Hubble's Law

Hubble's Constant

Plot distance vs. velocity. We find:
- Velocity proportional to distance
  - H_0 is called *Hubble's constant*
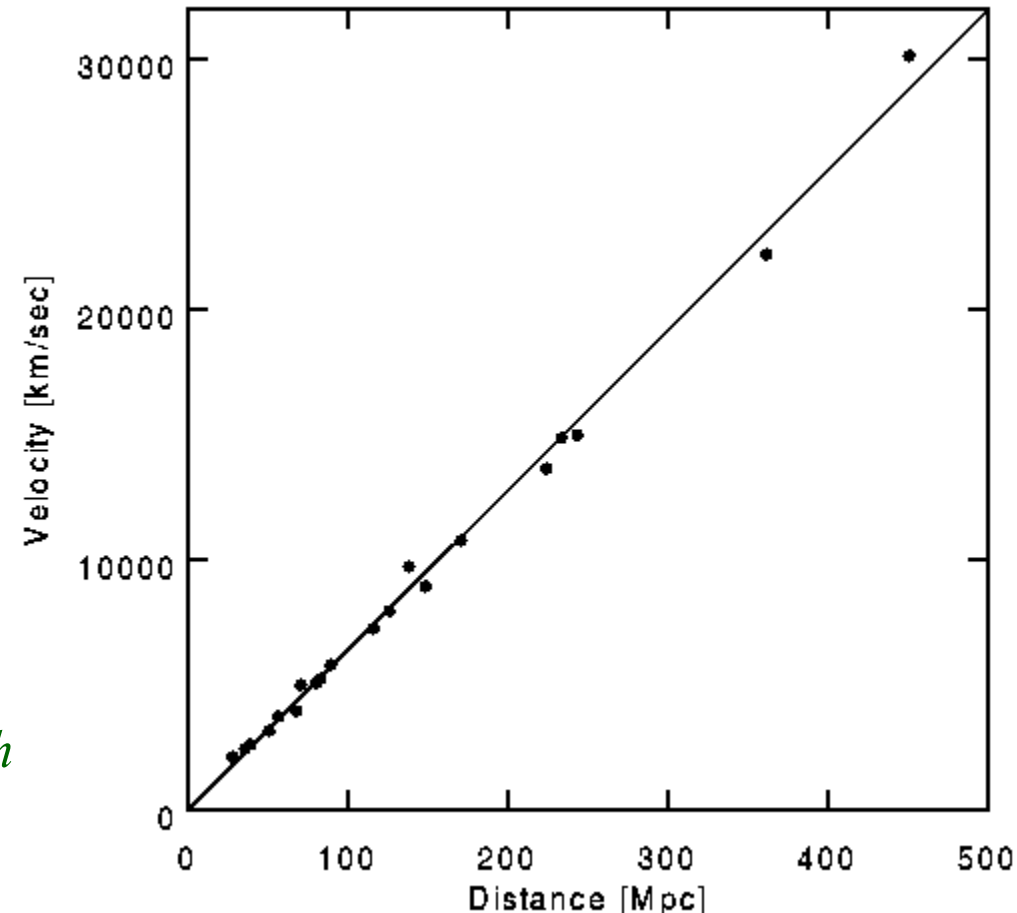- Best fit today (includes *many* assumptions)

$$v = H_0 d$$

$$H_0 = 67.7 \pm 0.4 \text{ km/s/Mpc}$$

Uncertainties:
- Until recently, we didn't know H_0 very well, so we would write

$$H_0 = 100h \text{ km/s/Mpc}$$

  - $h = 0.677 \pm 0.004$
  - I'll probably avoid h.
- Many published documents write answers in terms of h so that we remove the uncertainty in H_0

Past and Future History of Hubble's Constant

- In the mid 1900's, Hubble's constant was > 200 km/s/Mpc
  - Relied crucially on Cepheid Variable stars
- When it was realized that there were stars of different metallicity, and that the period-luminosity relationship was affected, H_0 suddenly decreased by factor of 2

- In the 1980's, there were two sets of techniques that measured Hubble's constant
  - One group gave values around 100 km/s/Mpc
  - The other group gave values around 50 km/s/Mpc
- Around 1990, it was realized that white dwarf supernovae were a great way to measure distances
  - We realized the value was around 70 km/s/Mpc
  - And we all lived happily ever after, until

- In the 2010's, various techniques yielded much more precise values
  - Methods using the cosmic distance ladder give 72±1 km/s/Mpc
  - Methods using fit to cosmological models give 68±1 km/s/Mpc

- **This discrepancy is currently unresolved**
- We will use the value

$$H_0 = 67.7 \pm 0.4 \text{ km/s/Mpc}$$

What's So Great About Hubble's Law?

