The Big Bang Theory

Ways of Labeling Past Events

Time in the Universe

- We can see the universe is expanding
- Extrapolating backwards, we conjecture that there is a time when everything was together

- We will call this event *The Big Bang*

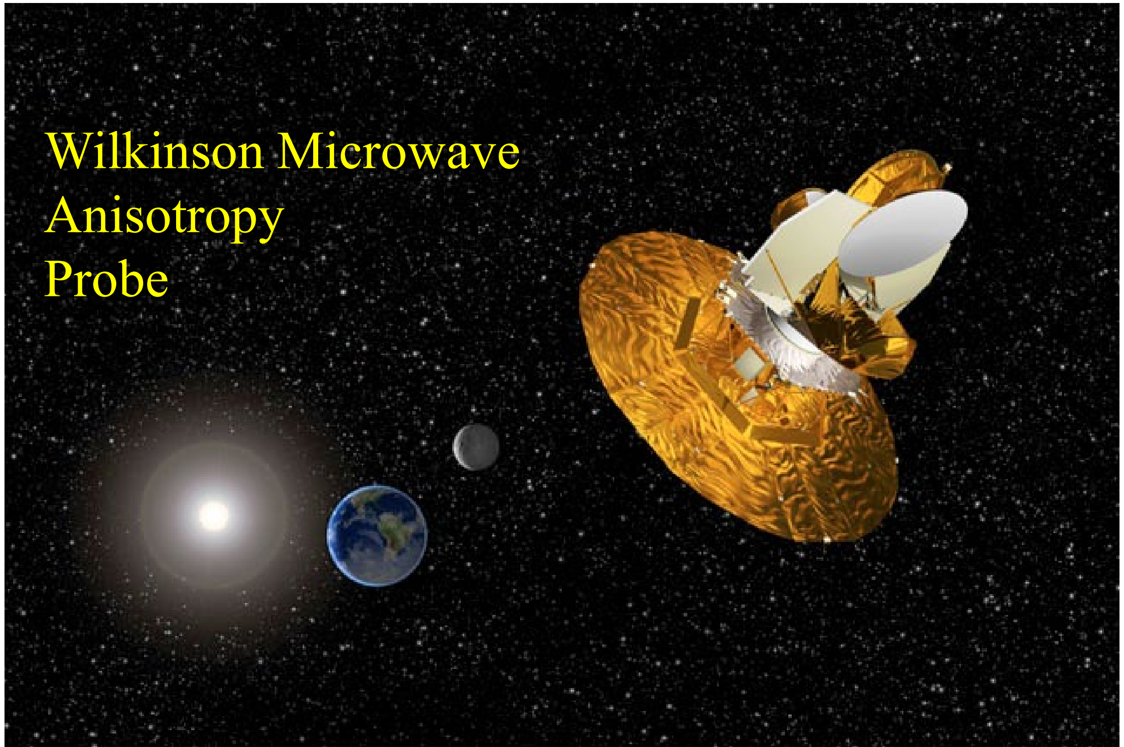
- We will call this time $t = 0$
  - Though in fact, we can't actually extrapolate all the way to $t = 0$
- The big bang theory really describes all the things that happen *after* $t = 0$

- As we work our way backwards, we will discover that conditions/temperatures/etc. become increasingly difficult to do experiments
- Therefore, we pass from knowledge to speculation the earlier we get

