Physics 310/610 – Cosmology

Solution Set C

1. **[15] Telescopes on Earth have background light coming from the atmosphere that limits how dim of objects they can see. The James Webb space telescope, based in space, with an effective diameter of *d* = 5.68 m, does not have this problem**
   (a) **[7] The dimmest objects detectable by the Webb have a bolometric magnitude of approximately m = 34. What is the corresponding brightness, in W/m²? Convert this into watts by multiplying by the collecting area of the telescopes.**

   Brightness is related to bolometric magnitude by the formula

$$F = 2.52\times10^{-8}\,\text{W/m}^2\left(10^{-\frac{2}{5}m}\right) = 2.52\times10^{-8}\,\text{W/m}^2\left(10^{-\frac{2}{5}\cdot 34}\right) = 6.33\times10^{-22}\ \text{W/m}^2\,.$$

The mirror has a radius of 2.84 m, so the total power is about

$$P = \pi R^2 F = \pi\left(2.84\ \text{m}\right)^2\left(6.33\times10^{-22}\ \text{W/m}^2\right) = 1.604\times10^{-20}\ \text{W}\,.$$

   (b) **[4] Assume this light is primarily in the infrared, with wavelength λ = 20 μm. Find the energy of a single photon with this wavelength.**

   The energy of a single photon with this wavelength would be

$$E = h\nu = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.998\times10^{8}\ \text{m/s}\right)}{20\times10^{-6}\,\text{m}} = 9.93\times10^{-21}\ \text{J}\,.$$

   (c) **[4] Find the number of photons falling on the Webb telescope per second, and the number collected in a one hour exposure.**

   The rate of photons would just be the power divided by the energy, so

$$\Gamma = \frac{P}{E} = \frac{1.604\times10^{-20}\ \text{J/s}}{9.93\times10^{-21}\ \text{J}} = 1.615\ \text{s}^{-1}\,,$$

$$N = \Gamma t = \left(1.615\ \text{s}^{-1}\right)\left(3600\ \text{s}\right) = 5810\,.$$

So the detection of such stars may rely, if done with a one hour exposure, on only a few thousand photons. Because no detector is 100% efficient, it might be using even fewer; furthermore, most nearby starts produce power primarily in the visible, not the infrared spectrum.

2. **[15] The nearest "star," α-Cen, is actually a double star. α-Cen A has apparent magnitude m_A = 0.01, and α-Cen B has apparent magnitude m_B = 1.33**
   (a) **[5] What is the ratio of the brightness of these two stars, F_A/F_B ? Assuming these two stars are at the same distance from us, can you conclude anything about their relative luminosities, L_A/L_B ?**

The flux is given by $F = k \cdot 10^{-\frac{2}{5}m}$. As we shall see, the constant *k* is irrelevant to this discussion. We therefore have

$$\frac{F_A}{F_B} = \frac{k \cdot 10^{-\frac{2}{5}m_A}}{k \cdot 10^{-\frac{2}{5}m_B}} = 10^{-\frac{2}{5}m_A + \frac{2}{5}m_B} = 10^{\frac{2}{5}(1.33-0.01)} = 3.37\,.$$

Now, the luminosity is related to the brightness by $F = L/4\pi d^2$, or $L = 4\pi d^2 F$. So we have

$$\frac{L_A}{L_B} = \frac{4\pi d^2 F_A}{4\pi d^2 F_B} = \frac{F_A}{F_B} = 3.37\,.$$

   (b) **[5] Suppose that you view the two stars together, so they look like a single star. What would be the corresponding magnitude for the two stars together?**

The total flux would be

$$F = F_A + F_B = k \cdot 10^{-\frac{2}{5}(0.01)} + k10^{-\frac{2}{5}(1.33)} = 0.991k + 0.294k = 1.285k\,.$$

We then equate this to the general formula relating brightness to apparent magnitude and solve for *m*:

$$F = 1.285k = k \cdot 10^{-\frac{2}{5}m}\,,$$
$$\log(1.285) = -\tfrac{2}{5}m\,,$$
$$m = -\tfrac{5}{2}\log(1.284) = -0.272\,.$$

   (c) **[5] These stars are each actually at a distance of *d* = 1.325 pc. What is each of their absolute magnitudes M_A and M_B?**

We use the general formula relating the two types of magnitudes and distances, namely, $m - M = 5\log d - 5$, or solving for *M*, $M = m + 5 - 5\log d$. We therefore have

$$M_A = m_A + 5 - 5\log d = 0.01 + 5 - 5\log(1.325) = 4.40,$$
$$M_B = m_B + 5 - 5\log d = 1.33 + 5 - 5\log(1.325) = 5.72.$$

Graduate Problems – No extra problems for PHY 610