- You can measure spectrum of almost any bright object
  - Don't have to wait for a supernova, or similar rare event!
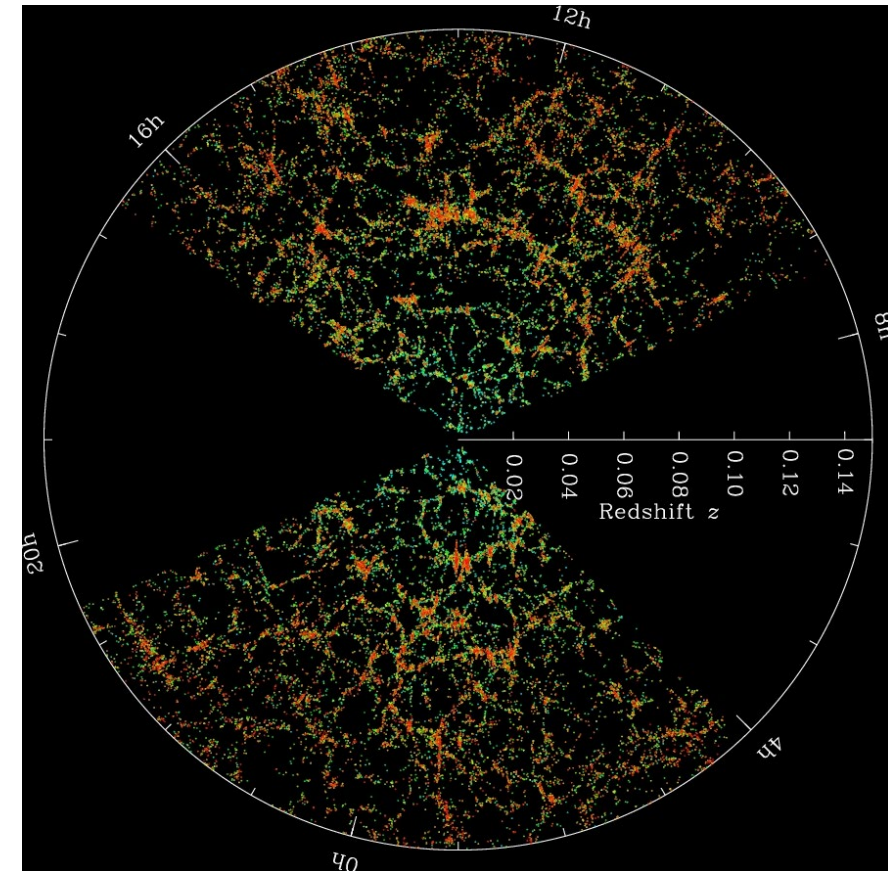
$$1 + z = \frac{\lambda_0}{\lambda}$$

- You can then use Doppler shift to get the velocity

$$1 + z = \sqrt{\frac{1 + v/c}{1 - v/c}} \approx 1 + \frac{v}{c}$$

- Then you use Hubble's Law to get the distance
  - Some caveats will be in order shortly

$$v = H_0 d$$

- Because this system is so robust, it is used to map out large portions of the universe

- Because velocity is proportional to z, which is directly measured, we can plot z and it will be proportional to distance

- For this reason, most such maps simply plot z instead of distance
  - This also bypasses any uncertainties in H_0

Sample Problem

A spectral line from a galaxy normally at 415.00 nm appears instead at 423.00 nm.
(a) How fast is the galaxy moving away from us?
(b) What is the distance to the galaxy?

$$1+z = \sqrt{\frac{1+v/c}{1-v/c}} \approx 1 + \frac{v}{c}$$

$$1+z = \frac{\lambda_{then}}{\lambda_{now}}$$

$$v = H_0 d$$

$$H_0 = 67.7 \pm 0.4 \text{ km/s/Mpc}$$

$$1+z = \frac{\lambda_{then}}{\lambda_{now}} = \frac{423.0}{415.0} = 1.0193 \approx 1 + \frac{v}{c}$$

