$

$\mathrm{eV} = 1.602\times10^{-19}\ \mathrm{J}$

$M_\odot = 1.989\times10^{30}\ \mathrm{kg}$

$\mathrm{y} = 3.156\times10^{7}\ \mathrm{s}$

$G = 6.674\times10^{-11}\ \mathrm{m^3/kg/s^2}$

Physical Constants

$k_B = 8.617\times10^{-5}\ \mathrm{eV/K}$

$k_B = 1.381\times10^{-23}\ \mathrm{J/K}$

$\hbar = 6.582\times10^{-16}\ \mathrm{eV\cdot s}$

$\hbar = 1.055\times10^{-34}\ \mathrm{J\cdot s}$

$\hbar c = 1.973\times10^{-7}\ \mathrm{eV\cdot m}$

Age of Universe

Matter

$$t = \frac{17.3\ \mathrm{Gyr}}{(z+1)^{3/2}}$$

Radiation

$$t = \frac{2.42\ \mathrm{s}}{\sqrt{g_{\mathrm{eff}}}}\left(\frac{\mathrm{MeV}}{k_B T}\right)^2$$

Temperature

$T_0 = 2.725\ \mathrm{K}$

Distance / Magnitudes

$$d = 10^{1+\frac{m-M}{5}}\ \mathrm{pc}$$

Part III: Calculation [100/120 points] (20 each)
PHY 310: Answer **five** of the following six problems
PHY 610: Answer **all six** of the following problems

26. The top quark is the heaviest particle in the standard model, with a mass of *m* = 173 GeV/*c*².

(a) What was the approximate temperature k_BT at the time when the top quark disappeared? Would this have been in the matter or radiation dominated era?

Particles disappear roughly when $3k_BT = mc^2$, so we have

$$k_BT = \tfrac{1}{3}mc^2 = \tfrac{1}{3}\left(173\text{ GeV}\right) = 57.7\text{ GeV}.$$

Since the radiation dominated era is whenever $k_BT > 1\text{ eV}$, this is clearly the radiation-dominated era.

(b) The fermions that were around at this time would have been the other quarks (12 spin states each for up, down …) and the leptons (18 total spin states). The bosons would have been the Higgs, $W^\pm$, *Z*, gluons, and photon (28 total). What was the value of g_{eff}?

There are a total of six quarks, but we are told to exclude the top quark, so the quarks contribute $12 \times 5 = 60$ spin states, and the leptons another 18 for a total of 78. The bosons would contribute 28 more. So, in total, we have

$$g_{\text{eff}} = g_B + \tfrac{7}{8}g_F = 28 + \tfrac{7}{8}\left(78\right) = 96.25.$$

(c) How old was the universe at this time? The top quark decays at a rate $\Gamma = 2.0\times10^{24}\text{s}^{-1}$. Would this decay have kept the top quark in thermal equilibrium?

We simply substitute into the relevant age formula, which yields

$$t = \frac{2.42\text{ s}}{\sqrt{g_{\text{eff}}}}\left(\frac{\text{MeV}}{k_BT}\right)^2 = \frac{2.42\text{ s}}{\sqrt{96.25}}\left(\frac{\text{MeV}}{57700\text{ MeV}}\right) = 7.41\times10^{-11}\text{ s}.$$

We then look at the product Γt to see if this process keeps it in thermal equilibrium

$$\Gamma t = \left(2.0\times10^{24}\text{ s}^{-1}\right)\left(7.4\times10^{-11}\text{ s}\right) = 1.5\times10^{14}.$$

The result is large, so obviously this could have kept things in thermal equilibrium.

27. In class we assumed that matter was gathered into galaxies at a late event, but could it have happened at the time of nucleosynthesis, *t* = 200 s?

(a) Would this have been during the matter or radiation dominated eras? What was the temperature at this time? If you need it, assume the current value of $g_{\text{eff}} = 3.36$.

