Dark Energy
Distance
How Light Travels

- How far away something is gets complicated at high z
  - How far it is now? How far it was then?
    How far light travelled? How distant it looks?
- The metric is $$ds^2 = -c^2 dt^2 + a^2 \left[d\psi^2 + f(\psi)^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right]$$
- We want to find the distance now d_0 corresponding to some red shift z
- Light will be coming to us along a constant θ and ϕ curve

$$d\theta = d\phi = 0$$

- Light follows lightlike curves – that is, $ds^2 = 0$

$$0 = -c^2 dt^2 + a^2 d\psi^2$$

$$\frac{d\psi}{dt} = \frac{c}{a}$$

Finding the Distance to an Object

$$ds^2 = a^2(t)\left[d\psi^2 + f(\psi)^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

$$\frac{d\psi}{dt} = \frac{c}{a}$$

- Useful, as before, to define x as the relative size of universe compared to now

$$x = \frac{a}{a_0} = \frac{1}{1+z}$$

- And therefore

$$\frac{d\psi}{dt} = \frac{c}{a_0 x}$$

- Now, we previously found, if all we have is matter, then

$$\frac{dx}{dt} = H_0\sqrt{\Omega x^{-1} + 1 - \Omega}$$

- Dividing these, we have

$$\frac{d\psi}{dx} = \frac{cH_0^{-1}}{a_0 x\sqrt{\Omega/x + 1 - \Omega}}$$

- Integrate both sides
- The distance is $a_0\psi$

$$d_0 = a_0\psi = cH_0^{-1}\int_{(1+z)^{-1}}^{1}\frac{dx}{\sqrt{x^2 + \Omega\left(x - x^2\right)}}$$

Understanding This Formula Qualitatively

$$d_0 = a_0\psi = cH_0^{-1}\int_{(1+z)^{-1}}^{1} \frac{dx}{\sqrt{x^2+\Omega\left(x-x^2\right)}}$$

Can we understand this formula?

- For small z, approximate integrand as 1
- Rearrange and this is Hubble's Law

$$d_0 \approx cH_0^{-1}\, x\Big|_{(1+z)^{-1}}^{1} = cH_0^{-1}\left(1-\frac{1}{1+z}\right)$$

$$d_0 \approx cH_0^{-1} z \qquad cz = H_0 d_0$$

- But the full formula includes corrections

How does the presence of matter (Ω) change things?

- As you increase Ω, you have more gravity

- Universe decelerates more quickly
- Universe has less time since fixed red shift z
- Shorter distance for fixed z

Luminosity Distance:

$$ds^2 = a^2(t)\left[d\psi^2 + f(\psi)^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

$$d_0 = a_0\psi = cH_0^{-1}\int_{(1+z)^{-1}}^{1}\frac{dx}{\sqrt{x^2 + \Omega\left(x - x^2\right)}}$$

- We actually don't measure the current distance d_0
- We get distances from standard candles
- We don't measure d_0, we measure the *luminosity distance* d_L:

$$d_L = \sqrt{\frac{L}{4\pi b}}$$

The distances d_0 and d_L differ in two ways:

$$A = 4\pi a_0^2 f(\psi)^2 = 4\pi d_0^2\, f(\psi)^2 / \psi^2$$

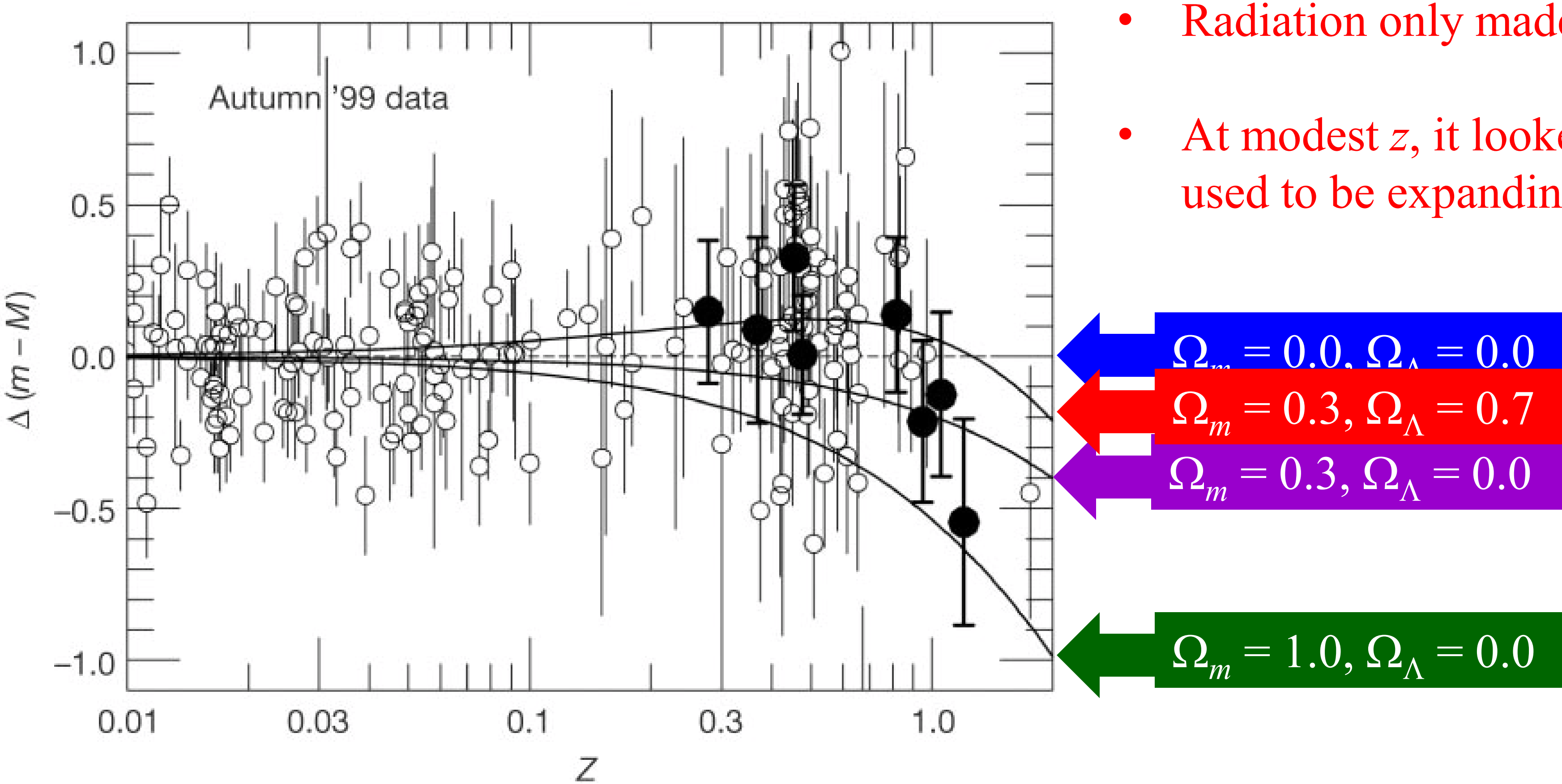
- The universe is curved
- Actual area of sphere is *not* $4\pi d^2$

- The object has a large redshift $1+z$
- Frequency decreases by $1/(1+z)$
- Each photon has energy decreased by $1/(1+z)$
- Rate at which photons are received decreased by $1/(1+z)$

$$d_L = (1+z)d_0 \cdot \begin{cases} \sinh\psi/\psi & \text{if } \Omega < 1 \\ 1 & \text{if } \Omega = 1 \\ \sin\psi/\psi & \text{if } \Omega > 1 \end{cases}$$

Type Ia Supernovae and Deceleration

- Measure luminosity distance d_L as a function of redshift
- Compare to model for different values of Ω:



- No model worked!
- Radiation only made it worse
- At modest z, it looked like the universe used to be expanding *quicker*!

Dark Energy

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2}$$

- Friedmann Equation has density ρ of stuff in the universe
- Multiply by a^2 $\quad \dot{a}^2 = \frac{8}{3}\pi G\rho a^2 - kc^2$
- To match supernova data, we need this velocity *increasing* today
- To have speed increasing, must have ρa^2 increasing with time
- Any matter which satisfies this constraint will be called ***dark energy***

Matter: (atoms, dark matter, neutrinos?)

- As universe expands, number of atoms decreases as a^{-3}
- So $\rho \sim a^{-3}$

Radiation: (light, neutrinos?, gravitons?)