- Any comments about $t = 0$ or even $t < 0$ are pretty much pure speculation

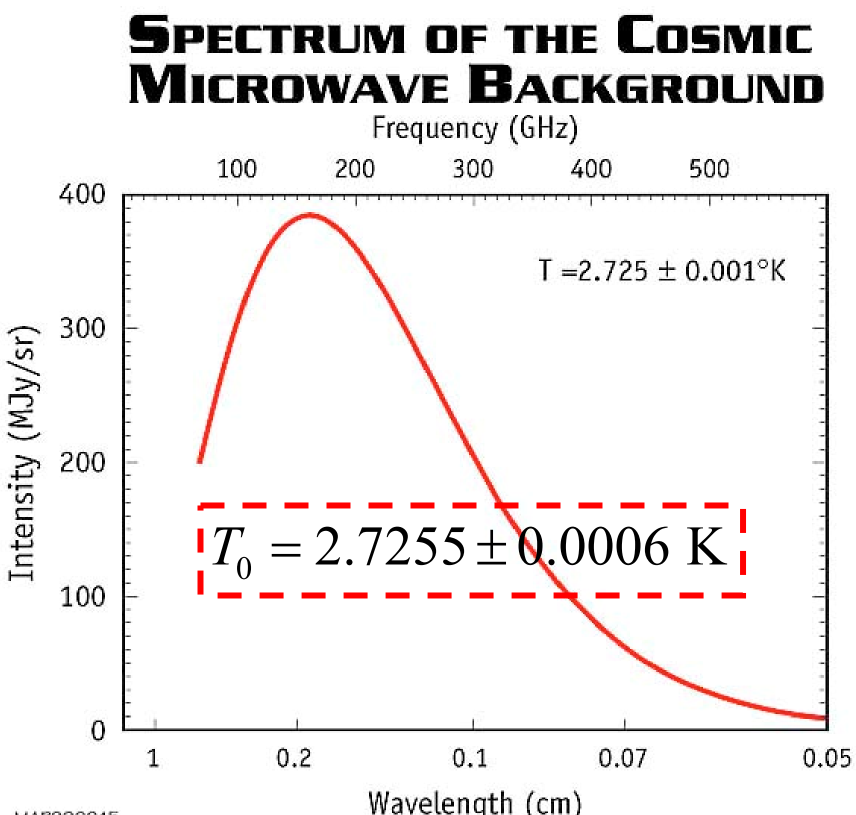
The Cosmic Microwave Background Radiation

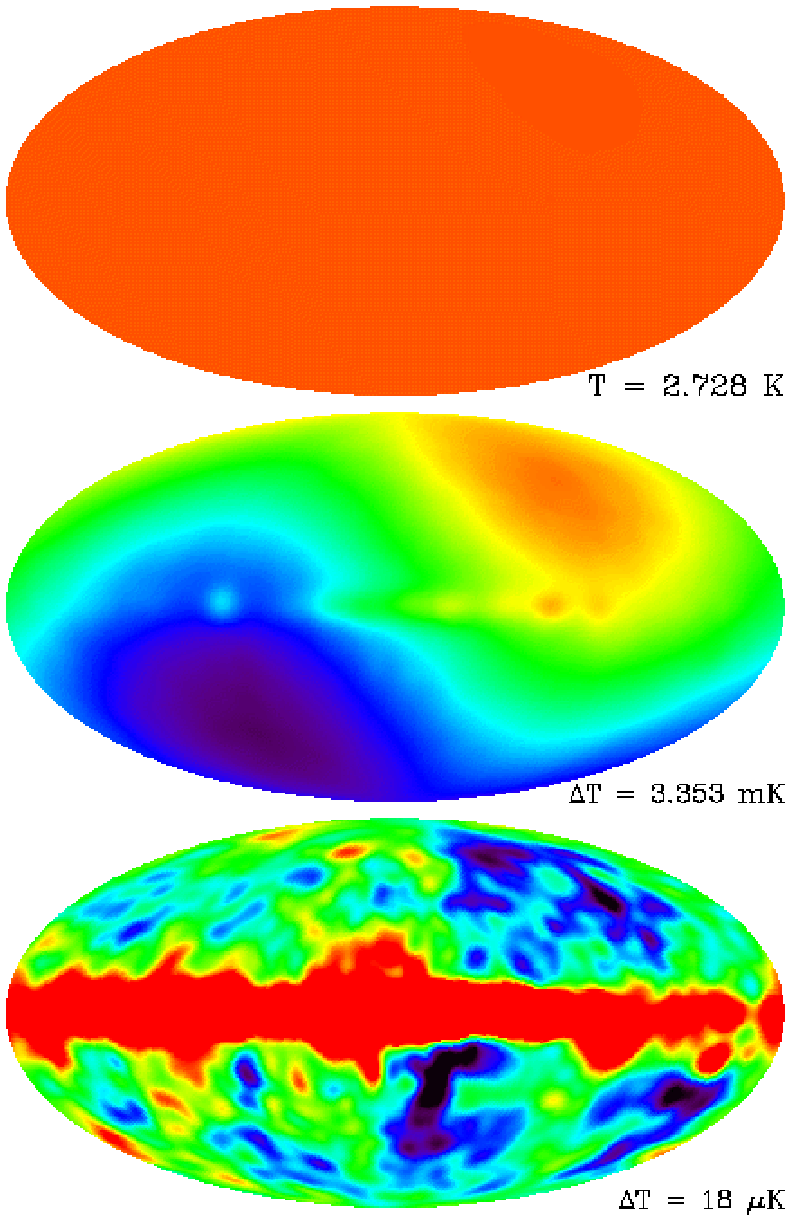
- In 1964, Arno Allan Penzias and Robert Woodrow Wilson discovered microwaves of extraterrestrial origin
- Since then, dedicated spacecraft have been sent up to study it in great detail
- These observations allow us to see *directly* back to when the universe was only a few thousand years old
- Indirectly, they allow us to study the universe almost back to the big bang



