Distance Methods

Types of Distance Methods

Geometric Distance Methods:

- Radar Distancing
- Parallax
- Light Echo Method

Standard Candle Distance Methods:

- Spectroscopic Parallax
- Cluster Fitting
- Tip of the Red Giant Branch
- Planetary Nebula Luminosity Function
- Cepheid Variable Stars
- Type Ia Supernovas

- Hubble’s Law

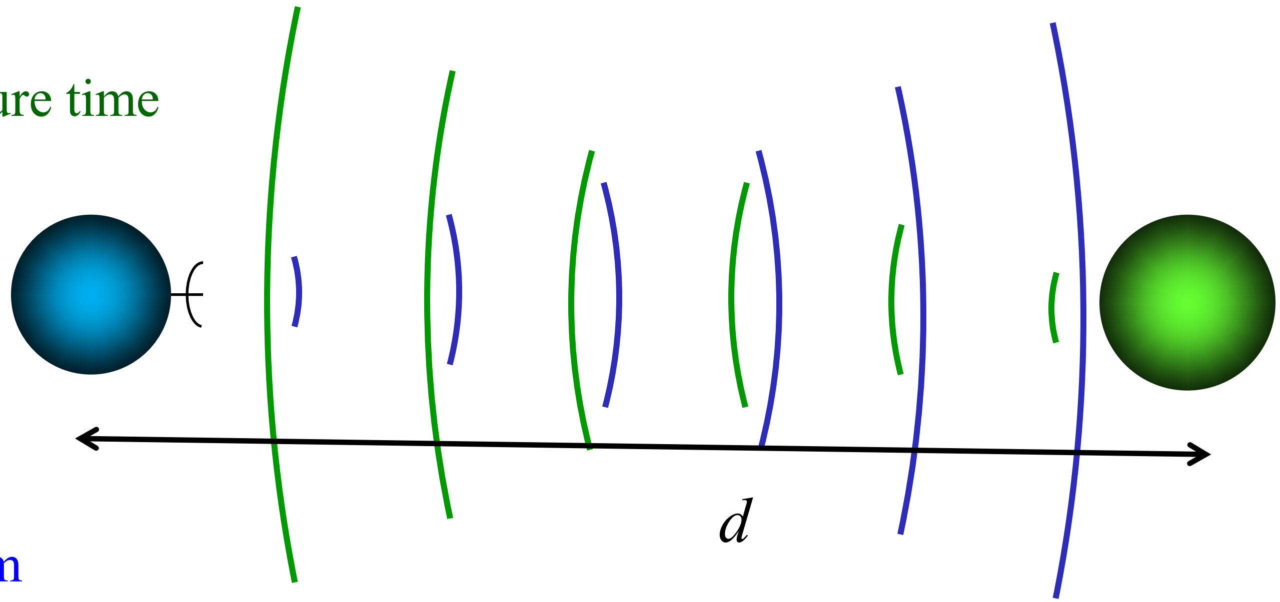
- *Geometric* distance methods rely on fundamental relationships between sizes, angles, etc.
- *Standard Candle* distance methods rely on objects that are believed to be consistently the same luminosity
- The methods are sometimes described as a *ladder*
  - You have to use the low rungs to get the higher rungs
  - Some rungs are sturdier than others

Geometric Methods

Radar Distancing

- Radio waves move at the speed of light *c*
- Bounce radio waves off a target and measure time to get an echo back
- If separation of two planets is *d*, then the time to see the signal is:

$$2d = ct$$

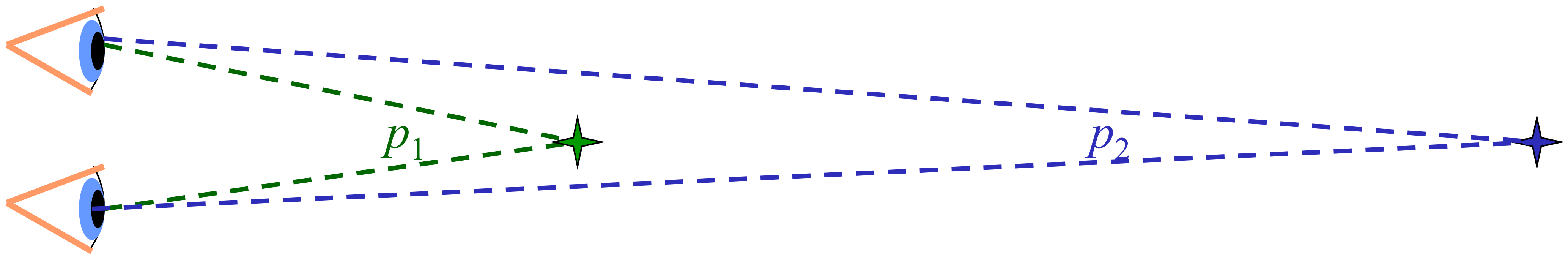


- Can only be used within the solar system
- Reliability limited only by the accuracy with which we measure time
  - Essentially no error
- This allows us to know the AU with high precision:

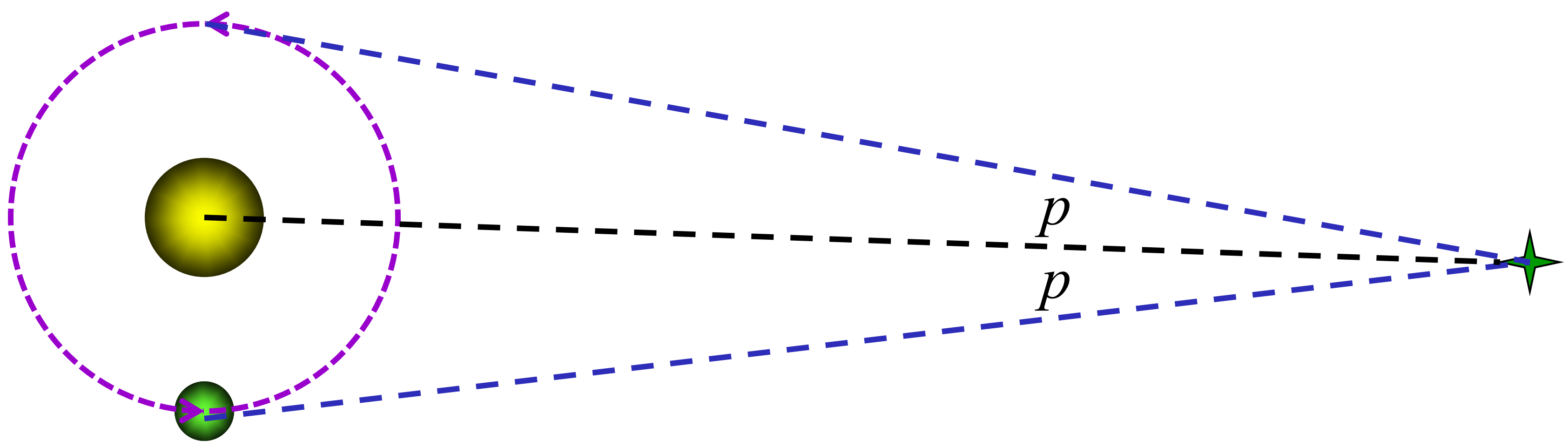
$$\text{AU} = 1.4960 \times 10^{8}\ \text{km}$$