$$v = 0.0193c$$

$$v = 5790 \text{ km/s}$$

$$d = \frac{v}{H_0} = \frac{5790 \text{ km/s}}{67.7 \text{ km/s/Mpc}}$$

$$d = 85 \text{ Mpc}$$

Failure of Hubble's Law at Small z

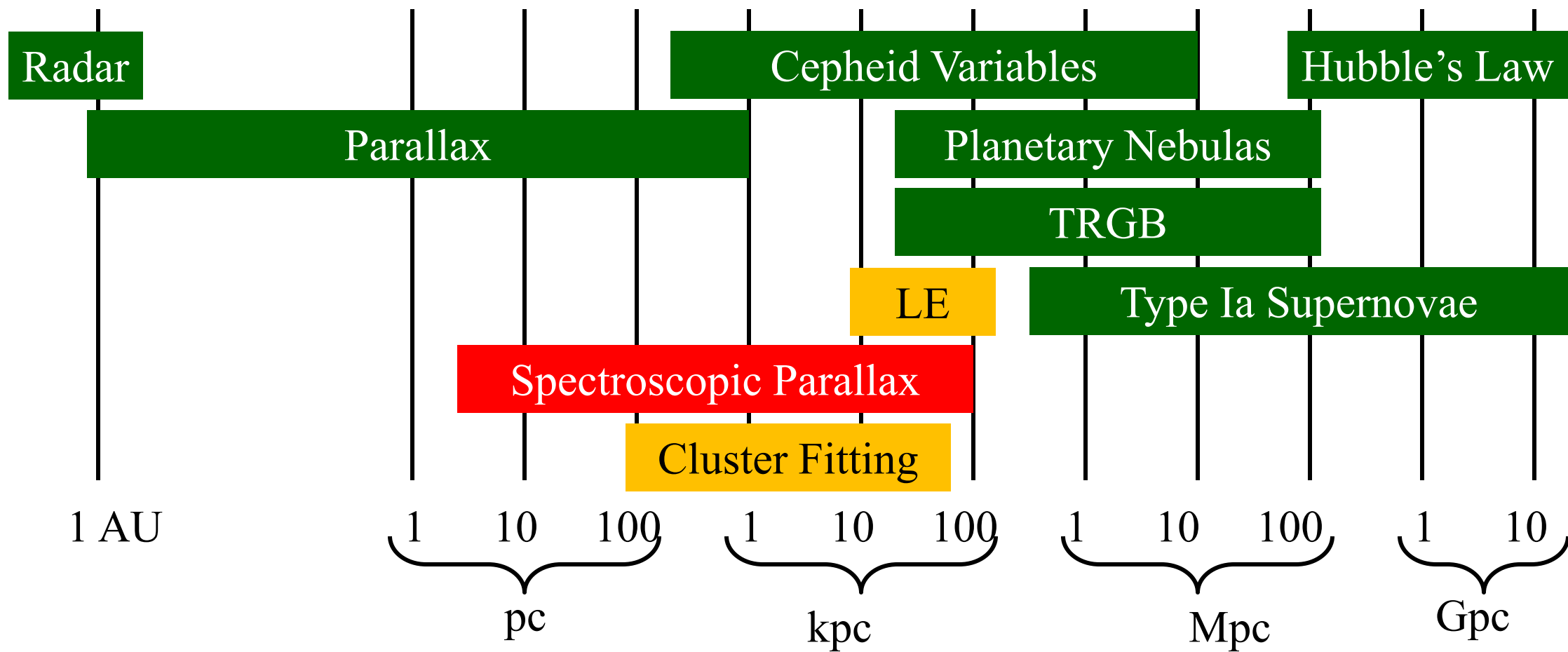Hubble's Law isn't perfect. In particular, for low z:

- Not everything expands
    - Galaxies, galaxy clusters, and smaller objects are not necessarily expanding
    - Galaxy superclusters expand more slowly than Hubble's Law implies

- Objects have extra velocities, called *peculiar velocities*
    - Because gravity changes velocities
    - Typically of order 500 km/s or so

- This causes errors, usually random, of magnitude v_p/H_0 in the distance
    - Typically 7 Mpc or so
    - *Poor* distance indicator for distances under 50 Mpc or so.

- We also have our own peculiar velocity that needs to be compensated for
    - We know how to do this

Failure of Hubble's Law at Large *z*

Hubble's Law isn't perfect. In particular for high *z*:

- As *z* gets large (> 0.2) objects are moving *relativistically*
  - Relativistic corrections
  - Distance gets confusing as well

- We are also looking back into the past
  - Bad: Expansion might not be constant
  - Good: We can study the universe in the past (!)

- Still, higher *z* corresponds to greater distance
  - If we can accurately measure distance to some high *z* objects, we can set up a calibration scale

- Bottom Line: Once a calibration is set up at high *z*, you can use modified Hubble's Law if you are careful

