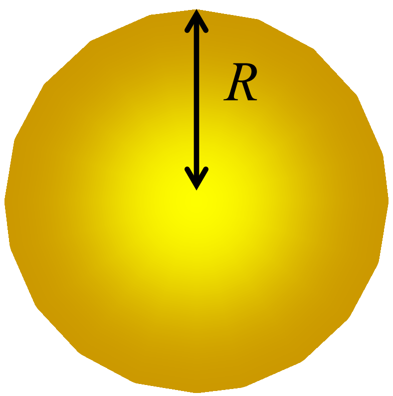
Stars

Star Basics

What Is a Star?

- A star is a sphere of plasma (gas) that produces heat via nuclear fusion
- The most important type of fusion turns hydrogen into helium
  - For most stars, this is the *only* source of energy



$$4\,{}^{1}H^{+} + 2e^{-} \rightarrow {}^{4}He^{++} + 2\nu_e + \textit{energy}$$

- The energy leaves the surface of the star in the form of electromagnetic radiation
- The surface of the star has approximately the same temperature all over
  - The power per unit area, or *flux* F over the surface of the star is approximately constant
- If the star has radius R, the total power (luminosity) is

$$L = FA$$

$$\boxed{L = 4\pi R^2 F}$$

Range of Mass of Stars

- Stars are formed from cool molecular clouds
  - We'll talk more about this later

- If the mass is too small, the star never starts nuclear fusion
  - So there's a minimum mass
  - Pretty well determined at about 0.075 $M_{\odot}$

- If the mass is too large, the star tends to blow itself apart
  - So there's a maximum mass
  - Hard to determine, but around 200 $M_{\odot}$

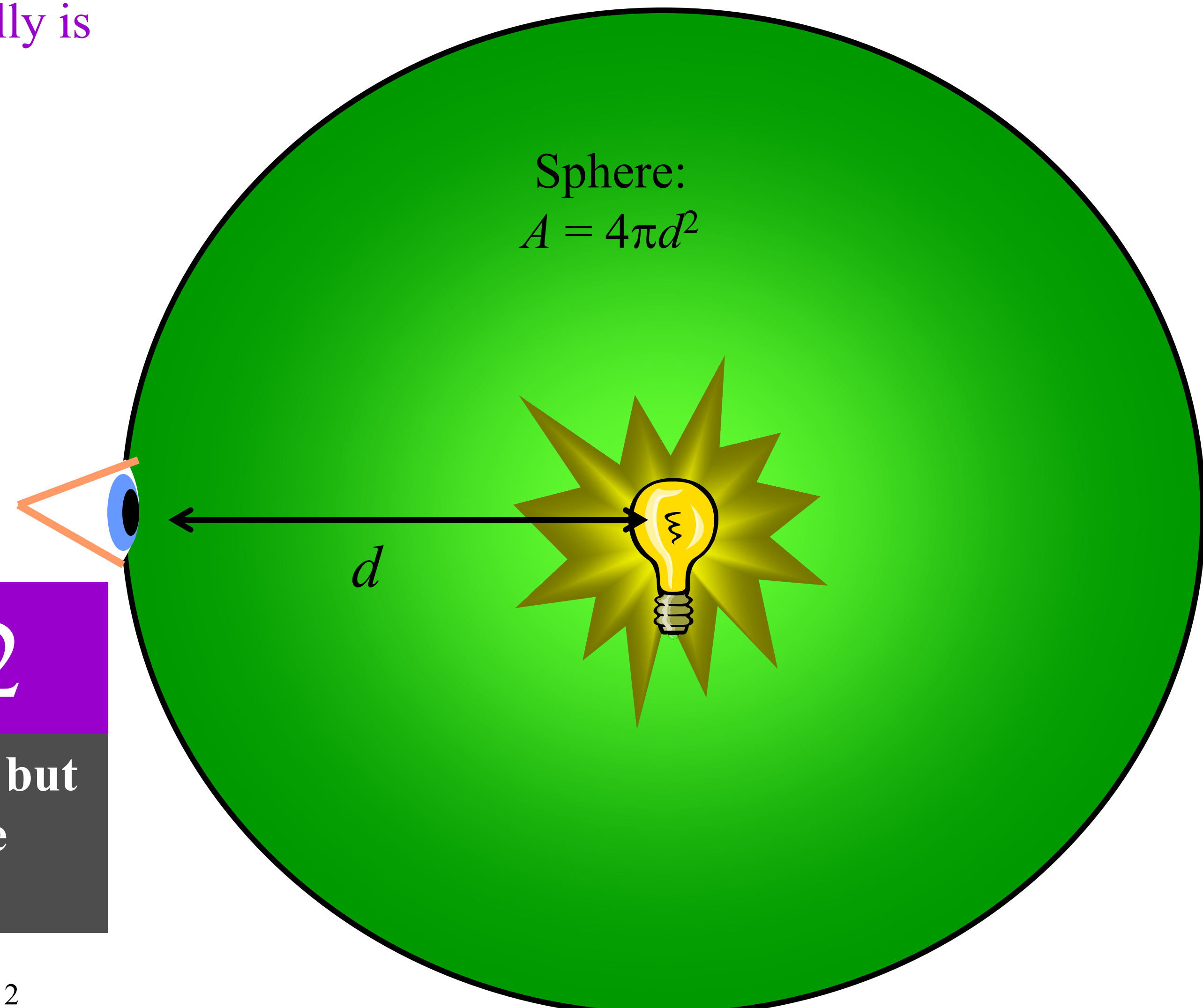
$$0.075 M_{\odot} < M < 200 M_{\odot}$$

- Therefore, all stars come in a range

- Low mass stars are *much more common* than high mass stars
  - The Sun is actually well above average
  - Stars near the upper limit are super rare
  - And they die very quickly

