Physics 310/610 – Cosmology

Homework Set H

1. In class we modeled both the thin disk and thick disk as stars with number density given by

$$n = n_0 e^{-R/h_r} e^{-|z|/h_z}$$

For the thin disk, assume $h_r = 3500$ pc and $h_z = 350$ pc. For the thick disk, $h_r = 3500$ pc and $h_z = 1200$ pc.

(a) Find a formula for the total number of stars in terms of n_0, h_r, and h_z. Make sure you are working in cylindrical coordinates!

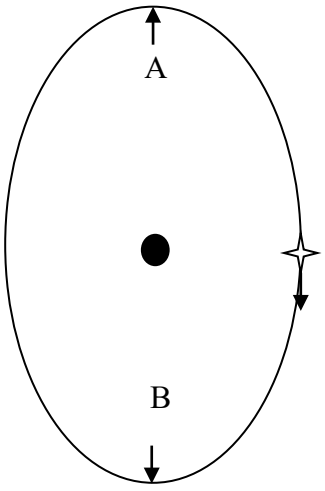
(b) What fraction of the stars are at a radius $R < R_0$ for any given R_0? Find the fraction of the disk stars closer than the Sun at $R_0 = 8300$ pc.

(c) The Sun is about at $z = 0$, and the local density of thin disk stars is about 0.14 pc^{-3}. Find the central density n_0, and the total number of stars in the thin disk. Multiply by 0.6 $M_\odot$ to get the approximate mass of the thin disk.

(d) The local density of thick disk stars is about 0.002 pc^{-3}. Repeat part (c) for the thick disk. What fraction of the disk stars are thick disk stars?

2. Find the Schwarzschild radius for a black hole with mass equal to the mass of the Earth (in cm), for the Sun (in km) and for the black hole at the center of our galaxy (in AU) assuming a mass of $M = 4.3 \times 10^6 M_\odot$. In the case of the Earth, draw a circle of approximately the correct radius for such a black hole on your paper. If it is too big to fit on your paper, or if it is too small to see, you have made an error.

PHY 610 – Do the following problem only if you are in PHY 610



3. An alien race with a similar situation to ours suspects there is a black hole at the center of their galaxy. They observe that it is circled by a star which they presume is moving in a perfectly circular orbit. The orbit does not *look* circular because the orbit it tilted with respect to the angle of observation. At right is the orbit as measured, with 1 cm representing 10 mas.

(a) Based on the shape of the observed orbit, what is the tilt of the orbit α compared to the normal? The angle is defined so that if $\alpha = 0°$ it would look like a perfect circle, and if $\alpha = 90°$ it would look like a flat line.

(b) A spectral line that would normally be at 589.00 nm is discovered to have a wavelength of 592.29 nm at point A and 585.73 at point B. At the position drawn, it is very close to the normal wavelength. What is the approximate speed of the star around the black hole?

(c) The star completes an entire orbit in 8.10 Earth years. What is the radius of the orbit, in AU?

(d) What is the mass of the black hole, in solar masses?

(e) What is the distance of the black hole?