End of Material for Midterm Test

Midterm Exam:
Wednesday October 13
2:00 – 4:00
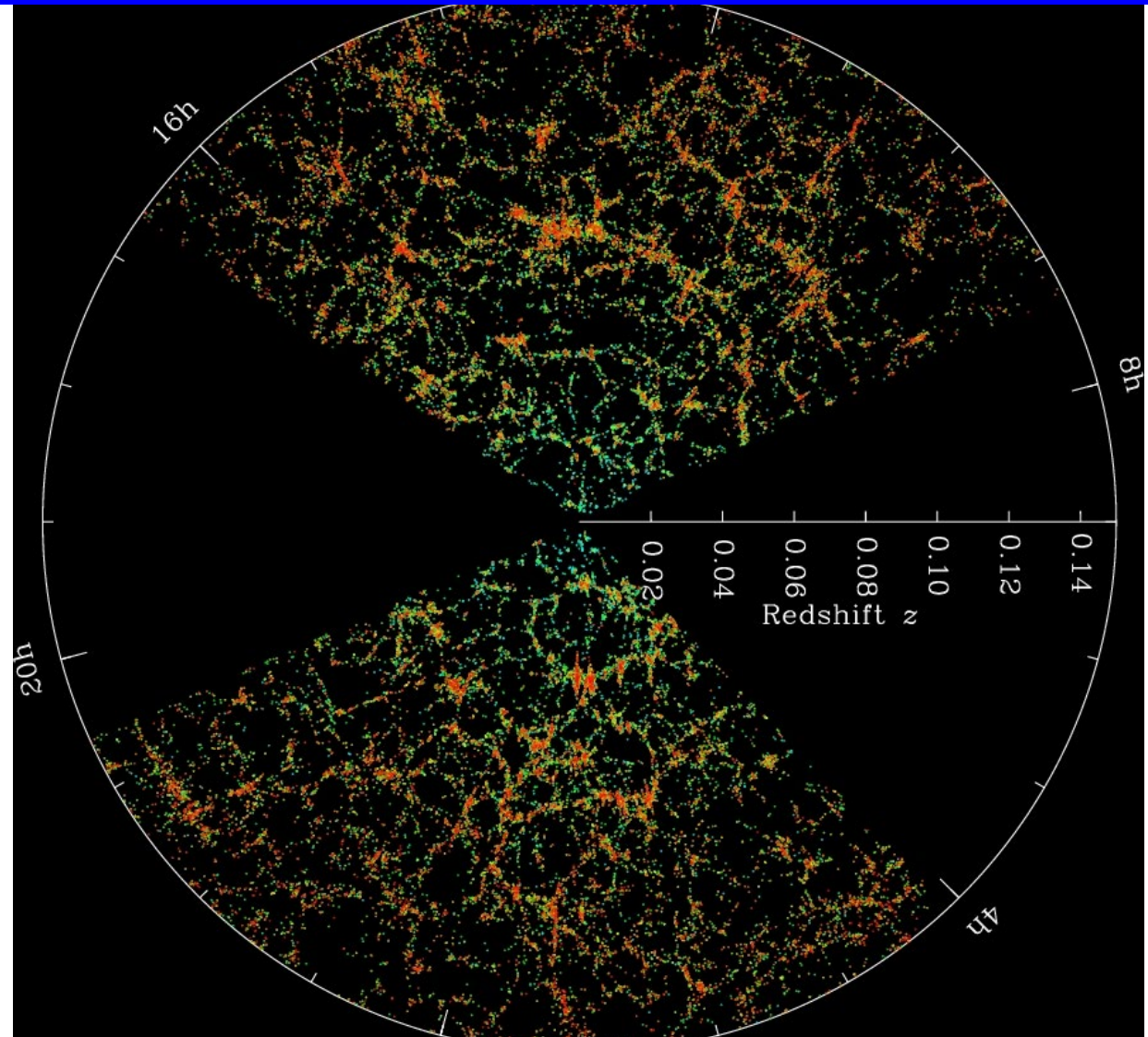Location TBA

List of equations for
Midterm is on the Web

Material for Exam:
- Pen/Pencil
- Paper
- Calulator
- 30 cm metric ruler
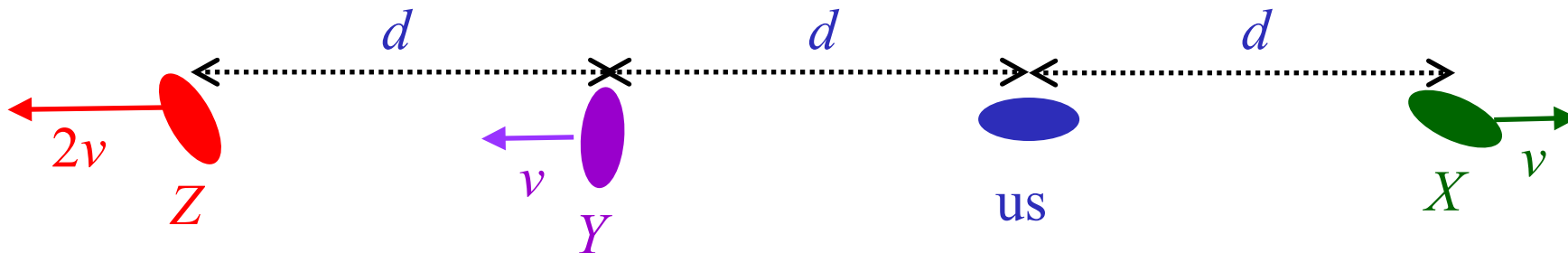
Hubble Expansion

Homogeneity and Isotropy

- On small scales, the universe is not at all uniform
  - Galaxies, Clusters, and Superclusters

- But on the largest scales, it seems to be roughly **homogeneous**
  - The same at all places

- It also looks **isotropic**
  - The same in all directions

- Many properties of the Universe can be worked out just from homogeneity and isotropy

Hubble's Law is Not Arbitrary

- For now, neglect relativity

- Assume the universe is homogenous and isotropic

- Suppose we see galaxy X at distance d moving away from us at speed v

- By isotropy, a galaxy Y on the other side of us at distance d must *also* be moving away at speed v
- But as viewed by galaxy X, galaxy Y is now moving away at speed $2v$ and is at a distance $2d$

- If the universe is homogenous, then galaxy Z, at distance $2d$ from us must also be moving at speed $2v$