Parallax (1)

- We use our two eyes to judge distances using a technique called *parallax*



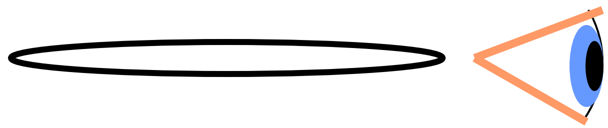
- The *difference* between the angle seen by each of the eyes is called the *parallax*
- It is limited by baseline, how far apart the two points you measure from are
- You can use the orbit of the Earth as a baseline

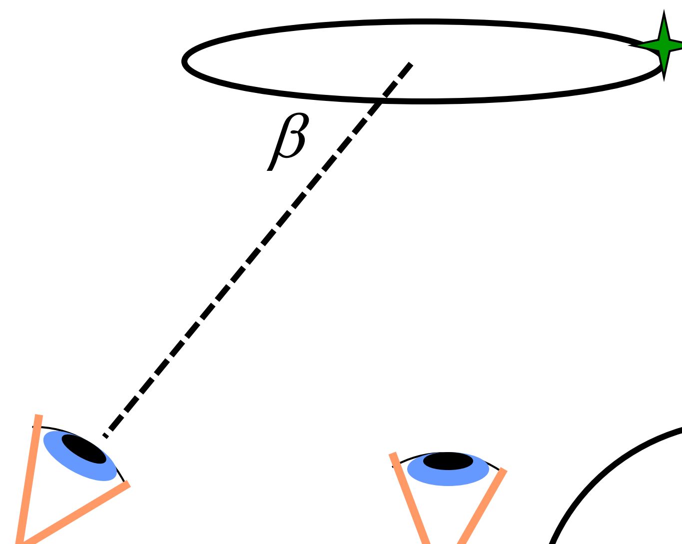


Parallax (2)

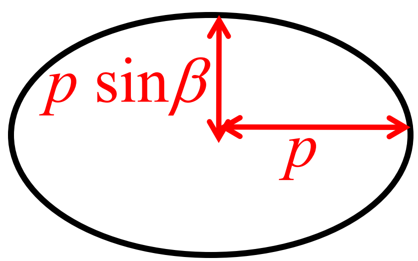
To understand what you will see, easiest to think of system as if *Earth* is still and *star* is moving in a circle:

- If you view it from the edge, it looks like a straight line
- If you view it from the bottom or top, it looks like a circle
- If you view it from an angle, it looks like an ellipse.

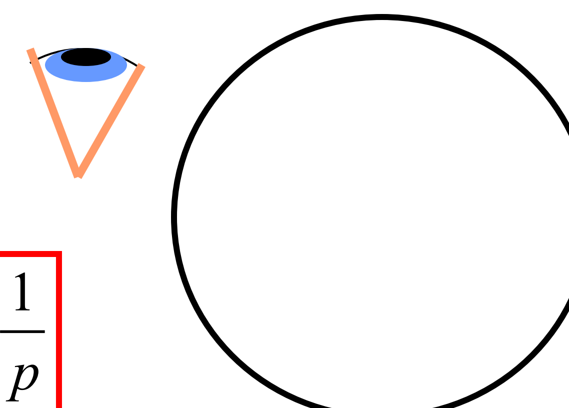




- The angular semi-major axis of this ellipse is the parallax
  - The other size depends on its ecliptic latitude β
- The *actual size* of the ellipse is 1 AU
  - It's really the Earth's orbit
  - We can determine distance:



$$d = \frac{s}{p} = \frac{1\text{ AU}}{p} = \left[(1\text{ AU}) \frac{1\text{ rad}}{1''} \right] \frac{1''}{p}$$



$$d = \frac{1}{p}$$

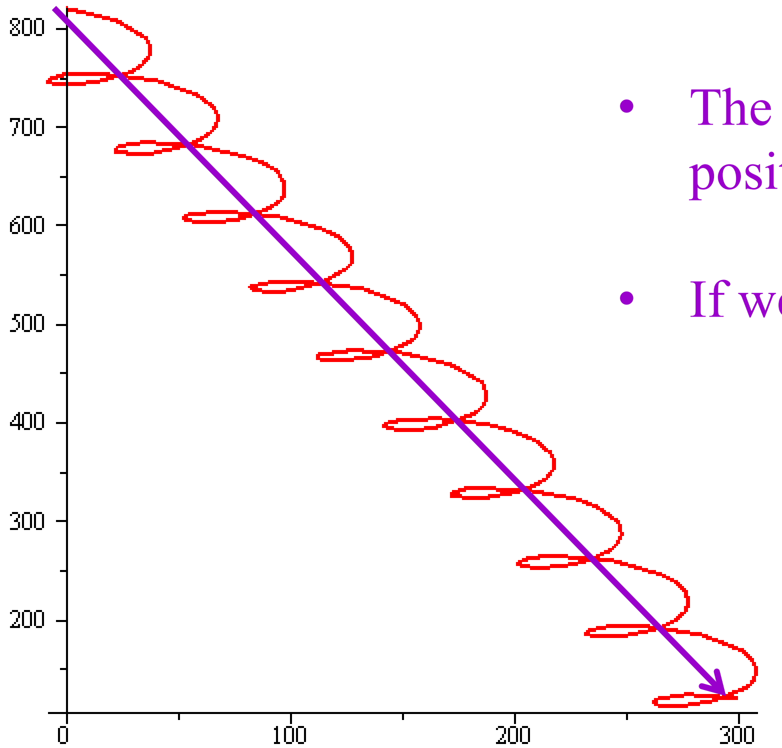
- This combination is called a *parallax-second* or ***parsec***

$$d = (1\text{ pc})(1'')/p$$

Proper Motion

Why it's not that simple:

- Actual paths of stars are more complicated
- Because the stars are also actually moving (relative to us)!