The CMBR – What We See

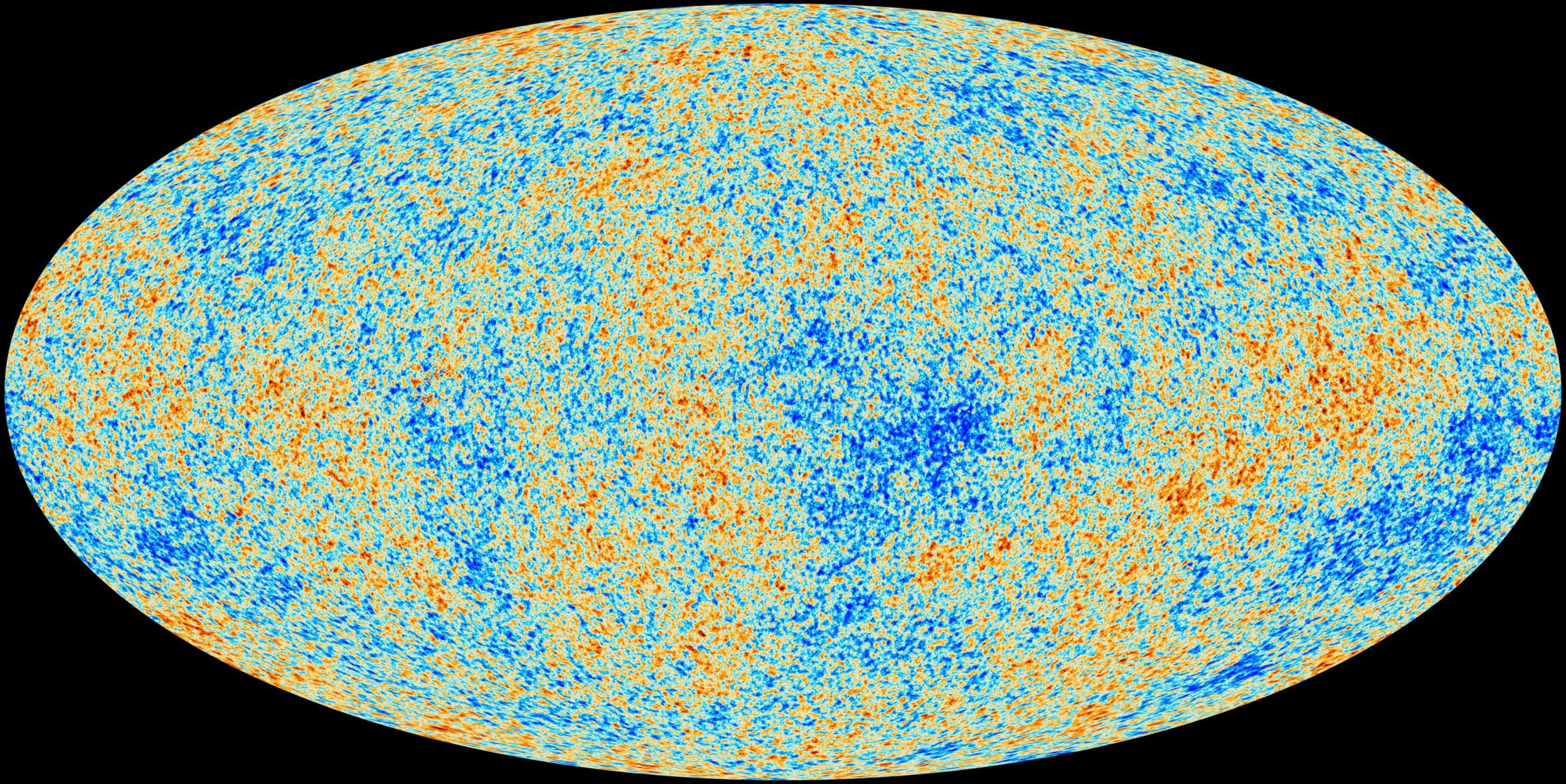
- The CMBR is almost a perfect thermal spectrum
- Furthermore, it is almost exactly isotropic
- Slightly hotter in one direction
- This is due to OUR peculiar velocity – about 370 km/s
    - Which can also be subtracted
- There are foreground sources that also have to be subtracted