- Therefore, if distances are expanding (or contracting), the speed *must* be proportional to distance – Hubble's Law is inevitable
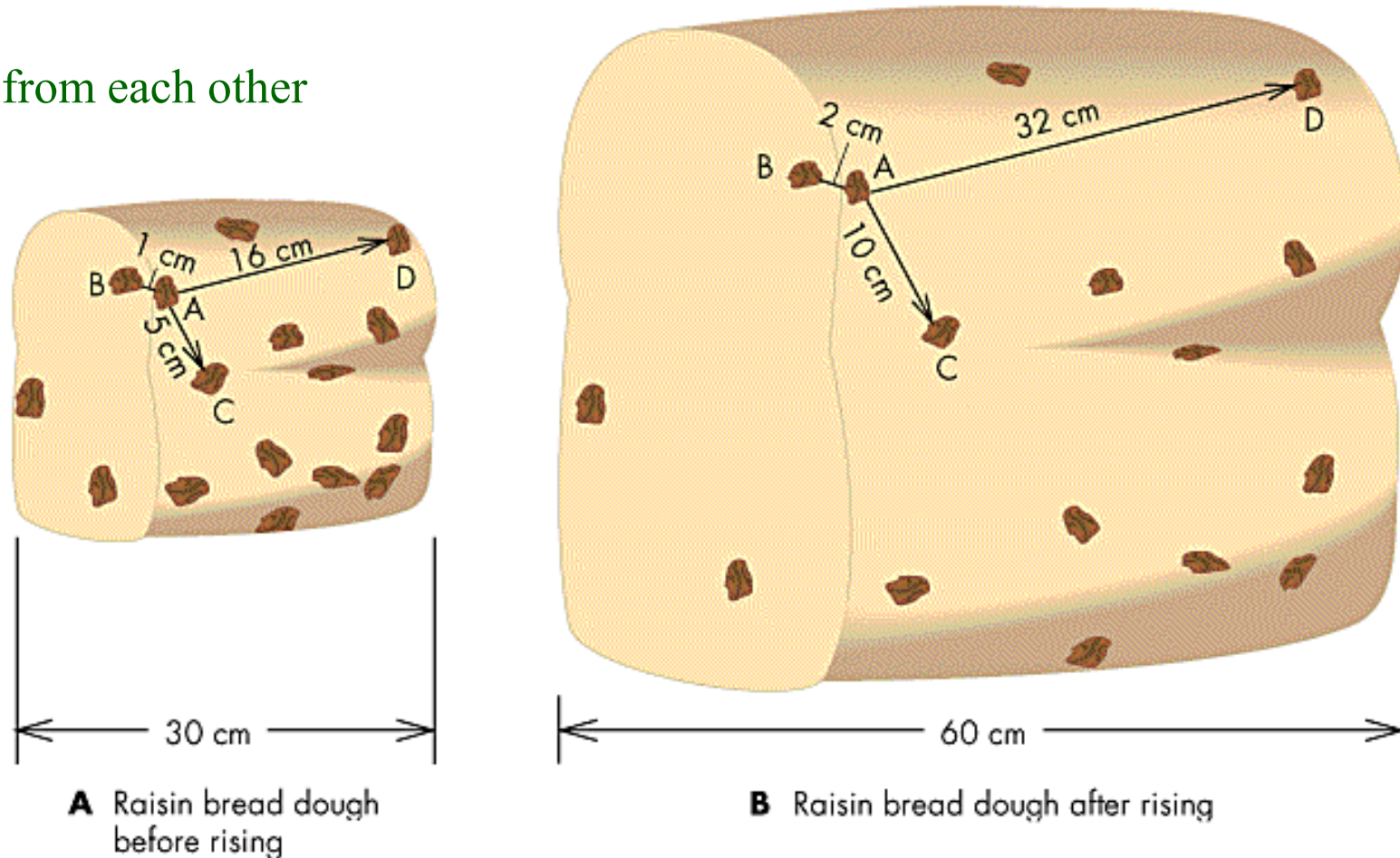
Two Ways of Thinking About Hubble Expansion

Hubble's Law can be thought of two ways:

- All galaxies are flying apart from each other

- The space between the galaxies is expanding

There is no special place in the universe
- It is meaningless to ask "where is it expanding from"
- All observers see the same thing

A Raisin bread dough before rising

30 cm

1 cm   16 cm

B   A   D

5 cm

C

B Raisin bread dough after rising

60 cm

2 cm   32 cm

B   A   D

10 cm

C

Age of the Universe: A Naïve Computation:

- Hubble's Law tells us relation between distance and velocity

$$\boxed{v = H_0 d}$$

- We can therefore figure out how long ago all this stuff was here:
  - Call t_0 the time when everything left here:

$$d = vt_0 = H_0 d t_0 \qquad H_0 t_0 = 1 \qquad t_0 = H_0^{-1} = 14.44 \pm 0.09 \text{ Gyr}$$

- This time is called the inverse Hubble time:

$$H_0 = 67.7 \pm 0.4 \text{ km/s/Mpc}$$

- Compare to oldest stars 13 ± 1 Gyr

- This is incorrect because the velocity of other galaxies is probably <u>not</u> constant