Matter-radiation equality was at several thousand years, so at 200 s we are certainly in the radiation dominated era. Rearranging the time-temperature relation, we have

$$\left(\frac{k_B T}{\text{MeV}}\right)^2 = \frac{2.42\text{ s}}{t\sqrt{g_{\text{eff}}}} = \frac{2.42\text{ s}}{(200\text{ s})\sqrt{3.36}} = 0.00660,$$

$$k_B T = \sqrt{0.00660}\text{ MeV} = 0.0812\text{ MeV} = 81.2\text{ keV},$$

$$T = \frac{81200\text{ eV}}{8.617\times10^{-5}\text{ eV/K}} = 9.43\times10^{8}\text{ K}.$$

(b) By what factor has the universe grown since this time?

In general, you have to keep track of how much reheating occurred, but since there was none between then and now, the temperature follows the usual rule where aT is constant, so that $aT = a_0 T_0$ and therefore

$$\frac{a_0}{a} = \frac{T}{T_0} = \frac{9.43\times10^{8}\text{ K}}{2.725\text{ K}} = 3.46\times10^{8}.$$

(c) The nearest large galaxy to us, the Andromeda galaxy, is currently approximately 760 kpc away. How far away would it have been at this time, in m, if it existed then?

We simply take the distance and divide by the amount the universe expanded, so we would have

$$d = d_0\frac{a}{a_0} = \frac{7.60\times10^{5}\text{ pc}}{3.46\times10^{8}} = (0.002197\text{ pc})(3.086\times10^{16}\text{ m/pc}) = 6.78\times10^{13}\text{ m}.$$

(d) Assume that at nucleosynthesis, the baryons were somehow gathered together at the speed of light. How far could they have traveled at the time of nucleosynthesis? Comparing with answer (c), could baryons have separated on the scale necessary to form galaxies at this time?

We simply assume that no "gathering" can occur faster than the speed of light, so the distance things could have been gathered at this time is limited to

$$d_{\max} = ct = (2.998\times10^{8}\text{ m/s})(200\text{ s}) = 6.00\times10^{10}\text{ m}.$$

This is three orders of magnitude too small to create objects separated on the scale of Andromeda and the Milky Way, so no, this is not a plausible scenario for gathering baryons.

28. The current mass density of radiation, matter, and cosmological constant (in atomic mass units u) are roughly

$$\rho_{m0} = 1.599 \text{ u/m}^3, \quad \rho_{\Lambda 0} = 3.593 \text{ u/m}^3, \quad \rho_{r0} = 4.697\times10^{-4} \text{ u/m}^3$$

The neutrinos have been included in the radiation in an appropriate manner.

(a) Write a formula for the density of these three concentrations ρ_m, ρ_Λ and ρ_r at all times in terms of the red-shift factor $z+1$.

We know that matter scales as $\rho_m \propto a^{-3}$, $\rho_r \propto a^{-4}$, and the cosmological constant contribution is constant. This means, for example, that $\rho_m = \rho_{m0}\left(a_0/a\right)^3 = \rho_{m0}\left(z+1\right)^3$. Using similar arguments for radiation, and keeping in mind that ρ_Λ is constant, we have

$$\rho_m = 1.599\left(z+1\right)^3 \text{ u/m}^3, \quad \rho_\Lambda = 3.593 \text{ u/m}^3, \quad \rho_r = 4.697\times10^{-4}\left(z+1\right)^4 \text{ u/m}^3 .$$

(b) Find the value of z +1 when the radiation equals the cosmological constant. At that time, find the density of the matter, cosmological constant, and radiation. What dominated the universe?

We set $\rho_\Lambda = \rho_r$ and solve for z + 1:

$$\begin{aligned}
3.593 \text{ u/m}^3 &= 4.697\times10^{-4}\left(z+1\right)^4 \text{ u/m}^3 . \\
\left(z+1\right)^4 &= \frac{3.593 \text{ u/m}^3}{4.697\times10^{-4} \text{u/m}^3} = 7650., \\
z+1 &= \left(7650\right)^{1/4} = 9.352.
\end{aligned}$$

We now simply substitute this value into the three formulas for the densities:

$$\begin{aligned}
\rho_m &= 1.599\left(z+1\right)^3 \text{ u/m}^3 = 1.599\times 9.352^3 \text{ u/m}^3 = 1308 \text{ u/m}^3, \\
\rho_\Lambda &= 3.593 \text{ u/m}^3, \\
\rho_r &= 4.697\times10^{-4}\left(z+1\right)^4 \text{ u/m}^3 = \left(4.697\times10^{-4}\right)9.352^4 \text{ u/m}^3 = 3.593 \text{ u/m}^3.
\end{aligned}$$

Of course, the last two numbers came out identical, because that's how we chose them. Obviously, matter dominated the universe, since it is the biggest number.