The CMBR – The True Variation

- You then subtract foreground sources
- What remains are a few ppm true fluctuations



Radiation in the Evolving Universe

- As the universe expands, the *density* of radiation energy changes
- Number density of radiation changes proportional to a^{-3}

$$n = n_0 \left(\frac{a_0}{a} \right)^3$$

- However, the wavelength *also* expands proportional to a
- This implies frequency decreases proportional to a^{-1}
- Each photon reduces its energy as a^{-1}

$$E = E_0 \left(\frac{a_0}{a} \right)$$

$$\rho_r \propto a^{-4}$$

- Total energy density proportional to a^{-4}
- Mass density proportional to a^{-4}

Today it is thermal at temperature T_0. What was it yesterday?

$$P(E) \propto \exp\left(-\frac{E}{k_B T} \right)$$

- Recall that for thermal distribution, probabilities satisfy
- *Number* of photons in any given state doesn't change
- But the energy of the states does change.

$$P(E) = P(E_0) \propto \exp\left(-\frac{E_0}{k_B T_0} \right) = \exp\left(-\frac{Ea}{k_B T_0 a_0} \right) = \exp\left(-\frac{E}{k_B T} \right) \quad \text{if} \quad T = T_0 \frac{a_0}{a}$$

- A thermal distribution remains thermal at all times
- To explain why it is thermal now, explain why it was thermal in the past

Three Ways of Labeling Past Events

- Past events can be labeled by what time t they happened (s or yr)

- They can also be labeled by their red-shift z, or $z + 1$
  - I won't be using this one much

- We can also label them in terms of temperature T (in K)
  - Or even multiply by Boltzmann's Constant to get energy $k_B T$ (in eV)

- The relation is simple: $$\frac{T}{T_0} = \frac{a_0}{a} = 1 + z$$

- This works well back to very early times
  - Imprecise at high z (10^9) due to electron-positron annihilation
  - Breaks down completely at inflation (10^{27}?)

- Beyond $z = 10^9$ we won't use z at all, but will use $k_B T$

Outline of History of Universe

Time	T or $k_B T$	Events
10^{-43} s	10^{18} GeV	Planck Era; time becomes meaningless?
10^{-39} s	10^{15} GeV	Inflation begins; forces unified
10^{-35} s	10^{14} GeV	Inflation ends; reheating; forces separate; baryosynthesis (?)
10^{-13} s	1500 GeV	Supersymmetry breaking, LSP (dark matter)
10^{-11} s	160 GeV	Electroweak symmetry breaking
14 μs	150 MeV	Quark Confinement
0.4 s	1.5 MeV	Neutrino Decoupling
1.5 s	0.7 MeV	Neutron/Proton freezeout
20 s	170 keV	Electron/Positron annihilation
200 s	80 keV	Nucleosynthesis
57 ky	0.76 eV	Matter-Radiation equality
370 ky	0.26 eV	Recombination
300 My	40 K	First Structure/First Stars
13.8 Gy	2.725 K	Today

The matter era

The Matter Dominated Era

Time–Red-Shift Relation

- In the recent past, the universe was dominated by matter
- The age of the universe *now* is given by:
- Recall: $x = a/a_0 = 1/(1+z)$
- Time-red-shift relation:
- For $z > 1$ or so, the Ω_m/x term dominates, ignore the others

$$t = H_0^{-1}\int_0^{(1+z)^{-1}} \left(\Omega_m/x + \Omega_\Lambda x^2 + 1 - \Omega_m - \Omega_\Lambda\right)^{-1/2} dx$$

$$t = H_0^{-1}\int_0^{(1+z)^{-1}} \left(\Omega_m/x\right)^{-1/2} dx = H_0^{-1}\Omega_m^{-\frac{1}{2}}\int_0^{(1+z)^{-1}} x^{\frac{1}{2}} dx = \tfrac{2}{3}H_0^{-1}\Omega_m^{-\frac{1}{2}}\left(1+z\right)^{-\frac{3}{2}}$$