- As universe expands, density of stuff decreases as a^{-3}
- Also, each photon gets red shifted to lower energy by factor a^{-1}
- So $\rho \sim a^{-4}$

Dark Energy: What is it?

Vacuum Energy Density, *aka* Cosmological Constant:

- According to particle physics, empty space has stuff constantly appearing and disappearing
- This means empty space has energy associated with it:
  - And therefore mass density
  - Unfortunately, particle physicists aren't very helpful with calculating how much energy density

$$u = \infty + ?$$

- The density of the vacuum doesn't change as universe expands
  - $\rho \sim 1$, independent of a
- This contributes another term, the *vacuum term* to the energy density
  - Its contribution is labeled Ω_Λ
- It was found that the data could be well fit with:

$$\Omega_m \approx 0.3$$
$$\Omega_\Lambda \approx 0.7$$

Dark Energy vs. the Cosmological Constant

- When Einstein first introduced his general theory of relativity, he first had what we now call Einstein's equations:

$$G_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

- He realized that this would describe a universe that was either contracting or expanding
  - Hubble had not yet discovered Hubble's Law
- To 'fix' it, he introduced another term, which included the cosmological constant Λ

$$G_{\alpha\beta} + \Lambda g_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

- Vacuum energy density, instead, introduces a new term in the stress-energy tensor:

$$T_{\alpha\beta}^{(\text{vac})} = -\rho^{(\text{vac})} g_{\alpha\beta}$$

- The two are equivalent if you set

$$\Lambda = \frac{8\pi G}{c^4} \rho^{(\text{vac})}$$

- There is no point in arguing which one is right

The ΛCDM Model

Dark Energy: What Does the Experiment Say?

It depends on what assumptions we use:

Assumption 1: Suppose we assume only that it behaves as a power law: $\rho \propto a^n$

- Including all data, experimental result:

$$n = 0.08 \pm 0.09$$

Note: normally given as *w*, where $n = -3(1+w)$

Assumption 2: Suppose we assume only that it is constant

- Including all data, experimental result:

$$\Omega_{\text{tot}} = 0.9993 \pm 0.0037$$

Assumption 3: Suppose we demand that it is constant, and $\Omega_{\text{tot}} = 1$

- Including all data, experimental result
- This model is called the ΛCDM model
- It has become the standard model for cosmology
- Λ stands for the vacuum energy density
- CDM stands for cold dark matter
  - "Cold" is a term we will clarify later