Distance, Luminosity and Brightness

- The *Luminosity L* is how bright a star actually is
  - Units: watts
- The *Brightness F* is how bright a star looks
  - Units: watts/m^2
- The *distance d* is how far away the star is
- These are related: $L = 4\pi d^2 F$



Sample Problem 1.2

Star X and star Y have the same brightness, but star *X* is ten times farther away. How do the star's luminosities compare?

$$L_X = 4\pi d_X^2 F_X$$

$$L_Y = 4\pi d_Y^2 F_Y$$

$$\frac{L_X}{L_Y} = \frac{d_X^2 F_X}{d_Y^2 F_Y} = \left(\frac{d_X}{d_Y}\right)^2 \frac{F_X}{F_Y} = 10^2 \cdot 1 = 100$$

Statistical Mechanics

The application of statistics to the properties of systems containing a large number of objects

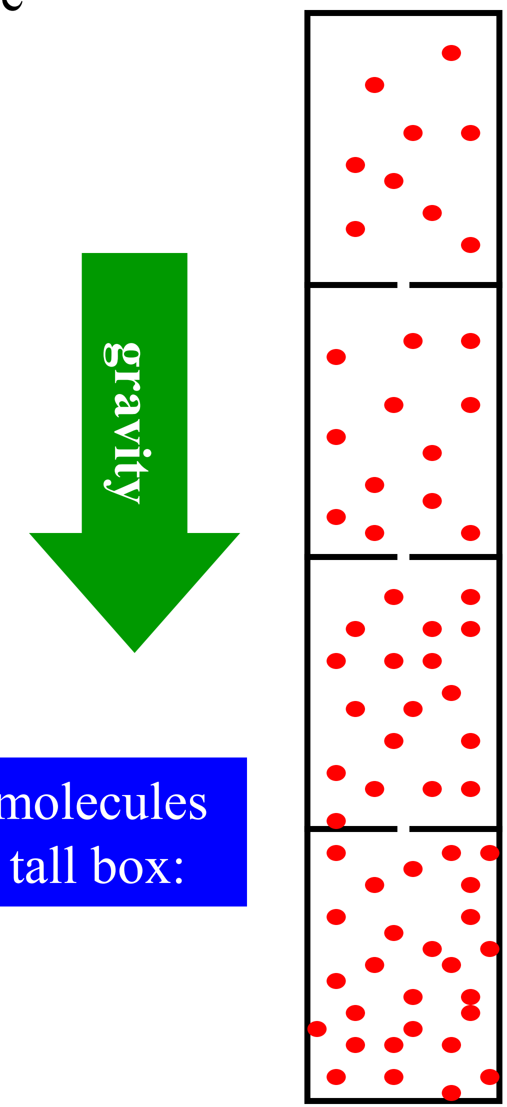
The techniques of statistical mechanics:

- When there are many possibilities, energy will be distributed among all of them
- The probability of a single "item" being in a given "state" depends on temperature and energy

$$P(E) \propto e^{-E/k_B T}$$

$$k_B = 1.3806 \times 10^{-23}\ \text{J/K} = 8.6173 \times 10^{-5}\ \text{eV/K}$$

Gas molecules in a tall box:



This page stolen from PHY 215

Black Body Radiation

- Stars are opaque in their interiors
  - The photons have many, many chances to scatter off of the electrons and atoms

- This causes the light to become completely randomized/thermalized

- Under such circumstances, we can apply *statistical mechanics* to calculate the probability of the wave having a certain amount of energy

- This will cause the light to end up in a distribution called *black body radiation*
  - Calculated (partially) in PHY 215

- The energy density per unit frequency or energy per unit wavelength is then given by

$$u_\lambda(\lambda) = \frac{8\pi}{\lambda^5}\frac{hc}{e^{hc/k_B T\lambda} - 1}$$

$$u_\nu(\nu) = \frac{8\pi}{c^3}\frac{h\nu^3}{e^{h\nu/k_B T} - 1}$$

- These can then be integrated to get the total energy density

$$u = \int_0^\infty u_\lambda(\lambda)\,d\lambda = \int_0^\infty u_\nu(\nu)\,d\nu$$