(c) How old was the universe at this time?

We use the formula for matter domination, which gives

$$t = \frac{17.3 \text{ Gyr}}{\left(z+1\right)^{3/2}} = \frac{17.3 \text{ Gyr}}{\left(9.352\right)^{1.5}} = 0.605 \text{ Gyr} = 605 \text{ Myr}.$$

29. Are the primordial neutrinos currently in thermal equilibrium? Assume neutrinos are massless, have a number density of $3.36\times10^{-8}\ \text{m}^3$, and a temperature of $T_\nu = 1.945\ \text{K}$. The cross section for two particles to interact weakly are typically given by $\sigma = AE_1E_2$ where $A = 4.85\times10^{-60}\ \text{m}^2\cdot\text{eV}^{-2}$ and E_1 and E_2 are the energies of the two particles.

(a) What is the typical energy of a neutrino at this temperature?

The typical energy of a massless particle at temperature *T* is given by $3k_BT$, so this gives us

$$E_1 \approx E_2 = 3k_BT_\nu = 3\left(8.617\times10^{-5}\ \text{eV/K}\right)1.945\ \text{K} = 5.03\times10^{-4}\ \text{eV}.$$

(b) Find a typical cross-section for neutrino-neutrino scattering.

We simply use the formula

$$\sigma = AE_1E_2 = \left(4.85\times10^{-60}\ \text{m}^2\cdot\text{eV}^{-2}\right)\left(5.03\times10^{-4}\ \text{eV}\right)^2 = 1.227\times10^{-66}\,\text{m}^2\ .$$

(c) Since we assumed neutrinos are massless, what is their velocity? Find the rate for neutrino-neutrino scattering today. You can use any appropriate approximation for the relative velocity $|\Delta v|$.

Each of the neutrinos will have velocity *c*, because they are massless. The *relative* velocity could be anything between 0 and 2*c*, because they could be colliding head-on or they could be nearly parallel. A nice round estimate would be $|\Delta v| = c$. We then get the rate of collisions using

$$\Gamma = n\sigma|\Delta v| = \left(3.36\times10^8\ \text{m}^{-3}\right)\left(1.227\times10^{-66}\,\text{m}^2\right)\left(2.998\times10^8\ \text{m/s}\right) = 1.236\times10^{-49}\ \text{s}^{-1}\ .$$

(d) Based on the approximate age of the universe today, how many scatterings of this type will a typical neutrino have undergone? Will this process keep it in equilibrium?

The universe is about 13.8 Gyr old, so we multiply this by the rate to give

$$N = \Gamma t = \left(1.236\times10^{-49}\ \text{s}^{-1}\right)\left(13.8\times10^9\ \text{y}\right)\left(3.156\times10^7\ \text{s/y}\right) = 5.38\times10^{-32}\ .$$

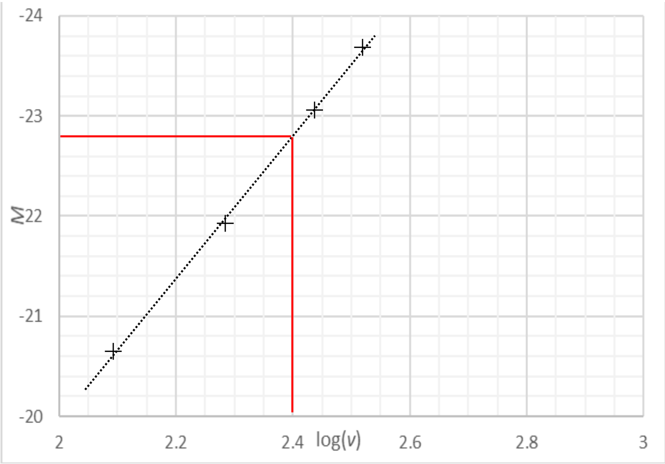
This number is much, much less than one, so neutrinos are not currently kept in equilibrium by this process. Neutrinos are approximately thermal because they *were* in thermal equilibrium early on, and if they are massless, they will remain in a thermal distribution.

