Physics 310/610 – Cosmology

Homework Set S

1. For this problem we are going to predict properties of an undiscovered particle(s) and predict its properties. We are going to make a table of the nine lightest spin 3/2 baryons
   (a) Looking at all the spin 3/2 baryons, divide them into categories based on their strangeness. Invent a generic name for each category. Note that there is a relatively small mass difference *among* the particles in each category.
   (b) Now make a table, with one row for each category. The columns should contain the following information (i) your generic name, (ii) strangeness, (iii) number of particles in that category (iv) average electric charge (v) average mass.
   (c) There is an additional row which corresponds to an undiscovered category, called Ω. By extrapolation, predict all of the entries of the additional row.
   (d) Look up the properties of the "Omega baryon," and see how well you did on your predictions. For any that are wrong, calculate the percent error.

2. For this problem we are going to need to calculate the *hypercharge* of various particles, which is defined as $Y = S + B$, the sum of strangeness and baryon number, and the *third component of isospin*, $I_3 = Q - \frac{1}{2}Y$, the sum of charge and half of hypercharge.
   (a) Find these quantities for the eight lightest spin-0 mesons. Plot the result as points on a 2D graph, with I_3 on the horizontal axis, and Y on the vertical axis. If two particles have the same value, designate it by drawing a circle around a dot.
   (b) Repeat for the eight lightest spin-½ baryons.

3. For each of the following particles (which are made only of up, down, and strange quarks), predict the quark content, based on their baryon number, charge, and strangeness. Denote your answers using *u*, *d*, or *s* for quarks, and $\bar{u}$, $\bar{d}$ and $\bar{s}$ for anti-quarks. It is possible for two particles to have the same quark content
   (a) Λ^0 (b) K^+ (c) $\bar{\Sigma}^+$ (anti-particle of Σ^-) (d) Σ^0 (e) π^-

Graduate Problem: Do the following problem only if you are in PHY 610.

4. There is a symmetry of strong interactions called *isospin*. We will explore its properties in the nucleons, a two state system consisting of protons $|p\rangle$ and neutrons $|n\rangle$
   (a) Define the I_3 operator as was done in problem 2. Write I_3 as a 2×2 matrix in $\{|p\rangle,|n\rangle\}$ space. Since I_3 is conserved in all interactions, it commutes with the Hamiltonian.
   (b) Consider the operator $2I_1$, which converts protons to neutrons and vice-versa. Write I_1 as a 2×2 matrix. Since protons and neutrons interact the same in strong interactions, I_1 commutes with the strong part of the Hamiltonian
   (c) Define I_2 by the commutator $[I_3, I_1] = iI_2$. Write I_2 as a 2×2 matrix.
   (d) You already have $[I_3, I_1]$. Work out $[I_1, I_2]$ and $[I_2, I_3]$.