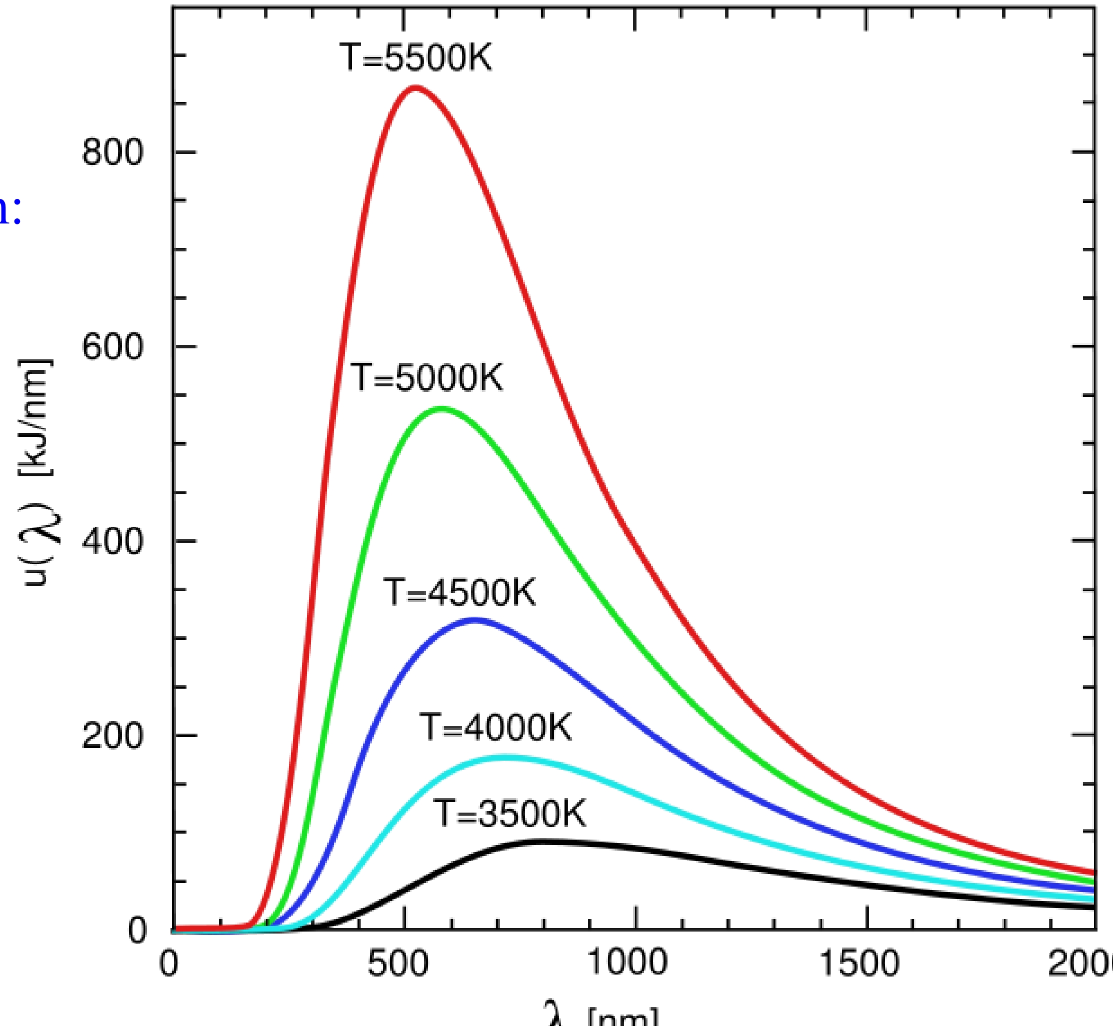
$$u = \frac{\pi^2}{15}\frac{(k_B T)^4}{(\hbar c)^3}$$

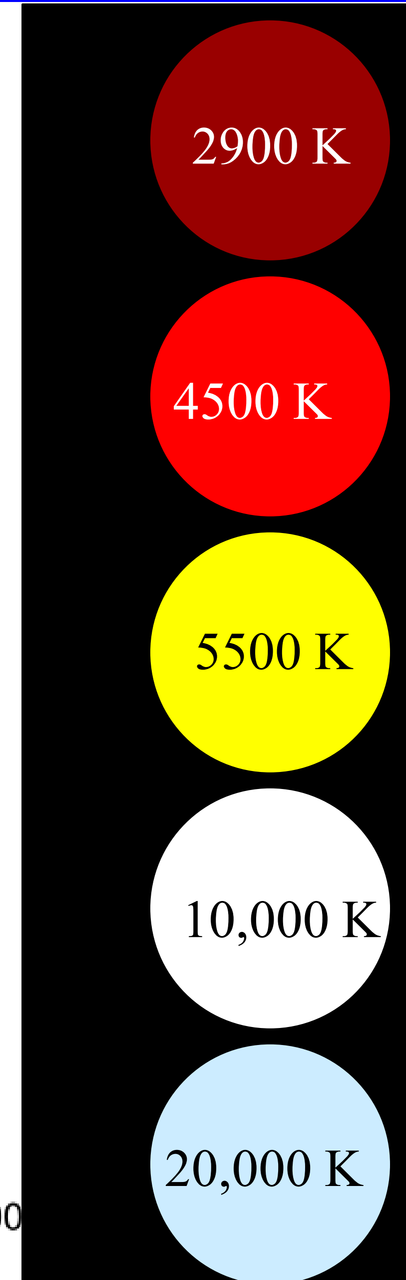
Wien's Law

$$u_\lambda(\lambda) = \frac{8\pi}{\lambda^5}\frac{hc}{e^{hc/k_B T\lambda} - 1}$$

$$\lambda_{\max} T = \frac{hc}{4.96511 k_B} = 2.8978 \times 10^{-3}\ \mathrm{m \cdot K}$$

- The spectrum depends on the temperature:
- We can find the peak wavelength:
- Color gives you a good idea of the temperature
- Colors go from dull red (cool) to electric blue (hot)