30. The Tully-Fisher relationship is a relationship for spiral galaxies between the asymptotic rotational velocity *v* and the absolute magnitude *M* of the galaxy.

(a) Calculate log(*v*) for each of the galaxies listed. Then plot log(*v*) vs. absolute magnitude *M*. You may put your answers directly on the provided table and chart.

Gal-axy	*v* km/s	log(*v*)	*M*
A	123	2.09	–20.64
B	332	2.52	–23.66
C	275	2.44	–23.03
D	191	2.28	–21.93

The values and the points have been incorporated into the table and on the graph at right.



(b) Explain roughly why the Tully Fisher relationship might be a good distance indicator. A trend line on the graph might help.

As you can see from the trend line drawn, there is a pretty good linear relationship between the logarithm of the velocity and the absolute magnitude. Hence by measuring the velocities, we can deduce the absolute magnitude, and then from the apparent magnitude we can get the distance.

(c) Galaxy *X* is a spiral galaxy with rotational velocity *v* = 250 km/s and apparent magnitude *m* = 12.03. Estimate the distance to galaxy *X*.

The velocity has a logarithm $\log(v) = \log(250) = 2.40$. Looking at the graph, you can see that the corresponding absolute magnitude is about $M = -22.80$. We can then find the distance using the formula

$$d = 10^{1+\frac{12.03-(-22.80)}{5}}\ \text{pc} = 10^{7.97}\ \text{pc} = 9.25\times10^{7}\ \text{pc} = 92.5\ \text{Mpc}.$$

	Mass (MeV/c^2)	spin	strange	type
Σ^0	1192	½	–1	baryon
Λ^0	1116	½	–1	baryon
n^0	938	½	0	baryon
π^0	135	0	0	meson
$\bar{K}^0$	498	0	–1	meson
ν	0	½	0	neutrino
γ	0	1	0	photon

31. The Σ^0 baryon almost always decays to two neutral particles chosen from the following list:

$$\bar{K}^0\pi^0,\quad \Lambda^0\nu,\quad \Lambda^0\gamma,\quad n^0\gamma,\quad n^0\bar{K}^0$$

(a) Which of these are impossible? For those that are possible, classify them as strong, weak, or electromagnetic.

Because all the particles are neutral, charge is automatically conserved. Since the Σ^0 is a baryon, there must be one baryon on the right, and since this is violated by the $\bar{K}^0\pi^0$ decay, this one is **impossible**. Since the Σ^0 is a fermion, we must have an odd number (one) of fermions on the right, but both of the particles in $\Lambda^0\nu$ are fermions, so this is also **impossible**. Energy is conserved, and for a decay that means the sum of the masses of the particles on the right must not exceed 1192 MeV. This is violated by the final decay, so $n^0\bar{K}^0$ is **impossible**.

The remaining two decays are possible and involve photons, which are not strongly interacting, so they can't be strong decays. The first one, $\Lambda^0\gamma$, does not violate strangeness, so it can proceed via **electromagnetic** decay, while the other, $n^0\gamma$, violates strangeness, and therefore is a **weak** process.

(b) Which one is probably the most likely to occur?

Strong processes are the fastest, then electromagnetic, and finally weak processes. Since there are no strong decays possible, it will decay in the electromagnetic channel, $\Sigma^0 \to \Lambda^0\gamma$.

(c) All of the baryons and mesons listed have only up, down and strange quarks and/or their anti-quarks. Which quarks or anti-quarks make up the Σ^0?

Since it is a baryon, it has three quarks, and with strangeness –1 it must have one strange quark with charge $-\frac{1}{3}$. To make the total charge come out zero, we must have $q = -\frac{1}{3} - \frac{1}{3} + \frac{2}{3}$, so the other quarks must be one up and one down, so the total quark content is **[uds]**.