- The average motion over many years is causing the apparent position of the star to change
- If we know the distance, we can measure the *tangential velocity*

$$s = \theta d \qquad \dot{s} = \dot{\theta} d$$

$$\boxed{v_t = \mu d}$$

Sample Problem

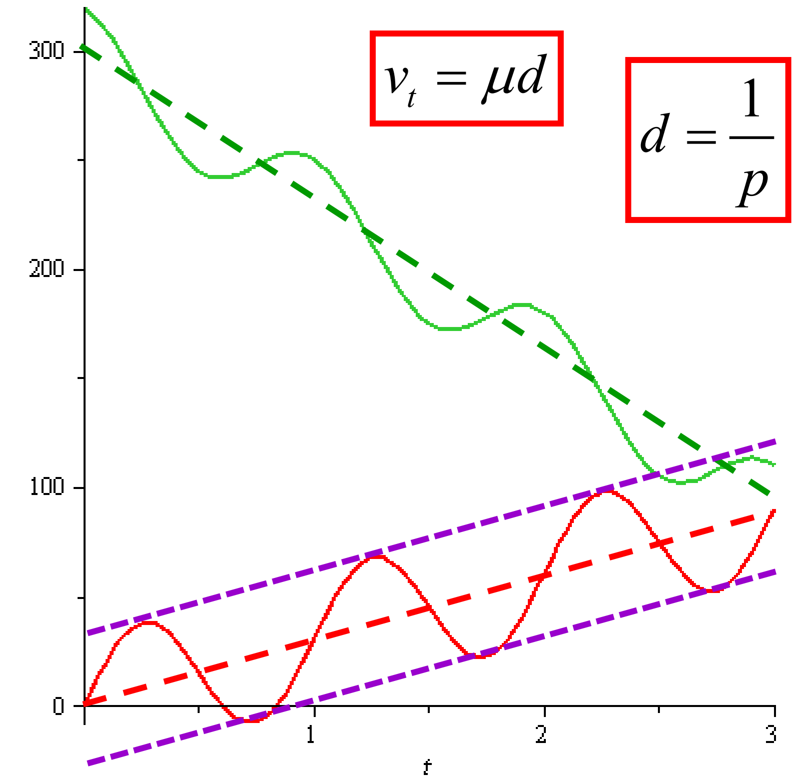
At right is plotted a star's variation in position in the sky in x (red) and y (green) over a three-year period in milliarcseconds. The red curve corresponds to the major axis of parallax. What is the:

(a) Angular velocity of proper motion, μ_x and μ_y

(b) Angular speed of proper motion μ

(c) Parallax in mas and distance in pc

(d) Transverse speed v_t



$$v_t = \mu d$$

$$d = \frac{1}{p}$$

$$\mu_x = \frac{90 \text{ mas}}{3 \text{ y}} = 30 \text{ mas/y}$$

$$\mu_y = \frac{-205 \text{ mas}}{3 \text{ y}} = -68 \text{ mas/y}$$

$$\mu = \sqrt{\mu_x^2 + \mu_y^2} = 74 \text{ mas/y}$$

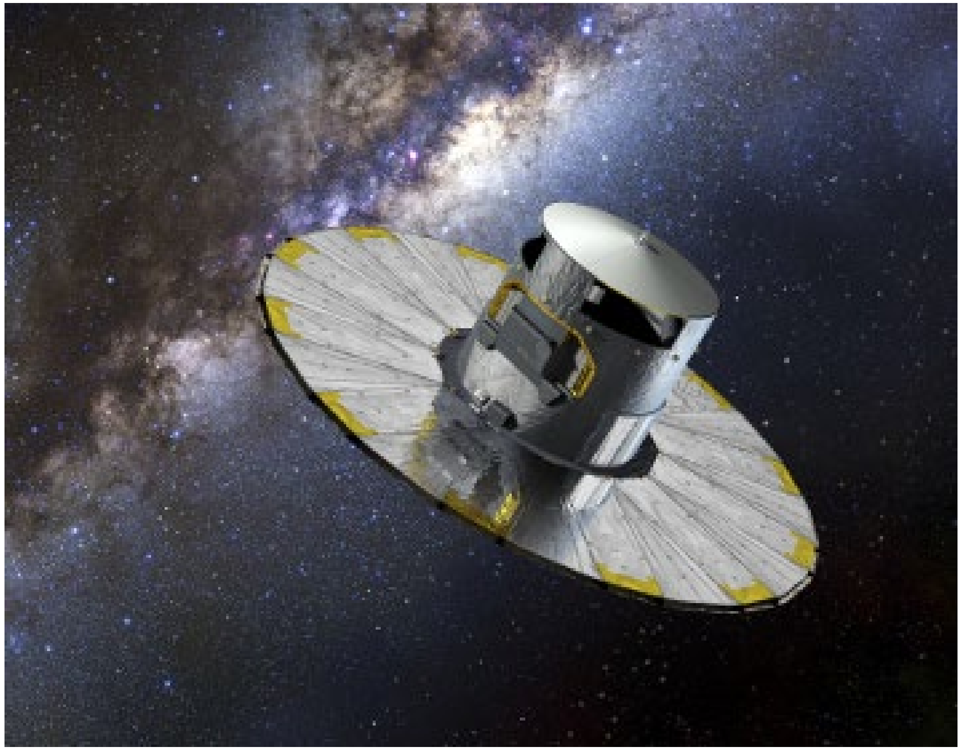
$$p = 32 \text{ mas} = 0.032''$$

$$d = \frac{1}{0.032} = 31 \text{ pc}$$

$$v_t = (31 \text{ pc})(74 \text{ mas/y}) = \frac{2.29'' \cdot \text{pc}}{\text{y}} \cdot \frac{\text{y}}{3.156\times10^7 \text{ s}} \cdot \frac{\text{AU} \cdot \text{rad}}{1'' \cdot \text{pc}} \cdot \frac{1.5\times10^8 \text{ km}}{\text{AU}}$$