$$t = \frac{17.26\text{ Gyr}}{\left(1+z\right)^{3/2}}$$

- Substitute $H_0 = 67.7$ km/s/Mpc and $\Omega_m = 0.3111$
- Complete actual relation:

$$t = \frac{2H_0^{-1}\sqrt{\Omega_m}}{3\left(1-\Omega_m\right)}\sinh^{-1}\left[\frac{1-\Omega_m}{\Omega_m\left(1+z\right)^{3/2}}\right]$$

- For small z, we neglected Ω_Λ
  - 12% error at $z = 1$, 1% error at $z = 4$
- For large z (z >1000) this formula is wrong because we neglected radiation

Time – Temperature Relations

$$t = \frac{17.26\ \text{Gyr}}{(1+z)^{3/2}}$$

$$\frac{T}{T_0} = \frac{a_0}{a} = 1 + z$$

- Rewrite in terms of temperature
- More meaningful to rewrite in terms of $k_B T$:

$$t = 17.26\ \text{Gyr}\left(\frac{T_0}{T}\right)^{3/2} = 17.26\ \text{Gyr}\left(\frac{k_B T_0}{k_B T}\right)^{3/2} = \frac{(17.26\ \text{Gyr})\left[(2.7255\ \text{K})(8.617\times10^{-5}\ \text{eV/K})\right]^{3/2}}{(k_B T)^{3/2}}$$

$$t = \frac{62.1\ \text{kyr}}{(k_B T/\text{eV})^{1.5}}$$

Recombination

Thermal or Not Thermal?

- For something to be in thermal equilibrium, it must interact with something
- Let Γ by the rate at which something interacts, say by scattering
- To thermalize, it must make *many* scatterings in the age of the universe t
- Number of scatterings is Γt

- If $\Gamma t < 1$, then no scattering
  - Distribution of particles will be unmodified; universe is transparent
- If $\Gamma t >> 1$, then lots of scattering
  - Universe is opaque, particles may become "thermalized"

Cross Section and Rates

- The cross-section σ is the area of the targets that are getting hit by some sort of projectile
  - Units of area: m^2

- The rate will also be proportional to the number density of targets *n*
  - Units of m^{-3}

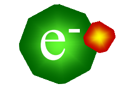
- It will also be proportional to the speed at which they come together
  - Units m/s

- Put it all together

$$\Gamma = n\sigma\left(\Delta v\right)$$

What Thermalizes Photons?

- Photons can scatter off of any charged particle
- Most likely with light particles, like electrons
- Cross section – comes from particle physics
  - k_e is Coulomb's constant
  - m is mass of electron
  - e is electron charge
- This assumes the electrons are free
  - If bound in atoms, cross-section is *much* lower



$$\Gamma = n\sigma(\Delta v)$$

$$\sigma_T = \frac{8\pi}{3}\left(\frac{k_e e^2}{mc^2}\right)^2$$

$$\sigma_T = 6.652\times 10^{-29}\ \mathrm{m}^2$$

What is current density of electrons in the universe?

- Density of "baryons", worked out in homework
- But only 75% of it is actually hydrogen
- Most of them are protons
  - Neutral universe → Comparable # electrons
- The density in the past was higher

$$n_{B0} = \frac{H_0^2\Omega_B}{\frac{8}{3}\pi G m_H} = 0.250\,\mathrm{m}^{-3}$$

$$n_{H0} = 0.188\ \mathrm{m}^{-3}$$

$$n_{e0} \approx 0.188\ \mathrm{m}^{-3}$$

$$n_e = \left(0.188\ \mathrm{m}^{-3}\right)(1+z)^3$$

When Did Photons Get Thermalized?