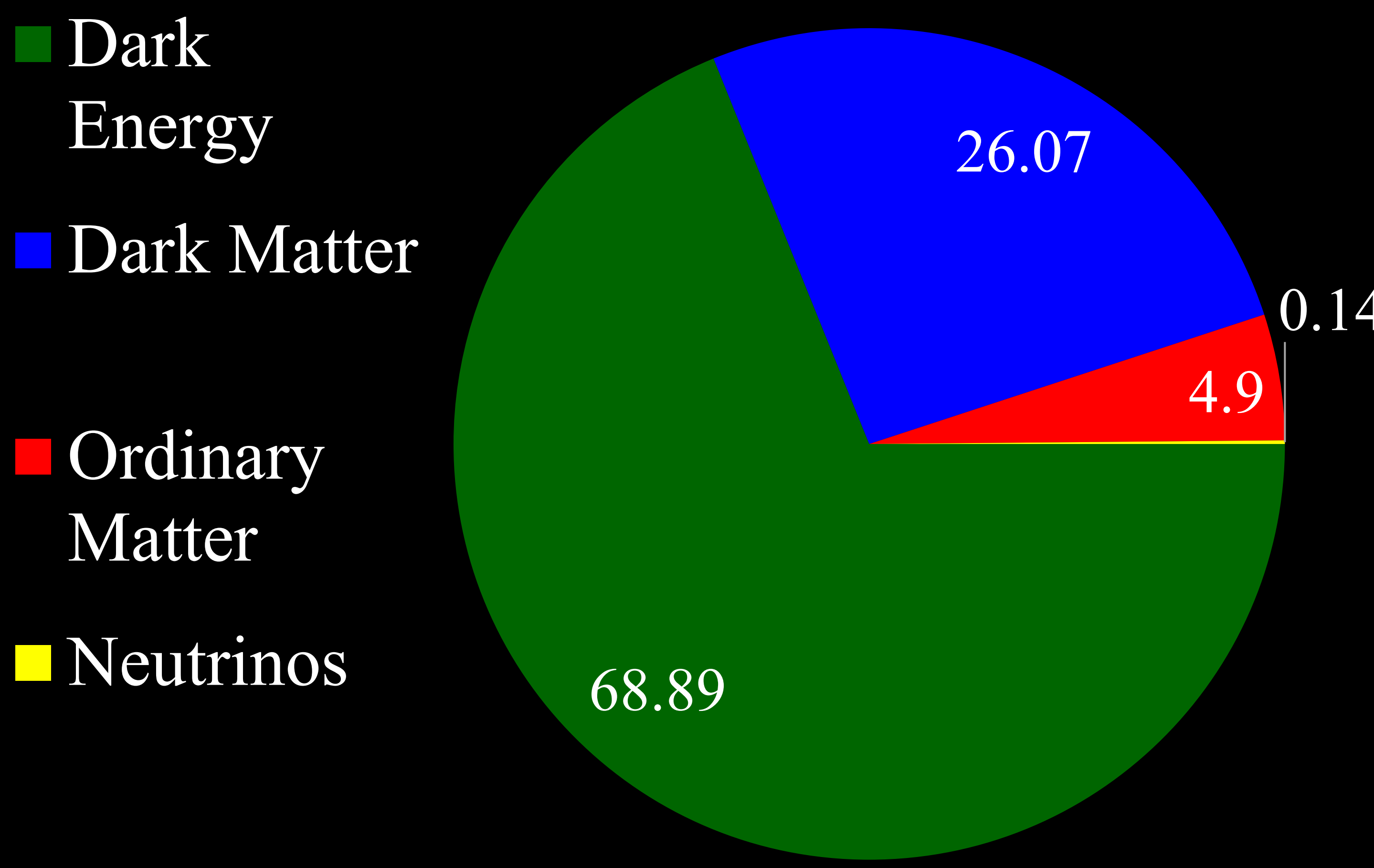
$$\Omega_b = 0.0490 \pm 0.0010$$
$$\Omega_d = 0.2607 \pm 0.0053$$
$$\Omega_\Lambda = 0.6889 \pm 0.0056$$

Composition of the Universe

$H_0 = 67.7 \pm 0.4$ km/s/Mpc

$t_0 = 13.79 \pm 0.02$ Gyr

$\Omega = 1$



Age of the Universe, Round 3 (1)

Assume:

$$\frac{\dot{a}^2}{a^2} = \tfrac{8}{3}\pi G\rho - \frac{kc^2}{a^2}$$

$$\tfrac{8}{3}\pi G\rho_{0i} = H_0^2\Omega_i$$

$$\rho = \rho_m + \rho_\Lambda$$

$$\tfrac{8}{3}\pi G\rho_{0m} = H_0^2\Omega_m$$

$$\tfrac{8}{3}\pi G\rho_{0\Lambda} = H_0^2\Omega_\Lambda$$

$$-kc^2/a_0^2 = H_0^2\left(1-\Omega_m-\Omega_\Lambda\right)$$

- Universe has only matter and dark energy
- Dark energy density is constant
- Matter scales as a^{-3}
- kc^2/a^2 scales as a^{-2}, of course
- Substitute in:

$$\tfrac{8}{3}\pi G\rho_m = H_0^2\Omega_m\left(a_0/a\right)^3$$

$$\tfrac{8}{3}\pi G\rho_\Lambda = H_0^2\Omega_\Lambda$$

$$-kc^2/a^2 = H_0^2\left(1-\Omega\right)\left(a_0/a\right)^2$$

$$\frac{\dot{a}^2}{a^2} = H_0^2\left[\Omega_m\left(a/a_0\right)^{-3} + \Omega_\Lambda + \left(1-\Omega_m-\Omega_\Lambda\right)\left(a/a_0\right)^{-2}\right]$$