$$v_t = 10.9 \text{ km/s}$$

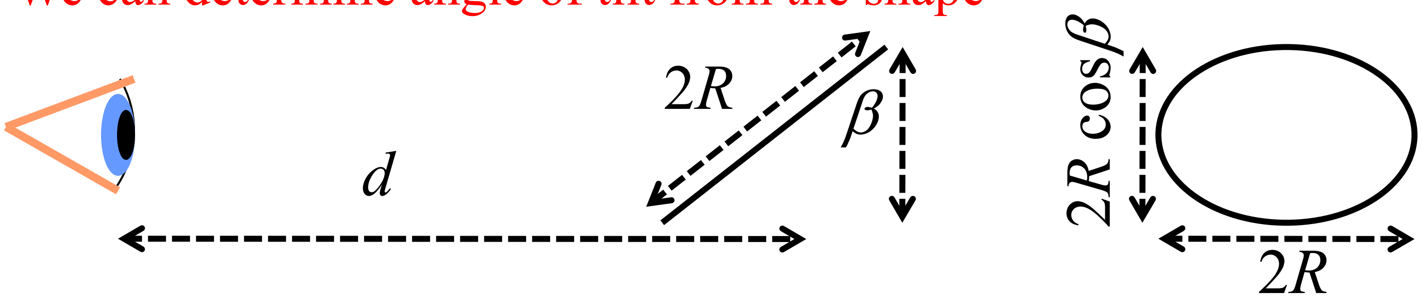
Gaia Spacecraft

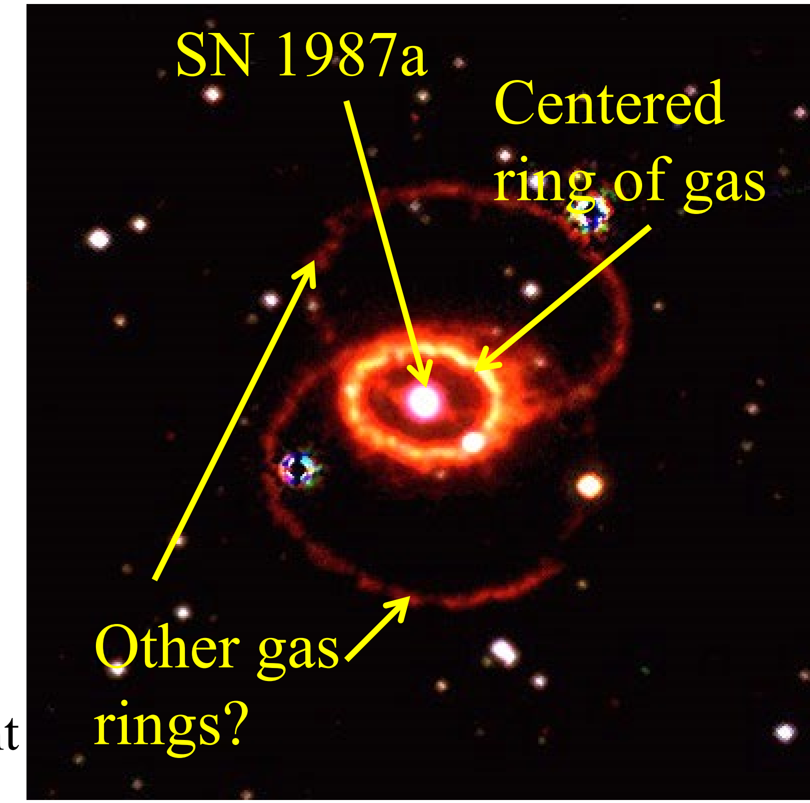


- European Spacecraft
- Launched in 2013
- Measures parallax of stars $m = 3$ to $m = 15$ with an error of 0.02 mas
- Measures parallax of stars down to $m = 20$ with an error of 0.2 mas
- Can measure distances to center of galaxy with better than 10% accuracy
  - Most recent data release in June 2022
  - Parallax and proper motion of 1.47 billion stars
  - More data expected in coming years

The Light Echo Method (1)

- Consider a very bright source of light that turns on suddenly
  - Like a supernova
- The bright ring is probably a circle centered on the supernova
- It looks like an oval because it is probably tilted compared to our point of view
- We can determine angle of tilt from the shape





- The light from the supernova comes straight to us at speed of light

$$t_S = d/c$$

- From the ring, it takes longer:
  - First it must go to the leading edge of the ring
  - Then it must come from the leading edge to our eyes

$$t_E = R/c + \left(d - R\sin\beta\right)/c$$

- We can measure the *difference* in time

$$\Delta t = \frac{R - R\sin\beta}{c}$$

The Light Echo Method (2)

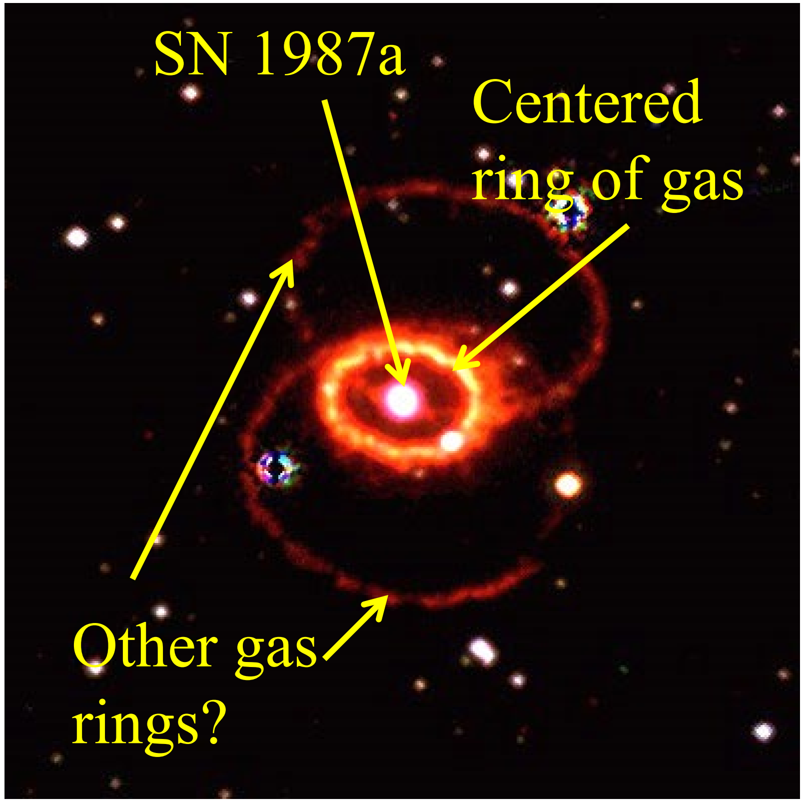
$$\Delta t = \left(R - R\sin\beta\right)/c$$

- We can now find the actual size of the object

$$R = \frac{c\Delta t}{1-\sin\beta}$$

- We can also measure the angular size of the object

$$R = \theta d \qquad d = \frac{R}{\theta} \qquad d_{\mathrm{SN1987a}} = 51\ \mathrm{kpc}$$



- Note many methods give distances only to *very specific* objects

- But many objects clearly are together
  - Probably at comparable distances

- Measuring distance to one object gives you all such distances
  - SN 1987a was in the Larger Magellenic Cloud

$$d_{\mathrm{LMC}} \approx 51\ \mathrm{kpc}$$

Standard Candles

A *Standard Candle* is any object that is consistently the same luminosity

- The luminosity is normally converted to an absolute magnitude M
- We can generally measure the apparent magnitude m
- We can then determine the distance d:

$$m - M = 5\log d - 5$$

$$d = 10^{1+\frac{m-M}{5}} \text{ pc}$$

To use standard candles, we must:

- Establish that they are standard candles, *i.e.*, show that they have consistently the same luminosity
- Calibrate the luminosity of one or a few representative members
  - Determine its distance d by some other method
  - Measure the brightness / apparent magnitude m
  - Find M from our distance formulas

Complications:

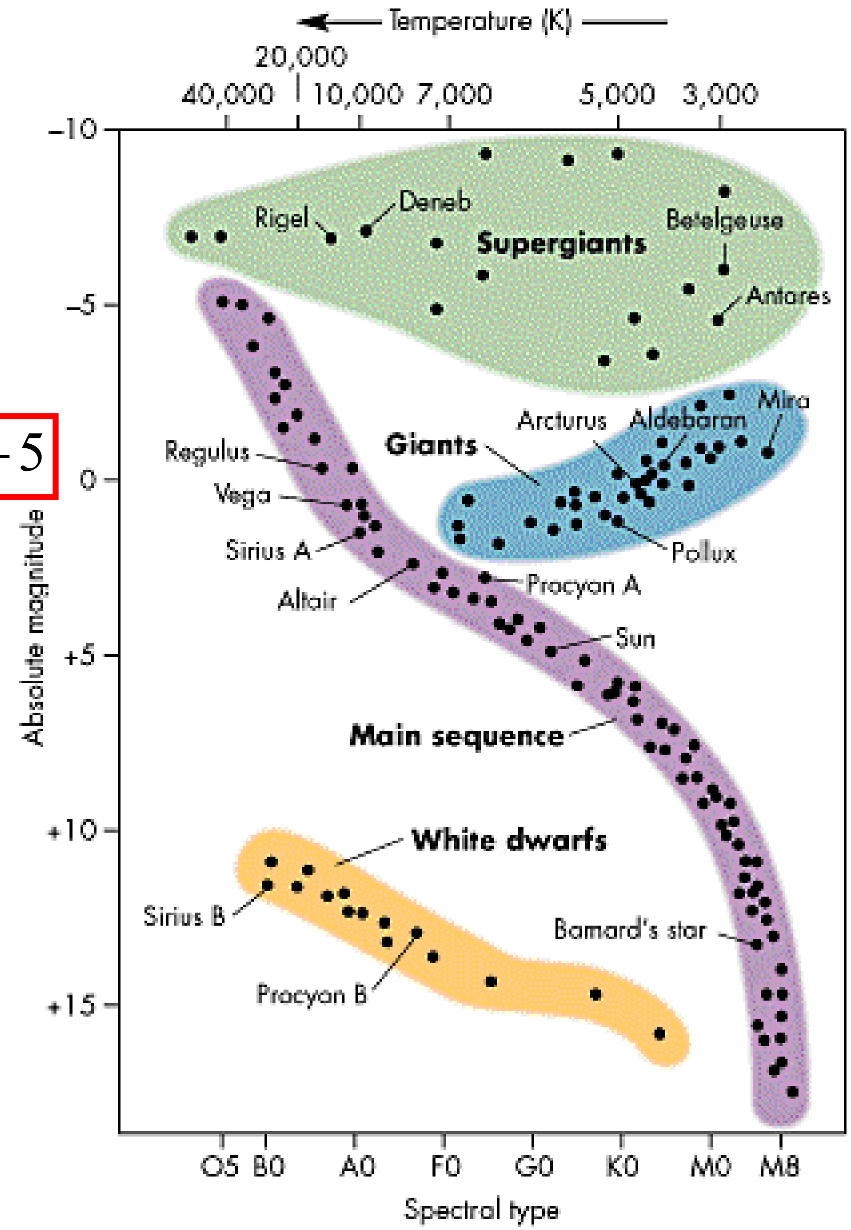
- There is often some spread in M:
  - Either introduces error or must be compensated for
- Any dust between us and a source will change m
  - Can be indirectly measured by comparing different filters

Spectroscopic Parallax

- Uses main sequence stars
  - These are 90% of all stars, so not a restriction
- Has nothing to do with parallax

- Study many nearby main sequence stars
  - Get their distances by parallax
- Measure their apparent magnitudes m
- Deduce their absolute magnitudes M
- Make a Hertzsprung Russell Diagram

$$m - M = 5\log d - 5$$



Now, to measure the distance to any M.S. star:
- Measure the apparent magnitude m
- Measure the spectral class (color)
- Use H-R diagram to deduce the absolute magnitude M
- Find the distance using

$$d = 10^{1+\frac{m-M}{5}} \text{ pc}$$

Sample Problem

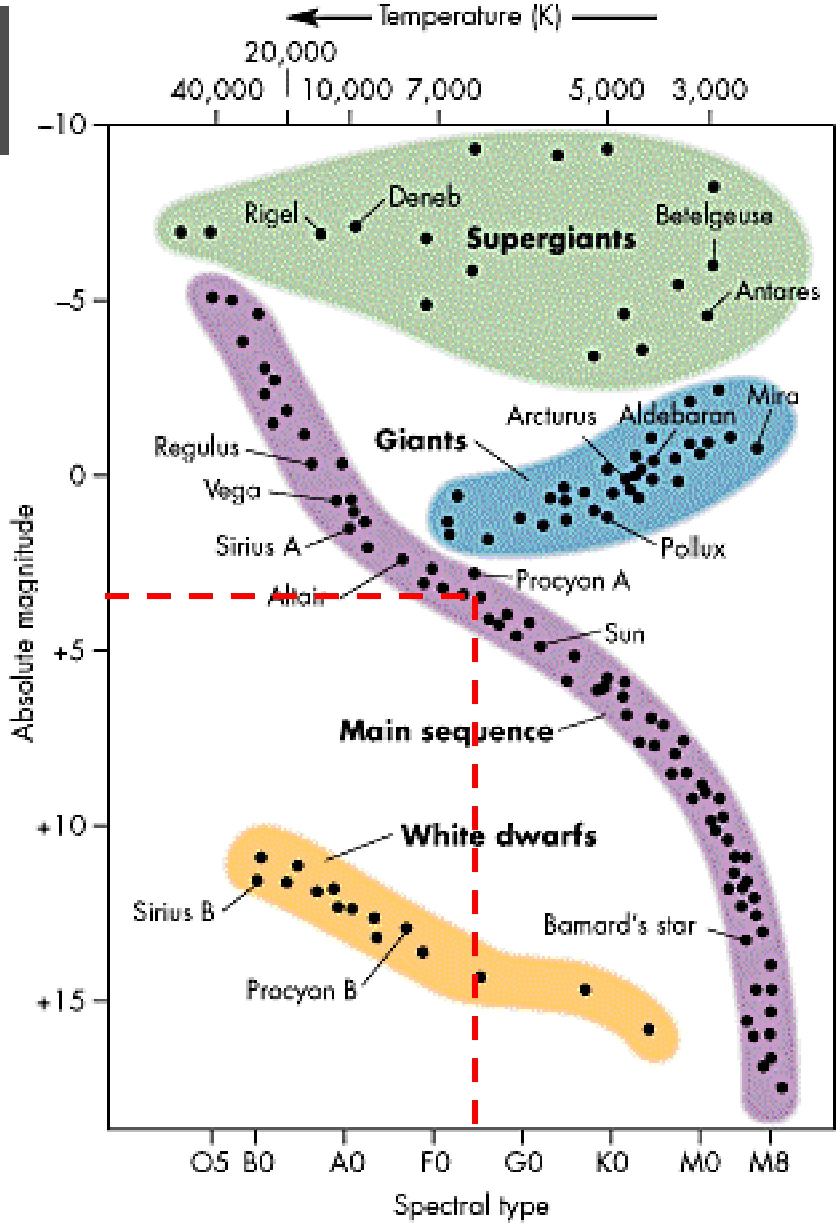
An F5 main sequence star has an apparent magnitude of $m = 14.6$. What is its distance?