Assuming gravity slows things down, should the actual age of the universe be grater than the inverse Hubble time, or less?
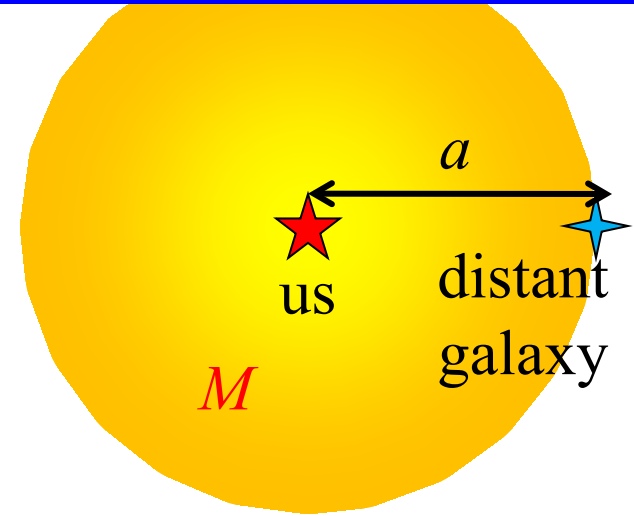
- Speeds are slowing down

- Speeds used to be greater

- Higher speed → Less time to get to where we are now

The Friedman Equation (1)

- Assume the universe is *homogenous* and *isotropic*
  - We can treat any point (us), as center of the universe
  - We can use spherical symmetry around us
- Let *a* be the distance to any specific distant galaxy
  - And let its mass be *m*
- By Gauss's Law, gravitational force is easy to find
  - Spherical symmetry tells us force is towards center
  - And caused by only mass closer than that galaxy

- Because expansion of the universe is uniform, stuff closer than galaxy always closer
  - *M* is constant
- Multiply both sides by 2*da*/*dt* and integrate over time

Don't forget the constant of integration!
- We could call it *k*, for example
- For technical/historical reasons, it is called – *kc*²
  - This makes *k* dimensionless

a

M

us   distant galaxy

$$F = -\frac{GMm}{a^2} = m\frac{d^2a}{dt^2}$$

$$\frac{d^2a}{dt^2} = -\frac{GM}{a^2}$$

$$\int 2\frac{d^2a}{dt^2}\frac{da}{dt}dt = -\int \frac{2GM}{a^2}\frac{da}{dt}dt$$

$$\left(\frac{da}{dt}\right)^2 = \frac{2GM}{a} - kc^2$$

The Friedmann Equation (2)

- Mathematical notation: Time derivatives

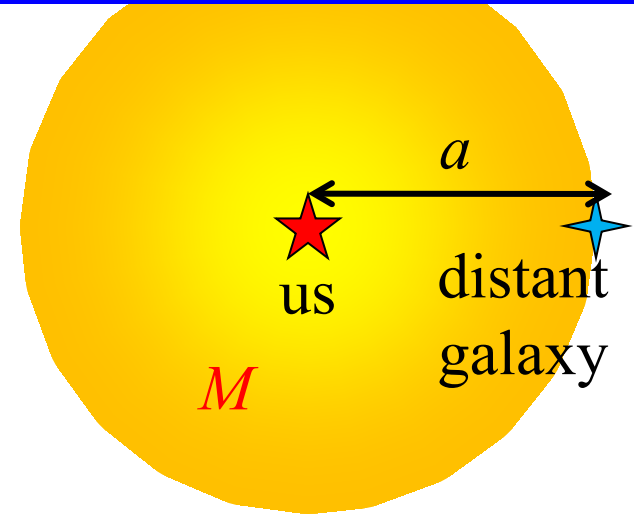$$\boxed{\dot{a} \equiv \frac{da}{dt}}$$

$$\dot{a}^2 = \frac{2GM}{a} - kc^2$$

$$\left(\frac{da}{dt}\right)^2 = \frac{2GM}{a} - kc^2$$

$$\frac{\dot{a}^2}{a^2} = \frac{2GM}{a^3} - \frac{kc^2}{a^2}$$

- Divide both sides by a^2
- Rewrite first term on right in terms of mass density ρ

$$M = \rho V = \frac{4}{3}\pi a^3 \rho$$

The resulting equation is called the *First Friedman Equation*
- The derivation was flawed
  - M does not necessarily remain constant
  - In relativity, other contributions to gravity
- The equation is correct, including relativity

$$\dot{a} = Ha$$

$$\boxed{\frac{\dot{a}^2}{a^2} = \frac{8}{3}\pi G\rho - \frac{kc^2}{a^2}}$$

$$\boxed{\frac{\dot{a}}{a} = H}$$

- The left side is the square of Hubble's constant
- Left side and first term on right independent of which galaxy you use
- We can pick a particular galaxy to make k bigger or smaller
  - Normally chosen so that $k = 0$, +1, or -1

a — us — distant galaxy — *M*

Ω

- We'd like to know what the value of k is

- Define the ratio:

- If we know how big Ω is, we'll know k

$$H^2 = H^2\Omega - \frac{kc^2}{a^2} \qquad \frac{kc^2}{a^2} = H^2(\Omega - 1)$$

- It is common to break Ω into pieces
  - Ordinary atoms and stuff: Ω_b
  - Dark Matter: Ω_d
  - Matter: $\Omega_m = \Omega_d + \Omega_b$
  - Radiation: Ω_r
  - Etc.