Sample Problem 1.3

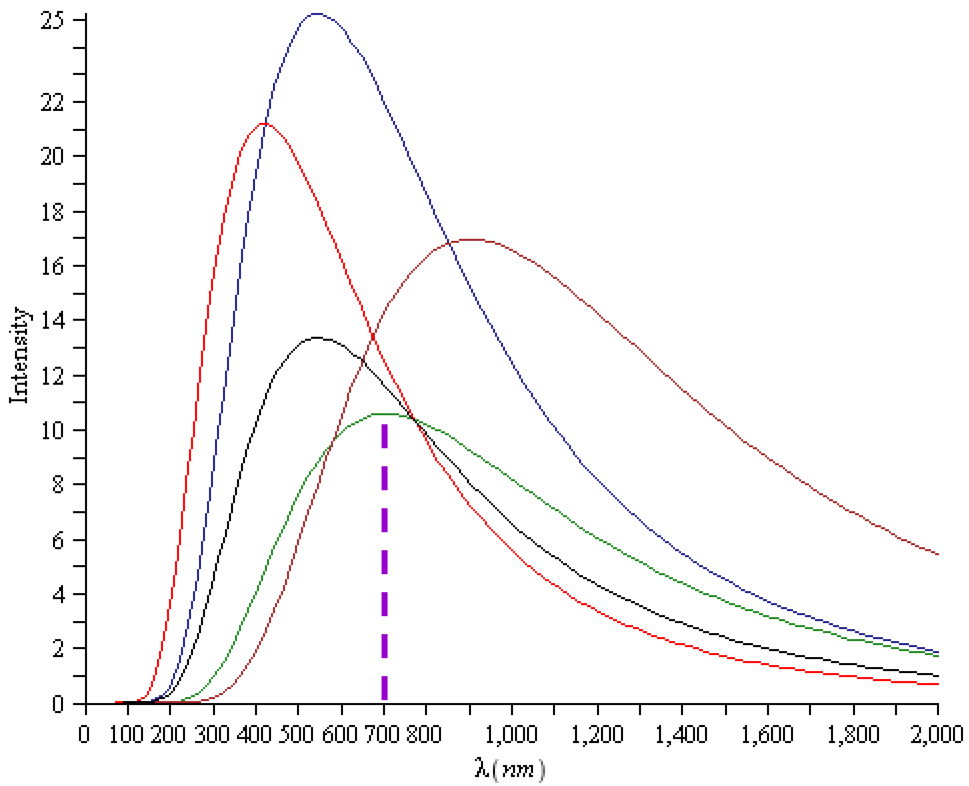
$$\lambda_{max} T = 2.8978 \times 10^{-3}\ \text{m} \cdot \text{K}$$

The graph at the right shows the light received from five stars

(a) Which star is the hottest?

(b) Which two stars have the same surface temperature?

(c) What is the temperature of the green star?



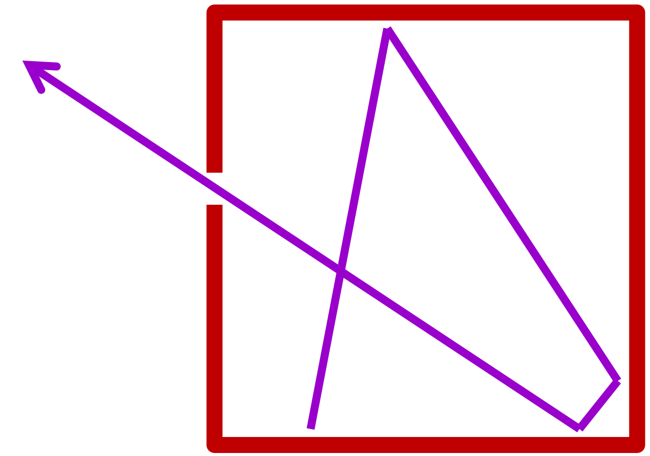
- The overall size of the curve depends on the size and distance of the star
- The peak/color of the star depends on the temperature
- Red star peaks at smallest wavelength / highest temperature
- Blue and black peak at the same wavelength
- Green curve peak is around 705 nm

$$T = \frac{2.8978 \times 10^{-3}\ \text{m} \cdot \text{K}}{705 \times 10^{-9}\ \text{m}} = 4110\ \text{K}$$

Stefan Boltzman Law

- We have formulas for the energy density
- For stars, we want to know rate at which energy escapes
  - Watts per square meter

Units: W/m^2



How much power comes out of a hole of area A in a black body at temperature T?

- Energy density is u
- It is light – moving at velocity c
- Half of it is moving the wrong way (×½)
- The half that is moving the right way is only moving partly in the right direction (×½)
- The resulting total power per unit area (flux):
- σ is called the Stefan-Boltzmann constant

$$F = \tfrac{1}{4}uc = \frac{\pi^2}{60} c \frac{(k_B T)^4}{(\hbar c)^3}$$

$$F = \sigma T^4$$

$$\sigma = 5.670 \times 10^{-8}\ \text{W/m}^2/\text{K}^4$$

- We can then find the total Luminosity:

$$L = 4\pi R^2 F$$

$$L = 4\pi R^2 \sigma T^4$$

- We can similarly find the flux per unit wavelength:

$$F_\lambda(\lambda) = \frac{2\pi}{\lambda^5} \frac{hc^2}{e^{hc/k_B T \lambda} - 1}$$

Stellar Spectra

Spectral Type

- Astronomers use letters to describe the temperature
  - Called the spectral type
- From hottest to coldest: O, B, A, F, G, K, M *
- Subdivided from 0-9, with 0 the hottest
  - Sun is a G2 star, for example