- Let $x = a/a_0$:

$$\frac{\dot{x}^2}{x^2} = H_0^2\left[\Omega_m/x^3 + \Omega_\Lambda + \left(1-\Omega_m-\Omega_\Lambda\right)/x^2\right]$$

$$\frac{dx}{dt} = H_0\sqrt{\Omega_m/x + \Omega_\Lambda x^2 + 1 - \Omega_m - \Omega_\Lambda}$$

$$\frac{dt}{dx} = H_0^{-1}\frac{1}{\sqrt{\Omega_m/x + \Omega_\Lambda x^2 + 1 - \Omega_m - \Omega_\Lambda}}$$

$$t_0 = H_0^{-1}\int_0^1 \frac{dx}{\sqrt{\Omega_m/x + \Omega_\Lambda x^2 + 1 - \Omega_m - \Omega_\Lambda}}$$

Age of the Universe, Round 3 (2)

$$t_0 = H_0^{-1} \int_0^1 \frac{dx}{\sqrt{\Omega_m / x + \Omega_\Lambda x^2 + 1 - \Omega_m - \Omega_\Lambda}}$$

- As Ω_m goes up, t_0 goes down
- As Ω_Λ goes up, t_0 goes up

- If $\Omega_m + \Omega_\Lambda = 1$, then

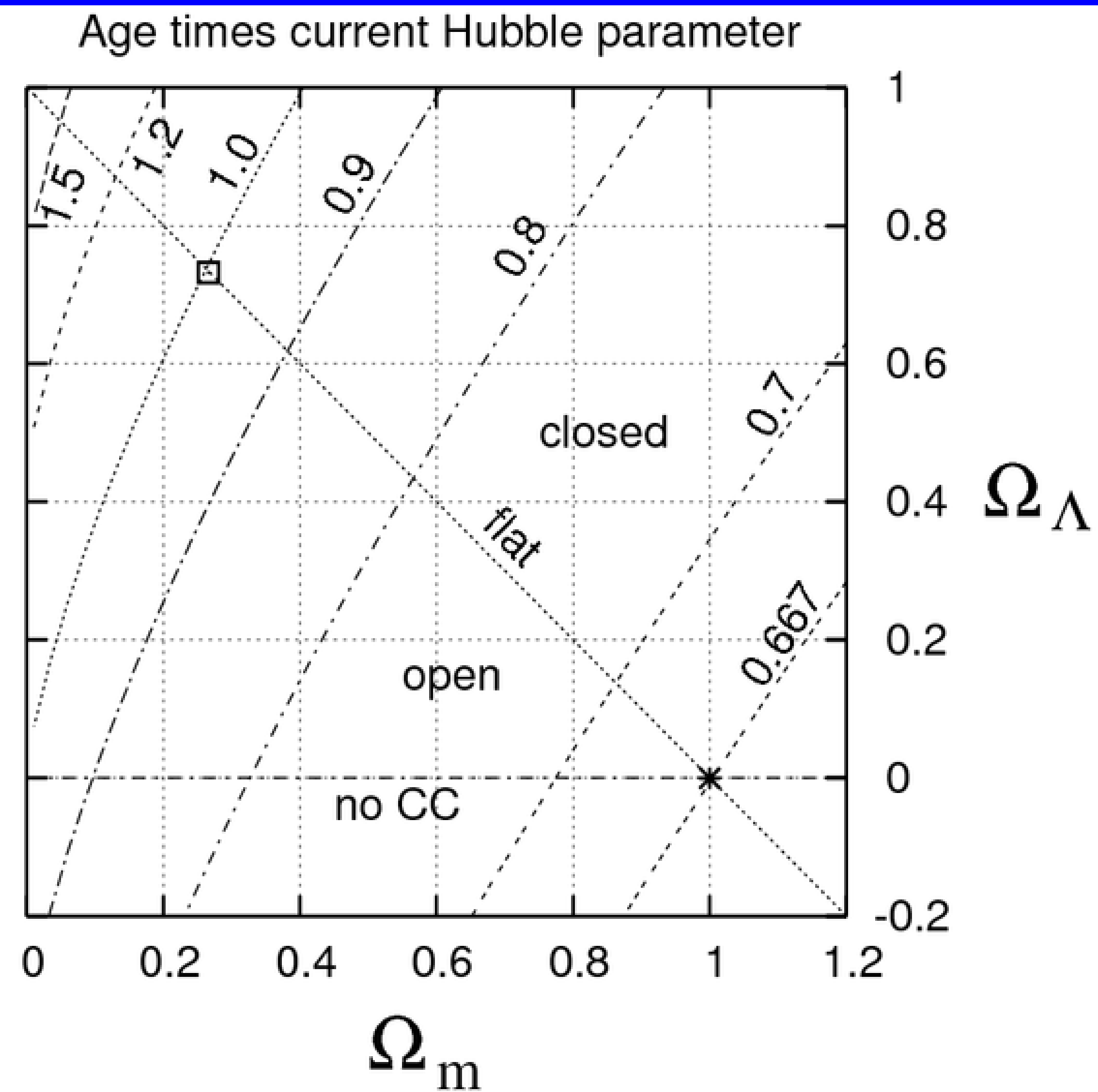
$$t_0 = H_0^{-1} \frac{2 \tanh^{-1} \sqrt{\Omega_\Lambda}}{3\sqrt{\Omega_\Lambda}}$$