- Relative velocity is c: $\Delta v = c$ $\Gamma = n\sigma(\Delta v)$ $\sigma_T = 6.652\times10^{-29}\ \mathrm{m}^2$
- Age of universe at red shift z is: $t = \dfrac{17.26\ \mathrm{Gyr}}{(1+z)^{3/2}}$ $n_e = (0.188\ \mathrm{m}^{-3})(1+z)^3$
- Number of collisions is:

$$\Gamma t = n\sigma(\Delta v)t \quad = (0.188\ \mathrm{m}^{-3})(1+z)^3(6.65\times10^{-29}\ \mathrm{m}^2)(2.998\times10^8\ \mathrm{m/s})\frac{17.26\ \mathrm{Gyr}}{(1+z)^{3/2}}\cdot\frac{3.156\times10^{16}\ \mathrm{s}}{\mathrm{Gyr}}$$

$$= 0.00204(1+z)^{3/2}$$

- At present, ($z = 0$), universe is transparent
- In the past ($z \approx 61$) universe is opaque *if* electrons were free
- Electrons are free now because of colliding galaxies, hot stars, etc.
- Before $z = 8.5$, most electrons were bound

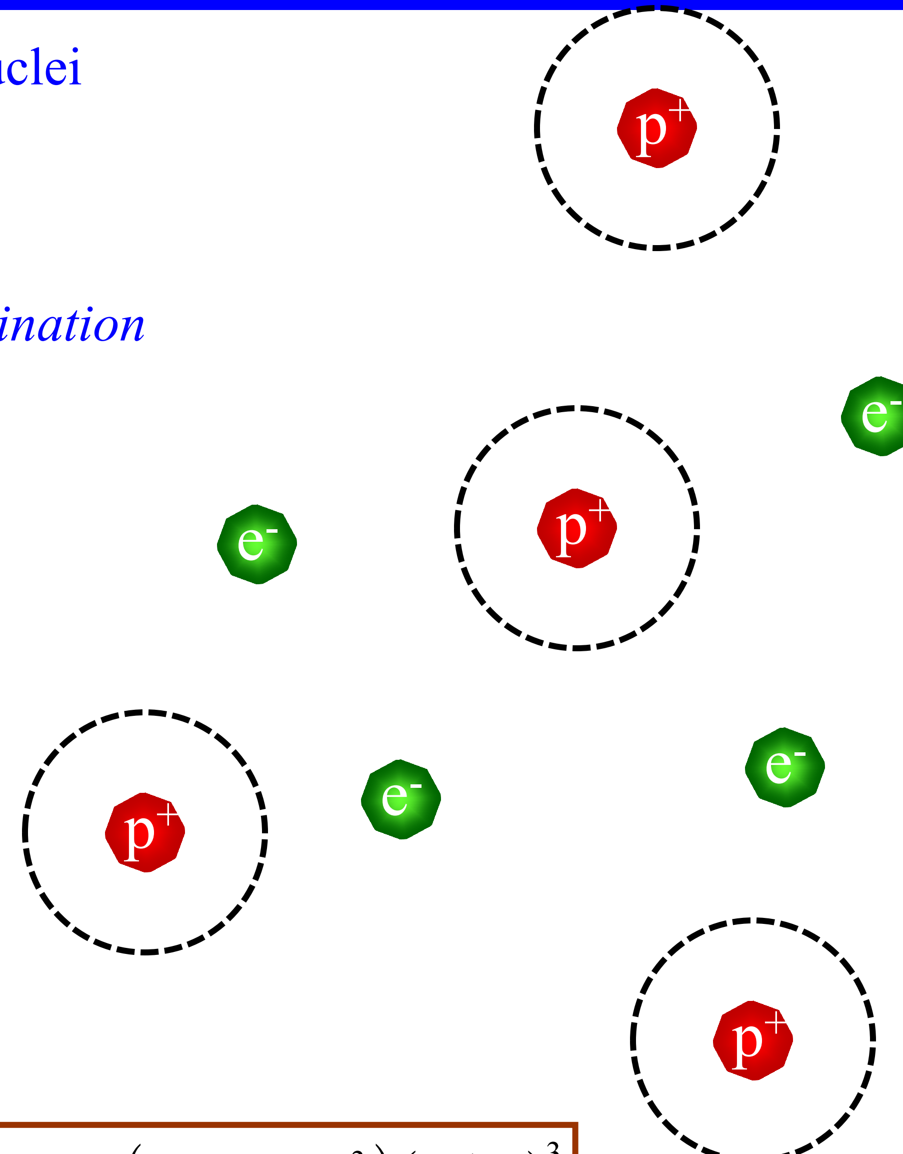
- Temperature at $z \approx 61$ was $T = T_0(1+z) \approx 169\ \mathrm{K}$
- Corresponding energy $k_B T = 0.0146\ \mathrm{eV}$
- Compare to hydrogen binding energy $E_b = 13.6\ \mathrm{eV}$