*Why I hate astronomers #1

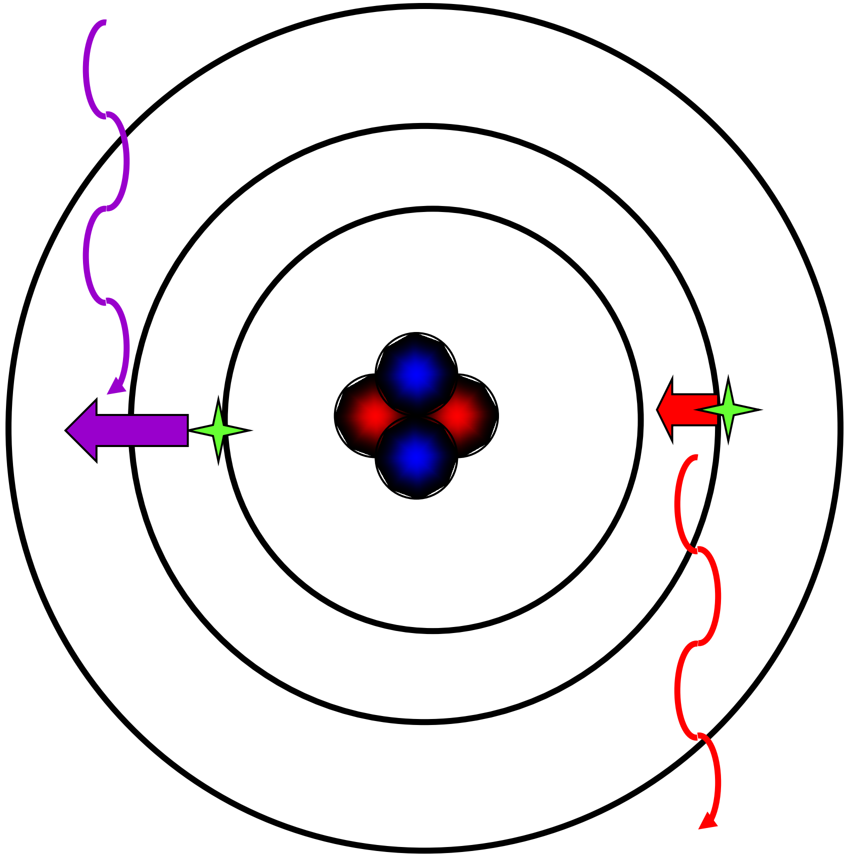
"Oh, be a fine girl, kiss me"

How can we determine spectral type?

- Wien's Law
  - Impractical
- Filters! Compare light in different ranges
  - V filter focuses on middle of visible range
  - B filter focuses on blue end of spectrum
  - R filter focuses on red end of spectrum
- In actuality, spectral lines are a better indication of *T*

Atoms

- The outer layers of stars are cool enough that electrons bind to nuclei to make atoms
- The electrons can be at a variety of energy "levels"
  - Hard to calculate for anything but hydrogen
  - Easy to measure here on Earth



- An incoming photon can be absorbed, and make the electron move to a higher energy level
  - But only if it has the right energy/wavelength

- An excited electron can also fall back, emitting a photon
  - But only if it has the right energy and wavelength

- These energies are the same for absorption/emission

These energies have a small spread, due to:

- Uncertainty principle
- Influence of nearby atoms (pressure broadening)
- Doppler broadening caused by moving atoms (aka thermal broadening)

Kirchoff's Laws

- Deep inside a star, the gas is so thick, everything gets completely thermalized, and you get a black body, or continuous spectrum

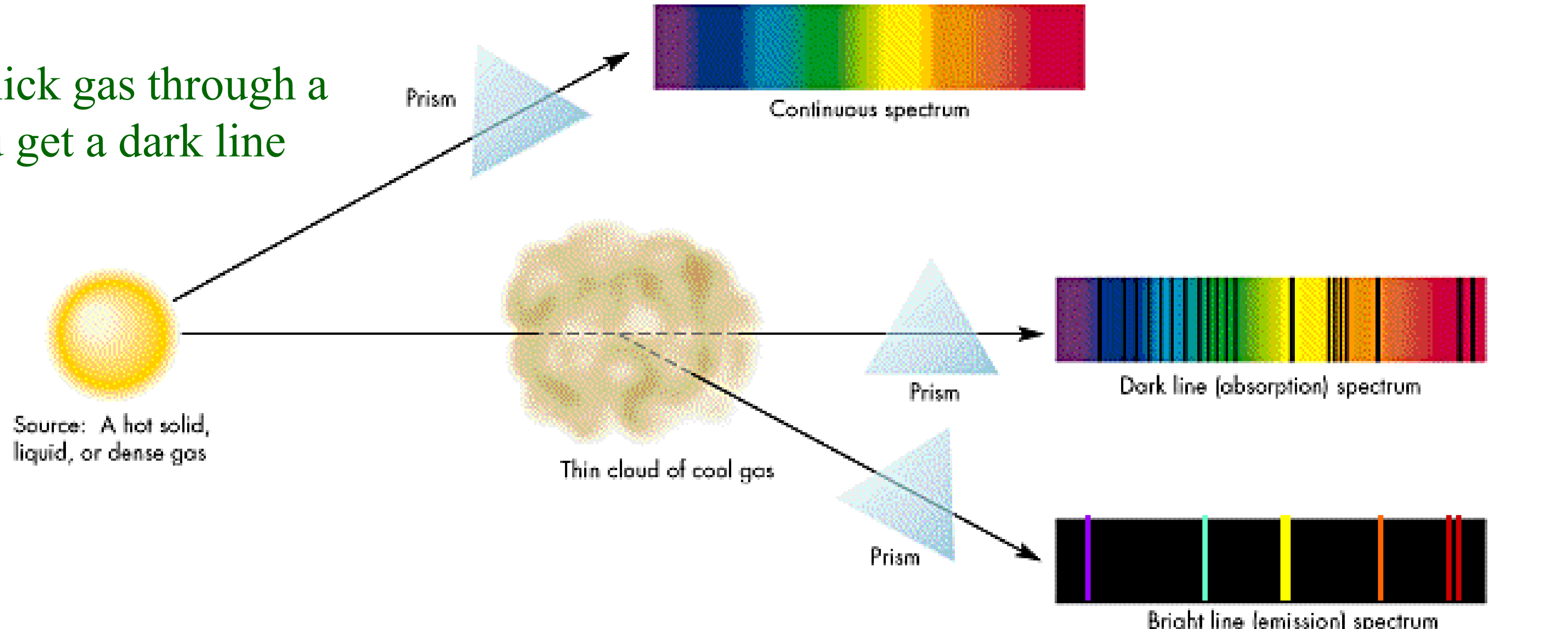
- But outside the star, things aren't so simple:

1) If you have a hot, thick gas, you get a continuous spectrum

2) If you have a hot, thin gas, you get a bright line spectrum

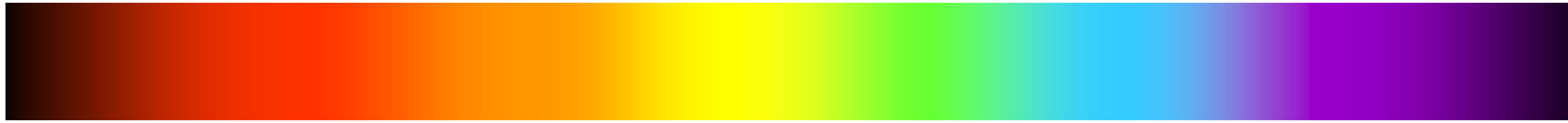
3) If you view a hot, thick gas through a thin, cooler gas, you get a dark line spectrum

- Stars are hot on the inside, cooler on the outside, so you get a dark line spectrum



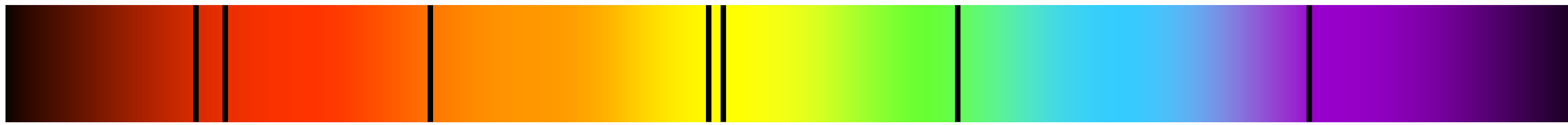
Kirchoff’s Laws Summary

- Hot thick gas:



- Hot thin gas:

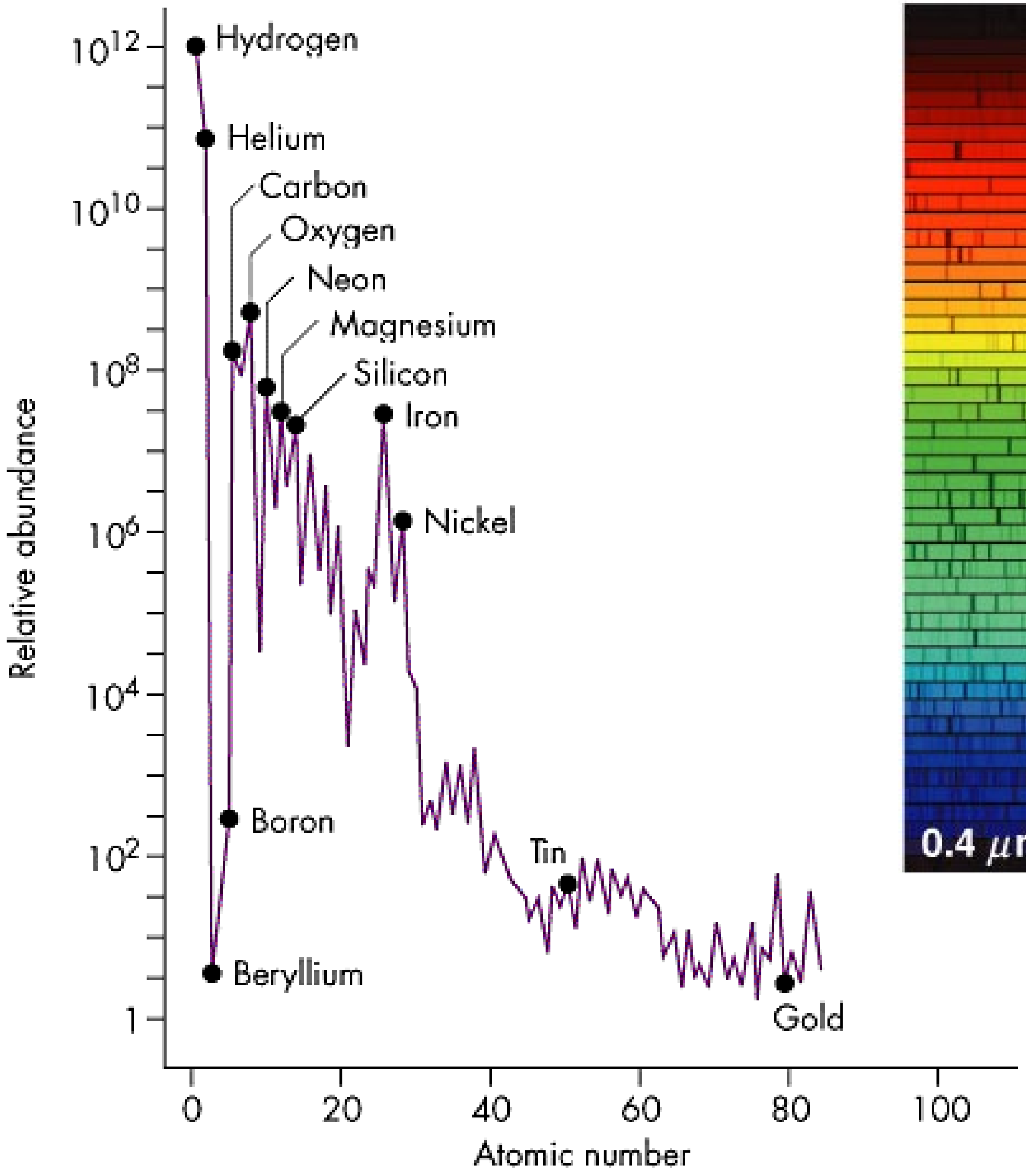
- Hot thick gas behind cooler gas:

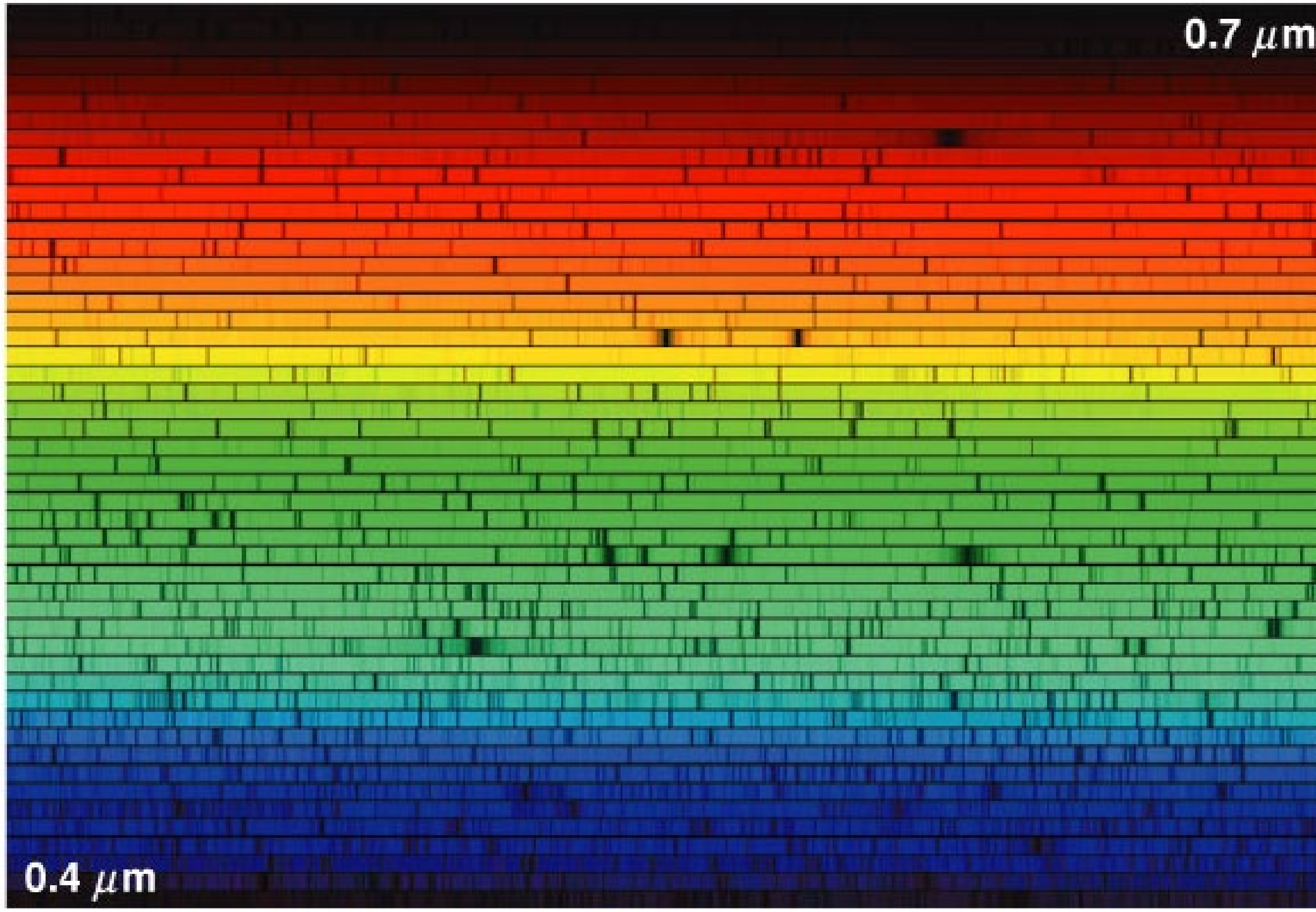


- Each element (hydrogen, helium, etc.) gives characteristic dark lines
- If you have two elements, you’d have two sets of lines