- Ω changes with time

- When referring to values today, we add the subscript 0
  - Sometimes

$$\frac{\dot{a}^2}{a^2} = \frac{8}{3}\pi G\rho - \frac{kc^2}{a^2} \qquad \frac{\dot{a}}{a} = H$$

$$\Omega \equiv \frac{\frac{8}{3}\pi G\rho}{H^2}$$

Dens.	Curv.	Name
$\Omega < 1$	$k = -1$	Open
$\Omega = 1$	$k = 0$	Flat
$\Omega > 1$	$k = +1$	Closed

$$\Omega_i \equiv \frac{\frac{8}{3}\pi G\rho_i}{H^2}$$

$$\Omega = \sum_i \Omega_i$$

$$\Omega(t_0) = \Omega_0$$

So, What is Ω?

Stuff	Cont. to Ω
Stars	0.013
Gas, Dust	0.036
Neutrinos	0.0014 – 0.14 (probably near low end)
Dark Matter	0.258 ± 0.011
Light	~ 10^{-4}
Total	**~ 0.308**

$$\Omega_b = 0.0486 \pm 0.0010$$

As we will see, we have missed a substantial fraction of Ω

Age of the Universe, Round 2 (1)

- What's the current density of stuff in the universe?

$$\frac{8}{3}\pi G \rho_0 = H_0^2 \Omega_0 \qquad\qquad -\frac{kc^2}{a_0^2} = H_0^2 \left(1 - \Omega_0\right)$$

$$\boxed{\Omega \equiv \frac{\frac{8}{3}\pi G \rho}{H^2}} \qquad \boxed{\frac{\dot{a}^2}{a^2} = \frac{8}{3}\pi G \rho - \frac{kc^2}{a^2}}$$

- For ordinary matter, density $\propto$ 1/Volume

$$\frac{8}{3}\pi G \rho = \frac{8}{3}\pi G \rho_0 \left(\frac{a_0}{a}\right)^3 = H_0^2 \Omega \left(\frac{a_0}{a}\right)^3$$

$$-\frac{kc^2}{a^2} = -\frac{kc^2}{a_0^2}\left(\frac{a_0}{a}\right)^2 = H_0^2 \left(1 - \Omega_0\right)\left(\frac{a_0}{a}\right)^2$$

- Substitute in:

$$\frac{\dot{a}^2}{a^2} = H_0^2 \Omega_0 \left(\frac{a_0}{a}\right)^3 + H_0^2 \left(1 - \Omega_0\right)\left(\frac{a_0}{a}\right)^2 \qquad \frac{\dot{x}^2}{x^2} = H_0^2 \Omega_0 x^{-3} + H_0^2 \left(1 - \Omega_0\right)x^{-2}$$

- Let $x = a/a_0$ $\quad \dot{x}^2 = H_0^2 \left(\Omega x^{-1} + 1 - \Omega\right)$

$$\frac{dx}{dt} = H_0 \sqrt{\Omega_0 x^{-1} + 1 - \Omega_0} \qquad \frac{dt}{dx} = H_0^{-1}\frac{1}{\sqrt{\Omega_0 x^{-1} + 1 - \Omega_0}}$$
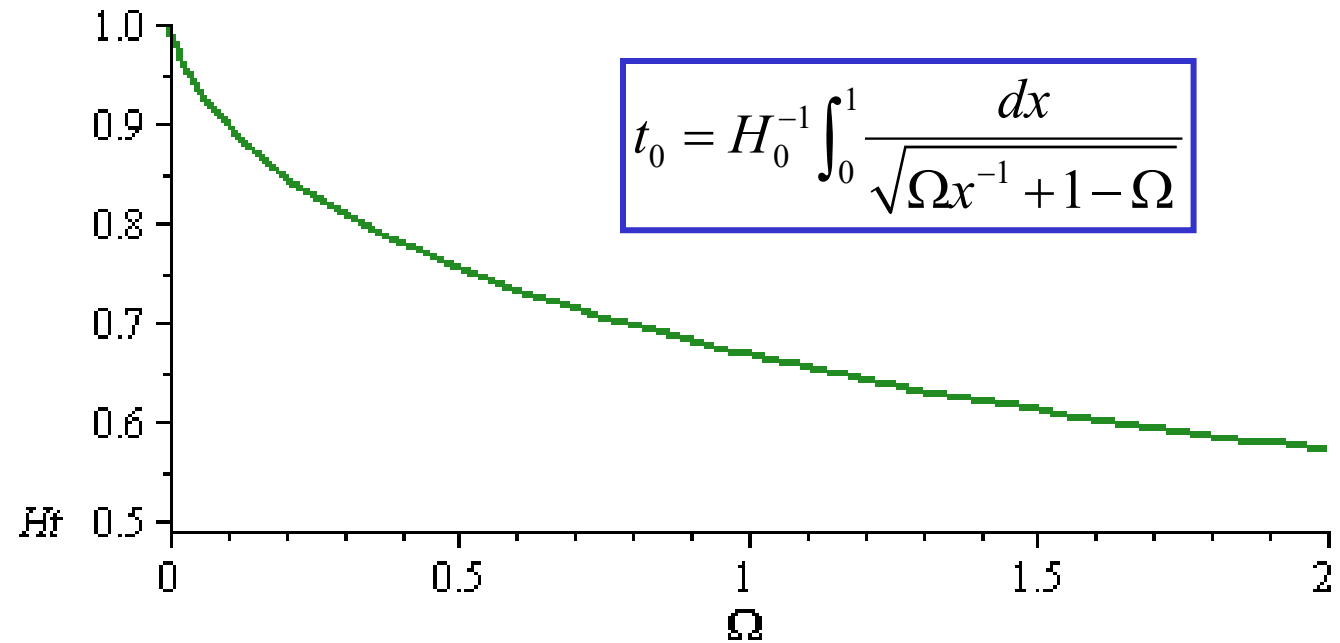
- For age of universe, let x run from 0 to 1