Outline of History of Universe

Time	T or $k_B T$	Events
10^{-43} s	10^{18} GeV	Planck Era; time becomes meaningless?
10^{-39} s	10^{15} GeV	Inflation begins; forces unified
10^{-35} s	10^{14} GeV	Inflation ends; reheating; forces separate; baryosynthesis (?)
10^{-13} s	1500 GeV	Supersymmetry breaking, LSP (dark matter)
10^{-11} s	160 GeV	Electroweak symmetry breaking
14 μs	150 MeV	Quark Confinement
0.4 s	1.5 MeV	Neutrino Decoupling
1.5 s	0.7 MeV	Neutron/Proton freezeout
20 s	170 keV	Electron/Positron annihilation
200 s	80 keV	Nucleosynthesis
57 ky	0.76 eV	Matter-Radiation equality
370 ky	0.26 eV	Recombination
300 My	40 K	First Structure/First Stars
13.8 Gy	2.725 K	Today

Recombination – What It Is

- The universe has thin gas of atoms, mostly hydrogen nuclei
- There are a matching number of electrons
- At high temperatures, the electrons prefer to be free
- As temperature drops, they are attracted to the nuclei
- The process where they come together is called *recombination*



- Before recombination, the universe is opaque
  - And therefore in thermal equilibrium
- After it, universe is transparent
- The microwave background shows us universe at recombination

- Throughout recombination, universe in thermal equilibrium
- We want to know when this happens.
- We already know density of hydrogen atoms

$$n_H \approx \left(0.188\ \mathrm{m}^{-3}\right)\left(T/T_0\right)^3$$

The Saha Equation (1)

- At modest values of z, the temperature was cold enough that hydrogen is atomic
- As z increases, temperature rises
  - Eventually, hot enough for atoms to be ionized
- Density of photons is so high, electrons are in thermal equilibrium with them
- Will electrons be free or bound? Depends on temperature
- For a thermal distribution, probability for each possibility is:

$$P(E) \propto \exp\left(-\frac{E}{k_B T}\right)$$

First: Bound electrons in hydrogen:

- Binding energy $E = -E_b = -13.6$ eV
- Density proportional to density of free protons:

$$P_b \propto \exp\left(\frac{E_b}{k_B T}\right) \qquad n_b \propto n_p \exp\left(\frac{E_b}{k_B T}\right)$$

Second: Free electrons:

- Kinetic energy

$$E = \frac{\mathbf{p}^2}{2m} = \frac{\hbar^2 \mathbf{k}^2}{2m}$$

- Density requires us to add up all intermediate states

$$n_F \propto \int \frac{d^3\mathbf{k}}{(2\pi)^3} \exp\left(-\frac{\hbar^2 \mathbf{k}^2}{2m k_B T}\right) = \left(\frac{k_B T m}{2\pi\hbar^2}\right)^{3/2}$$

- We need the ratio of these expressions

$$\boxed{\frac{n_b}{n_F} = n_p \left(\frac{k_B T m}{2\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{E_b}{k_B T}\right)}$$

The Saha Equation (2)

We want to know what fraction x of electrons are free (same as free protons)

$$\frac{n_b}{n_F} = n_p \left(\frac{k_B T m}{2\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{E_b}{k_B T}\right)$$

- Let x be the fraction of hydrogen atoms that are free

$$n_F = n_H x = n_p$$

- Same as density of free protons
- Density of bound electrons is

$$n_b = (1-x) n_H$$

- Substitute this all in:

$$\frac{1-x}{x^2} = n_H \left(\frac{k_B T m}{2\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{E_b}{k_B T}\right)$$

- Binding energy $E_b = 13.6$ eV
- Density of hydrogen is

$$n_H \approx \left(0.188\ \text{m}^{-3}\right)\left(T/T_0\right)^3$$

- Substitute it all in:

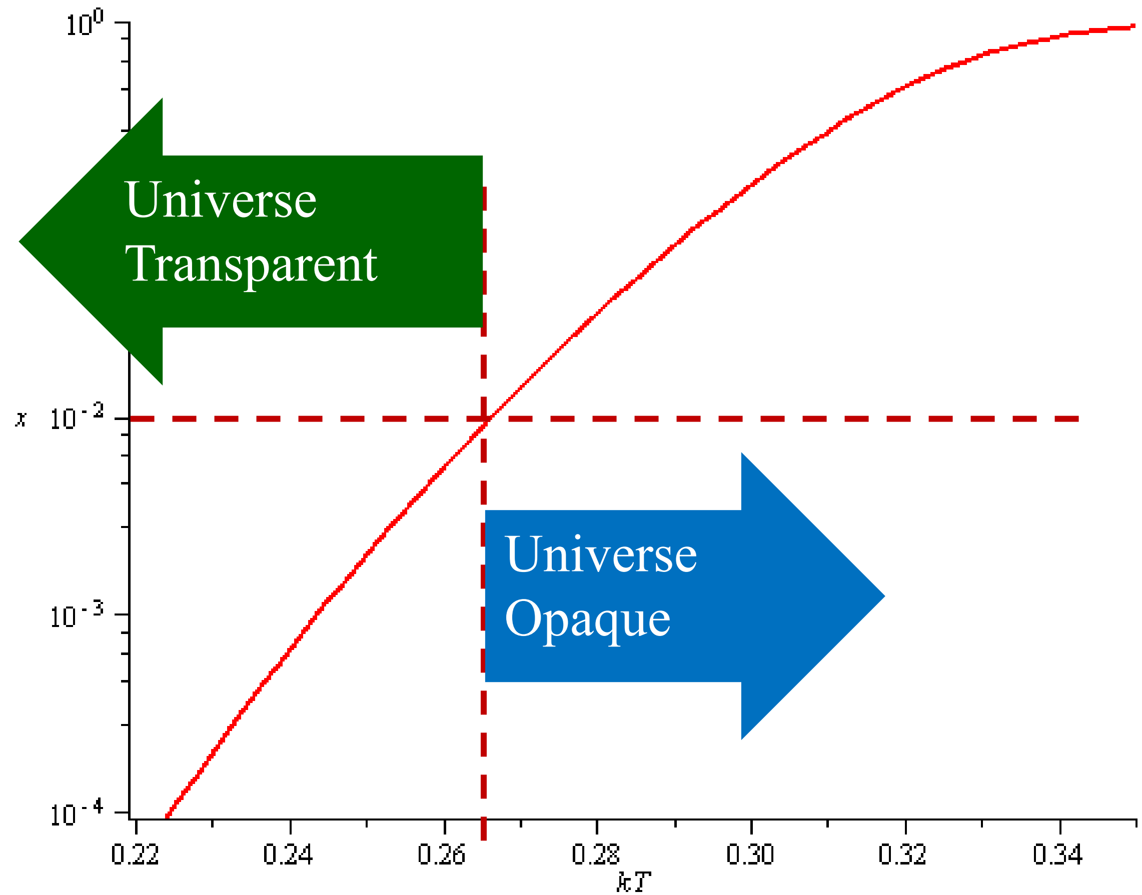
$$\frac{1-x}{x^2} = \frac{\left(k_B T/\text{eV}\right)^{3/2}}{2.05\times10^{17}} \exp\left(\frac{13.6\ \text{eV}}{k_B T}\right)$$

- Need to bind about 99% (?) of electrons to make universe transparent

The Saha Equation (3)

$$\frac{1-x}{x^2} = \frac{\left(k_B T/\text{eV}\right)^{3/2}}{2.05\times 10^{17}} \exp\left(\frac{13.6\ \text{eV}}{k_B T}\right)$$

- My calculations indicate universe becomes transparent at $k_B T = 0.265$ eV
- For some reason this number came out a little wrong
  - Actual recombination is at $k_B T = 0.256$ eV



Recombination – The Results

- Universe becomes transparent at about $k_B T_* = 0.256$ eV
- This corresponds to $z^* = 1090$ or $T = 2973$ K
- Age of universe can be approximated as:

$$t_* \approx \frac{17.26 \text{ Gyr}}{(1+z)^{3/2}} = 480{,}000 \text{ yr}$$

- This is inaccurate because we are too close to radiation era

$$z_* = 1090$$
$$T_* = 2973 \text{ K}$$
$$k_B T_* = 0.256 \text{ eV}$$
$$t_* = 373{,}000 \text{ yr}$$

Time	T or $k_B T$	Events
373 ky	0.26 eV	Recombination

- The cosmic microwave background shows us the temperature of the universe at this time
- We still have to explain the small fluctuations (much later)