- If $\Omega_\Lambda = 0.6889$, then

$$t_0 = 0.954 H_0^{-1} = 13.8 \text{ Gyr}$$

- Compare oldest stars 13 ± 1 Gyr
- Best estimate:

$$t_0 = 13.79 \pm 0.02 \text{ Gyr}$$



No Age Problem

What Dominates the Universe?

$$\frac{\dot{a}^2}{a^2} = \frac{8}{3}\pi G\rho - \frac{kc^2}{a^2}$$

$$\frac{\dot{a}^2}{a^2} = \frac{8}{3}\pi G\left(\rho_m + \rho_r + \rho_\Lambda\right) - \frac{kc^2}{a^2}$$

What dominates it now?

- Matter: $\Omega_m = 0.31$ (significant)
- Radiation $\Omega_r = 10^{-4}$ (tiny)
- Dark Energy: $\Omega_\Lambda = 0.69$ (dominant)
- Curvature kc^2/a^2: $1 - \Omega_{tot} < 0.01$ (small)

These change with time for two reasons:

- Scaling – universe scale factor *a* changes
- Conversion: one type turns into another
  - Stars cause matter → radiation

When were each of these dominant?

How do each of these scale?

- Matter: $\rho_m \propto a^{-3}$ ← Near Past
- Radiation: $\rho_r \propto a^{-4}$ ← Distant Past
- Dark Energy: $\rho_\Lambda \propto 1$ ← Future
- Curvature: $\propto a^{-2}$ ← *Never*

At any time, except maybe now, we can ignore curvature ($\Omega = 1$)

$$\frac{\dot{a}^2}{a^2} = \frac{8}{3}\pi G\rho$$

The Future of the Universe

$$\frac{\dot{a}^2}{a^2} = \frac{8}{3}\pi G\rho \qquad \Omega_i = \frac{\frac{8}{3}\pi G\rho_{i0}}{H_0^2}$$

- Starting soon, we can ignore all stuff except vacuum energy density

$$\frac{\dot{a}^2}{a^2} = \tfrac{8}{3}\pi G\rho_\Lambda = \tfrac{8}{3}\pi G\rho_{\Lambda 0} = \Omega_\Lambda H_0^2 \qquad \frac{\dot{a}}{a} = H_0\sqrt{\Omega_\Lambda} \qquad \frac{da}{dt} = \left(H_0\sqrt{\Omega_\Lambda}\right)a$$

$$a(t) \propto \exp\left(H_0 t\sqrt{\Omega_\Lambda}\right)$$

Universe grows exponentially

- The *time* for one *e*-folding of growth is
  - This is *longer* than current age of universe

$$t_\Lambda = H_0^{-1}\Omega_\Lambda^{-1/2} = 17.3\text{ Gyr}$$

- On the long time scale we are "alone"
  - Except for Local group
- In a few 100 Gyr, rest of universe will look empty
  - Red shifted to invisibility

Alternate Futures?

- Previous slide assumes that the dark energy is:
  - Vacuum energy density (no scaling)
  - Eternal (no decay) [this one probably true]

We know very little about dark energy:

- If it scales as $\rho_\Lambda \propto a^n$, then $n = -0.08 \pm 0.09$

If n is small but negative:

- Universe expands as a power law, not exponential
- Functionally the same

If n is small but positive:

- As universe expands faster and faster, ρ_Λ gets bigger and bigger
- The faster it goes, the faster it accelerates
- In finite time, $\rho_\Lambda \propto a^n$, becomes infinite
- H becomes infinite
- All objects in the universe get ripped apart
- “The Big Rip”
- At least 100 Gyr in the future