$$M \approx +3.5$$

$$d = 10^{1+\frac{m-M}{5}}\ \text{pc}$$

$$d = 10^{1+\frac{14.6-3.5}{5}}\ \text{pc} = 10^{3.22}\ \text{pc}$$

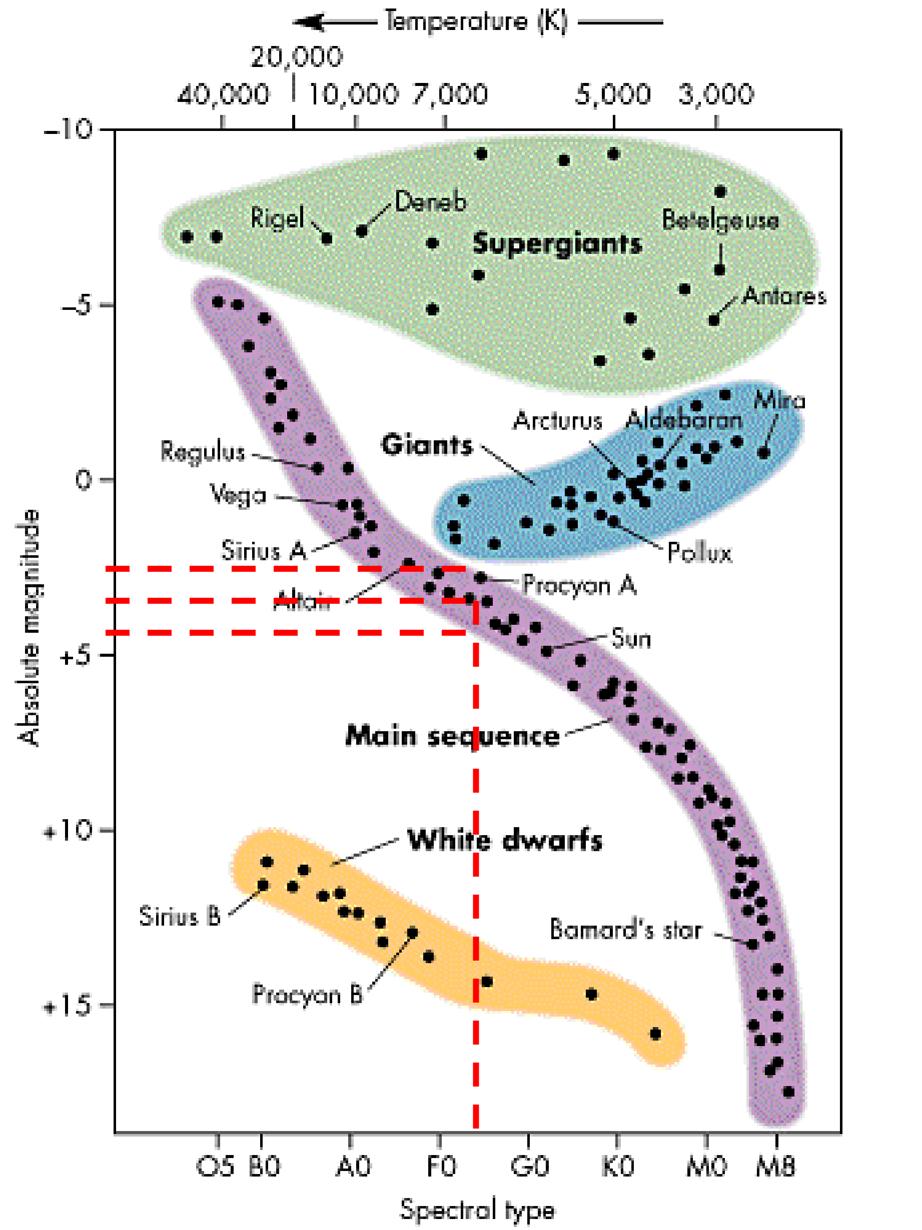
$$d = 1700\ \text{pc}$$



Problems with Spectroscopic Parllax

- Main sequence stars are not exceptionally bright
  - You can't see them at vast distances
  - Must use other methods

- The main sequence is a band, not a line
  - Metallicity varies significantly
    - Can be measured in the spectrum and compensated for
  - Age varies significantly
    - Difficult to compensate for with a single star
    - Use clusters!