- Integrate to get the age of the universe:

$$\boxed{t_0 = H_0^{-1}\int_0^1 \frac{dx}{\sqrt{\Omega x^{-1} + 1 - \Omega}}}$$

Age of the Universe, Round 2 (2)

$$t_0 = H_0^{-1} \int_0^1 \frac{dx}{\sqrt{\Omega x^{-1} + 1 - \Omega}}$$

- If $\Omega_0 = 0.308$, then
  - $t_0 = 11.6 \pm 0.16$ Gyr

- Theorists like $\Omega_0 = 1$
- If $\Omega_0 = 1.000$, then
  - $t_0 = 9.61 \pm 0.13$ Gyr

- The oldest stars are 13 ± 1 Gyr old

The Age Problem

The Scale Factor

$$k = -1, 0, \text{ or } +1$$

$$\frac{\dot{a}^2}{a^2} = \frac{8}{3}\pi G\rho - \frac{kc^2}{a^2}$$

$$\frac{\dot{a}}{a} = H$$

$$\frac{a}{a_0} = \frac{d}{d_0}$$

- The distance *a* is called the *scale factor*
- All distances increase with the same ratio
- This applies to anything that is not bound together
  - Even to waves of light!

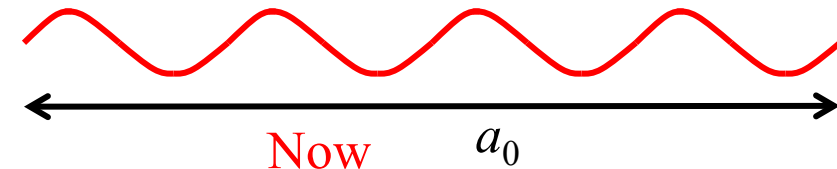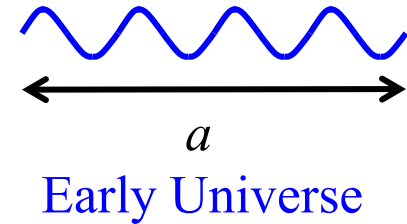Red shift can be thought of as a stretching of universe
- Or is it Doppler shift?
  - Either view is correct; there isn't a right answer
- This view *does* give us another relationship:

$$\frac{a}{a_0} = \frac{\lambda}{\lambda_0} = \frac{1}{1+z}$$

$$\frac{a}{a_0} = \frac{1}{1+z} = \frac{d}{d_0}$$

a

Early Universe

Now a_0

- Most common way to label something in the distant past: *z*
  - Bigger *z* means earlier

Annoying Notational Problems

- The scale factor, describing the size of the universe, goes by different names
- Many older sources call it $R(t)$
- More commonly, it is now called $a(t)$

- I like to stick with the book. So what does Ryder call it?
  - Her convention: $a(t_0) \equiv 1$, and R_0 is how big it is now

- I *refuse* to use this messed up notation

- Therefore, we are going to use the convention I'm most used to

$$\text{scale factor} = R_0 a(t)$$

$$\boxed{\text{scale factor} = a(t)}$$

Ω and the Shape of the Universe

- What's the circumference of a circle of radius *r*?
- According to Einstein, Space time can be curved!
  - Shape of space depends on density parameter: Ω

- If $\Omega = 1$ then universe is *flat* and $C = 2\pi r$

- If $\Omega > 1$ then universe is *closed* and $C = 2\pi a \sin(r/a)$

- If $\Omega < 1$ then universe is *open* and $C = 2\pi a \sinh(r/a)$

- Surface area of a sphere:

$$A = \begin{cases} 4\pi r^2 & \Omega = 1, \\ 4\pi a^2 \sin^2(r/a) & \Omega > 1, \\ 4\pi a^2 \sinh^2(r/a) & \Omega < 1. \end{cases}$$

- A closed universe is finite

$\Omega_0 > 1$

$\Omega_0 < 1$

$\Omega_0 = 1$

MAP990006