Physics 310/610 – Cosmology

Solution Set U

1. **[15] There is an additional problem with neutrinos as dark matter: It turns out the Pauli Exclusion Principle makes it hard to fit them in a galaxy. The computation is a bit complex, but we can approximate it pretty well as follows:**
   (a) The local density of dark matter in the neighborhood of the Sun is around 0.4 GeV/*c*²/cm³. Assuming all three neutrinos weigh $3.7\ \mathrm{eV}/c^2$ each, what is the local number density *n* of neutrinos in m⁻³? Then divide by three to get the number density of just one of the three types of neutrinos.

The density is simply $n = \rho/m$, so

$$\sum_{i=1}^{3} n_{\nu i} = \frac{0.4\ \mathrm{GeV}/c^2/\mathrm{cm}^3}{3.7\ \mathrm{eV}/c^2}\left(\frac{100\ \mathrm{cm}}{\mathrm{m}}\right)^3 \frac{10^9\ \mathrm{eV}}{\mathrm{GeV}} = 1.08\times10^{14}\ \mathrm{m}^{-3}$$

We then divide by three to get $n_{\nu i} = 3.60\times10^{13}\ \mathrm{m}^{-3}$.

(b) Suppose I placed each neutrino in a box of volume *L*³. What would be the size of the box such that the density of neutrinos would match part (a)?

If we have one neutrino per box of volume L^3, then the density is $n = L^{-3}$, so

$$L = n^{-1/3} = \left(3.60\times10^{-13}\ \mathrm{m}^{-3}\right)^{-1/3} = 3.03\times10^{-5}\ \mathrm{m}.$$

(c) According to quantum mechanics, to fit a particle in a box of size *L* would require it to acquire a momentum of $p = \pi\hbar/L$. Work out the corresponding momentum. The most convenient units for this would be eV/c.

There are a lot of complications we are ignoring here, such as ignoring spin, treating the particles as if they are in one dimension, etc. If you do the entire calculation correctly, you would find that this formulas is only about 2% wrong. Using the formulas we were given, we have

$$p = \frac{\pi\hbar}{L} = \frac{\pi\hbar c}{cL} = \frac{\pi\left(197.33\ \mathrm{eV}\cdot\mathrm{nm}\right)}{c\left(3.03\times10^{-5}\ \mathrm{m}\right)\left(10^9\ \mathrm{nm/m}\right)} = 0.0205\ \mathrm{eV}/c\,.$$

(d) What is the corresponding velocity of the neutrinos? Compare to the approximate escape velocity of a galaxy like ours, probably around 600 km/s.

Since pc is less than the rest mass, the particle is non-relativistic, and the velocity is given by

$$v = \frac{p}{m} = \frac{0.0208 \text{ eV}/c}{3.7 \text{ eV}/c^2} = .00553c = .00553\left(299,800 \text{ km/s}\right) = 1660 \text{ km/s}\,.$$

This is significantly higher than escape velocity. Basically, the Pauli exclusion principle makes it hard to pack this many neutrinos into this small a volume.

2. [15] Suppose the universe is filled with matter in some regions, and anti-matter in others. Let's imagine the segregation occurred at the time of quark confinement, so somehow the quarks congregated together, and so did the anti-quarks. How large could these regions be?

(a) Assuming the quarks and anti-quarks are gathered at the speed of light, what is the largest region *at the time* over which quarks could be gathered together?

Quarks, like other particles, can travel no faster than the speed of light. Checking our notes, quark confinement occurred when the universe was about $t = 1.4\times10^{-5}\text{ s}$. If such a segregation occurred at this time, the quarks can move no farther than

$$d = ct = \left(3.00\times10^{8}\text{ m/s}\right)\left(1.4\times10^{-5}\text{ s}\right) = 4200\text{ m}.$$

(b) Assuming the universe has kept *aT* constant since then (a fair approximation), how much has the universe grown since then? How large is this region now?

Again consulting notes, the temperature at the time was $k_B T = 150\text{ MeV}$. The universe has grown by a factor of the ratio of temperatures, which is about

$$\frac{a_0}{a} \approx \frac{T}{T_0} = \frac{k_B T}{k_B T_0} = \frac{\left(150\text{ MeV}\right)\left(10^{6}\text{ eV/MeV}\right)}{\left(8.617\times10^{-5}\text{ eV/K}\right)\left(2.725\text{ K}\right)} = 6.39\times10^{11}$$

As a consequence, the region will have grown to a size

$$d_0 = d\frac{a_0}{a} = \left(4200\text{ m}\right)\left(6.39\times10^{11}\right) = 2.68\times10^{15}\text{ m}.$$

This is about 0.09 pc.

(c) Find the baryonic mass of spheres of size given in part (b) using the density of baryons today $\rho_{b0} = 4.196\times10^{-28}\text{ kg/m}^3$.

We now multiply the volume of a sphere times the mass density, which gives us

$$M = \tfrac{4}{3}\pi d_0^3 \rho_{b0} = \tfrac{4}{3}\pi\left(2.68\times10^{15}\text{ m}\right)^3\left(4.196\times10^{-28}\text{ kg/m}^3\right) = 3.38\times10^{19}\text{ kg}.$$

(d) As a basis of comparison, look up the Earth's mass. Is it plausible that this mechanism might produce pockets of matter as observed today?

The mass of the Earth is $5.97\times10^{24}\text{ kg}$. So we would have pockets of matter and anti-matter, but they would be smaller than the Earth. This is not plausible, since we know the universe is matter on a scale much larger than the Earth.

Graduate Problems: There are no problems for PHY 610 on this homework.