Cluster Fitting (1)

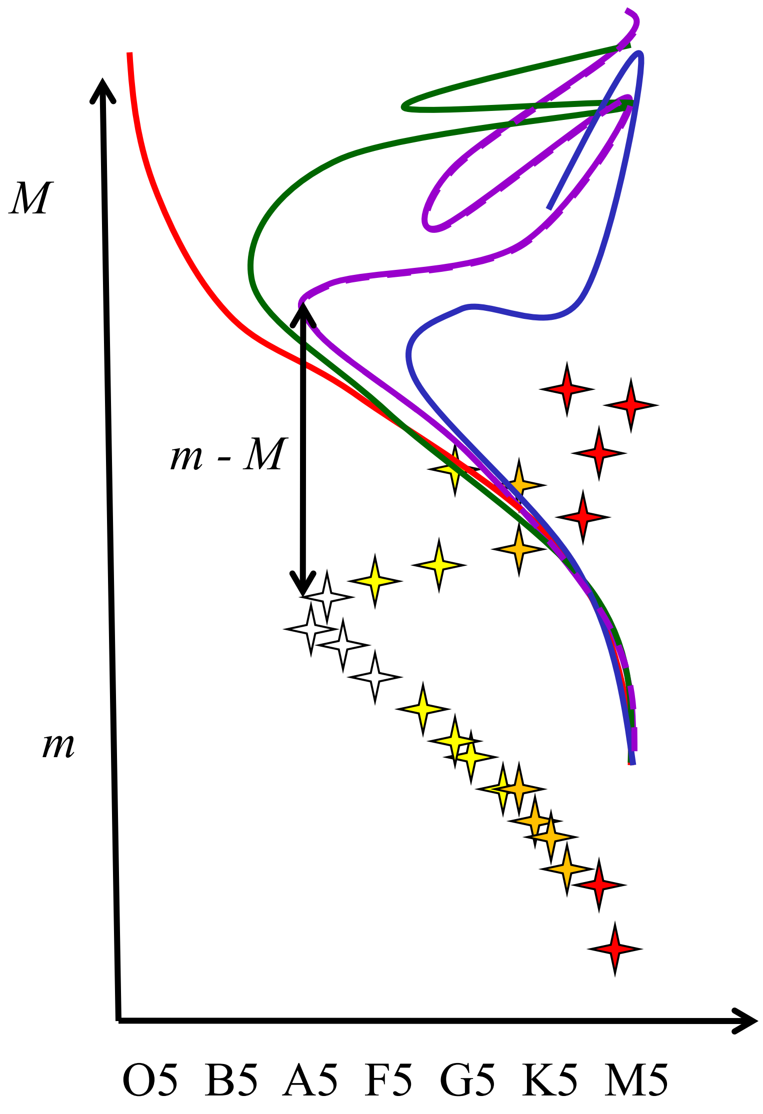
- Spectroscopic parallax on steroids
- Applies to clusters of stars
  - Many stars with similar composition and magnitude
- Plot the apparent magnitude vs. spectral type
- Measure composition – metallicity

- Build a computer model predicting what a set of stars would look like with this composition
- Plot the *absolute magnitude* vs. spectral type

- Age the computer-generated stars until the graph has the same shape
- Turn off point tells you when to stop
- Compare the *absolute magnitude* of the result with the *apparent magnitude* of the actual cluster

- Find the distance from

$$d = 10^{1+\frac{m-M}{5}} \text{ pc}$$



Cluster Fitting (2)

Advantages

- More accurate than spectroscopic parallax
- Statistics of many stars helps eliminate errors

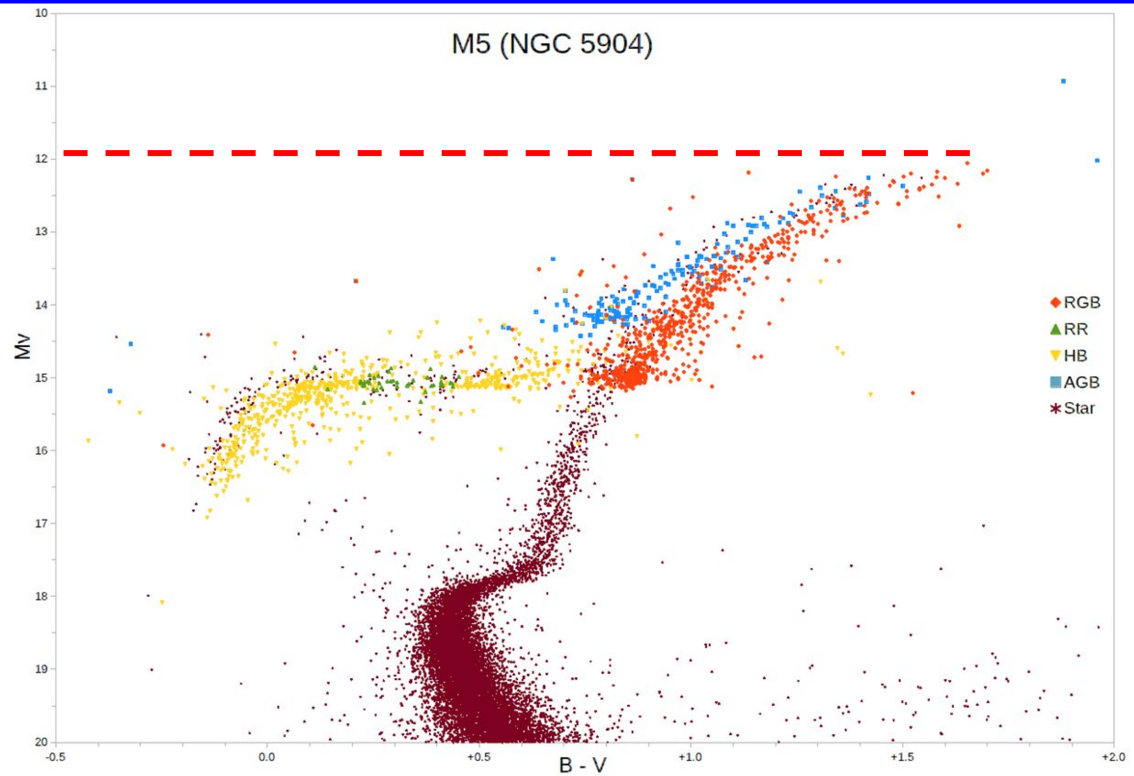
Disadvantages

- Relies heavily on main sequence stars
- These stars are relatively dim
- Cannot be used beyond our galaxy

Tip of the Red Giant Branch (TRGB)

- Look at the brightest stars in the red giant branch
- They will have a range of luminosities
- But there is a cutoff or highest luminosity (most negative absolute magnitude)
- If you look in the infrared, it seems to be almost independent of metallicity

$$M_t = -4.10 \pm 0.1$$



- Make a HR diagram for a collection you want to know distance to
- Determine brightness at the tip m_t
- Find the distance $d = 10^{1+\frac{m_t - M_t}{5}}\ \text{pc}$ $\quad d = 10^{1+\frac{11.9+4.1}{5}}\ \text{pc} = 16\ \text{kpc}$

