Physics 310/610 – Cosmology

Homework Set O

1. In class, we found that in the future, the size of the universe will grow exponentially, $a \propto \exp(H_\Lambda t)$.
   (a) Using our best estimates of H_0 and Ω_Λ, find H_Λ in Gyr^{-1}. A good estimate of the distance to the edge of the visible universe at that time would be $d_{\max} = c/H_\Lambda$. Find $d_{\max}$ in Gpc.
   (b) At present, the nearest galactic cluster to the local group is about at a distance of 3.3 Mpc. Assuming it participates in the general expansion of the universe, how far in the future will it be until it reaches the distance $d_{\max}$.
   (c) We know about the big bang largely because of the cosmic microwave background radiation. Find the peak wavelength for the $\lambda_{\max}$ for the current cosmic microwave background radiation. This radiation is theoretically undetectable when $\lambda_{\max}$ exceeds $d_{\max}$, due to the expansion of the Universe. How long in the future will this occur?

2. Estimate the age of the universe (in convenient multiples of the year), the red shift z, the temperature T in K, and the characteristic energy $k_B T$ for each of the following events:

Event	z	**T (K)**	**$k_B T$ (eV)**	**Age**
Reionization	10.5			
Room Temp		300.		
Recombination			0.256	

   (a) Reionization of the universe at $z = 10.5$.
   (b) Universe is at room temperature $T = 300$ K.
   (c) Recombination $k_B T = 0.256 \text{ eV}$.

3. The number density of photons in a thermal distribution is given by

$$n_\gamma = \frac{2\zeta(3)}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3 \quad \text{where} \quad \zeta(3) = \sum_{n=1}^{\infty} \frac{1}{n^3} \approx 1.202$$

   (a) Find a general formula for the average energy of a photon, given by $\bar{E} = u/n$. Hint: your instructor uses the approximation $\bar{E} = 3k_B T$.
   (b) Find the current density of background photons in the universe, and the ratio of photons to baryons, n_B/n_γ.

Graduate Problem: Only do this problem if you are in PHY 610

4. The 4d metric (assuming the universe is flat) is given by

$$ds^2 = -c^2 dt^2 + a^2(t)\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

where in the future, $a(t) \approx a_0 \exp(H_\Lambda t)$, where a_0 is the size of the universe now, and t is the time starting from now.

(a) Suppose we have an incoming photon moving directly towards us (photons always have $ds = 0$). Find an equation for dr/dt.

(b) Solve the equation from part (a) so you can get $r(t)$ for an incoming photon

(c) Show that at *any* time in the future, there is a distance d_{max} such that a photon leaving from d_{max} at time t will never reach us. The distance to an object at time t is given by $ra(t)$. You should find that d_{max} is independent of time.