Advantages

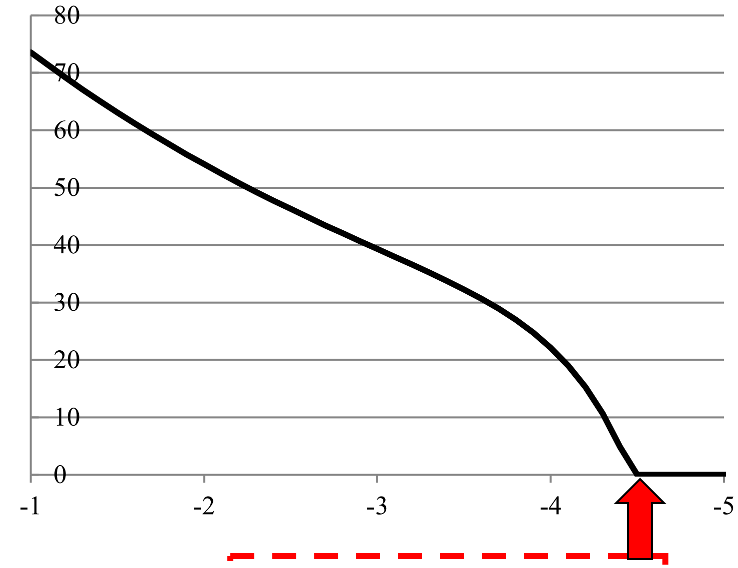
- Apparently it is insensitive (in the infrared) to age and metallicity

Disadvantages

- They still aren't *that* bright

Planetary Nebula Luminosity Function

- Planetary nebulas come in a variety of luminosities
- But the distribution seems to be almost independent of where they come from
  - Very little dependence on the metallicity
- The maximum luminosity can be determined from nearby planetary nebulae:



$$M^* = -4.47 \pm 0.05$$

- Find an object with several (many?) planetary nebulas
- Make a histogram of number vs. apparent magnitude
- Fit to curve – determine maximum brightness m^*

- Find the distance $$d = 10^{1+\frac{m^*-M^*}{5}}\ \text{pc}$$

Advantages

- Can see these brighter objects at larger distances

Disadvantages

- They aren't *that* bright
- You can only get distance to large objects – like galaxies

Cepheid Variable Stars (1)

- In their giant stages, certain stars begin to pulsate
  - Known as *Cepheid Variable Stars*
- The bigger the star is, the slower its pulsation
- The bigger the star is, the more luminous it is
- There is a relationship between the period and the luminosity/absolute magnitude

$$M = -2.43\log(P) - 1.62$$

P is period in days

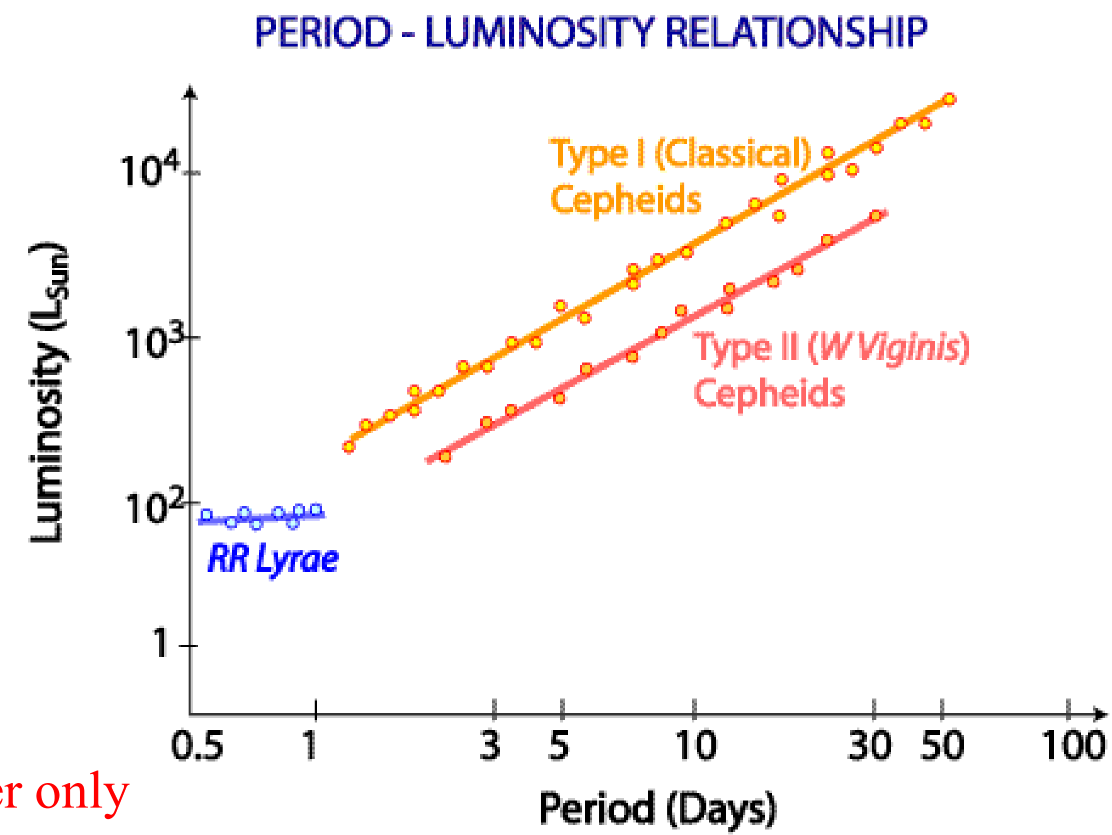
- Measure the period of a pulsating Cepheid variable star
- Use this formula to determine the <u>average</u> visible absolute magnitude M_V
- Measure its <u>average</u> apparent magnitude m_V
- Determine the distance from

$$d = 10^{1+\frac{m-M}{5}}\ \text{pc}$$

Cepheid Variable Stars (2)

Advantages

- Quite accurate method
- Bright, comparable to planetary nebulas
- You only need one



Disadvantages

- Still somewhat rare stars – clusters or bigger only
- Metallicity changes the relationship
  - Most stars near us (type I) have high metallicity
  - Some stars have much lower metallicity
  - Must be compensated for

Type Ia Supernovae

- All type Ia supernovae are approximately 1.4 M_{Sun} white dwarfs that blow up the same way.
- They should all have the same maximum luminosity

$$M_{\max} = -19.3 \pm 0.03$$

- Find a type Ia supernovae where you want it
- Measure its maximum apparent brightness *m*
- Find the distance using:

$$d = 10^{1+\frac{m-M}{5}} \text{ pc}$$

Disadvantages
- They *aren't really* standard candles:
- There is a spread in the maximum magnitude
- There is an experimental correlation between how fast they fade and their maximum magnitude
  - Can be used to compensate for this problem
- They are very rare – difficult to calibrate

Advantages
- Quite accurate method
- Spectacularly bright

Mixed:
- So far away, other effects become important
  - Relativistic speeds, curvature of universe