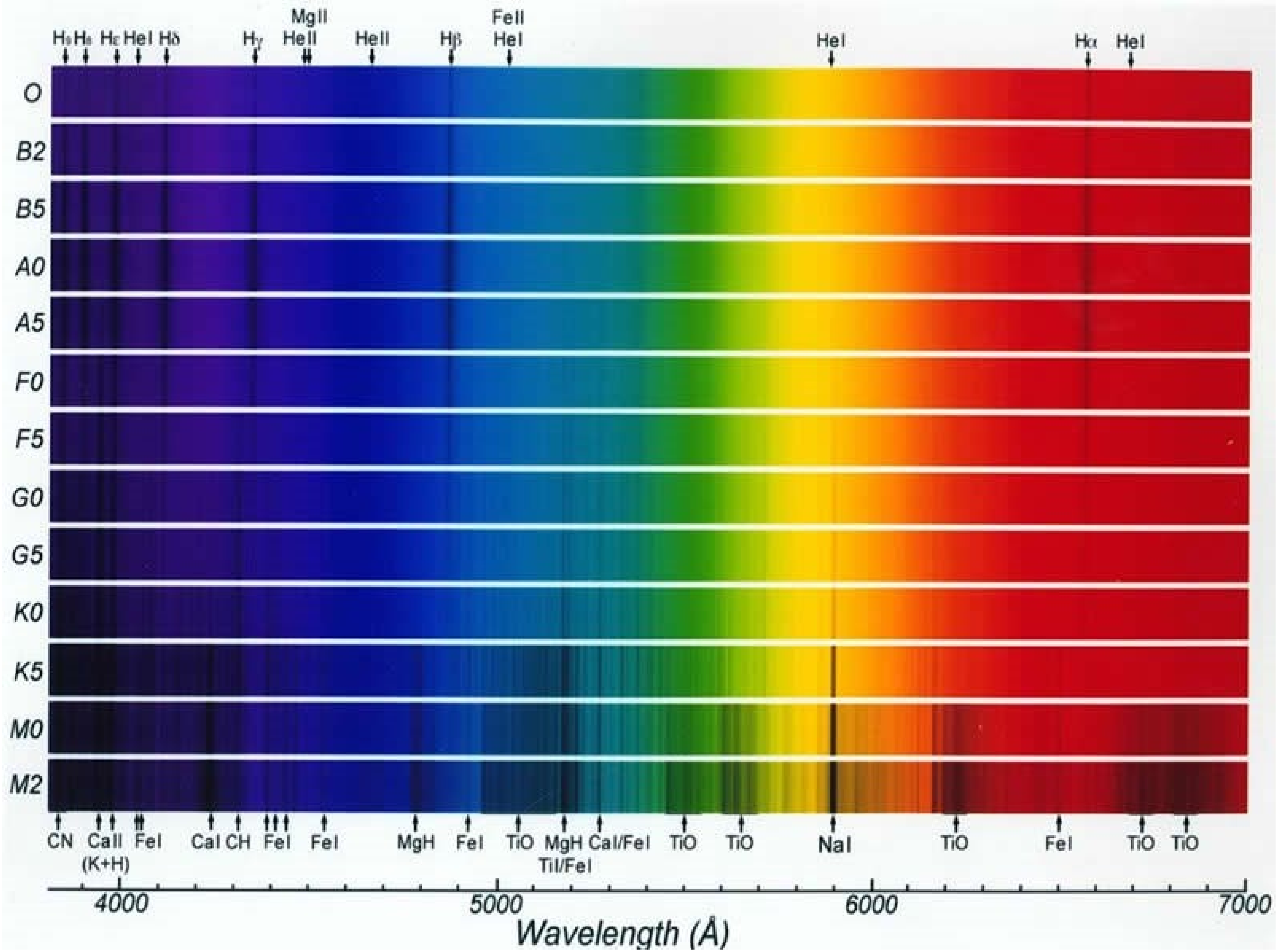
- The combination of lines tells you what the star is composed of
- The strength (darkness) of the lines tells you the fraction of each element
  - It also depends on the temperature
- The width of the lines tells you the pressure
  - A hint about the mass, size, etc. of the star

The Sun's Spectrum





Other Stars' Spectra



Composition of Stars

Most stars have outer composition made of hydrogen, helium, other things:

- Hydrogen mass fraction: called *X*
  - Typically 70-80%
- Helium mass fraction: called *Y*
  - Typically 20-30%
- Everything else: called *Z* or *metallicity**
  - Ranges from 10^{-8} up to a few %

*Why I hate astronomers #2

During *most* of their life, stars do not mix their composition very much

- Surface composition represents (roughly) composition at the time of their birth

- Exceptions: dead stars and dying stars
- The oldest stars tend to have very low metallicity
  - Suggests that even in early universe, there was hydrogen and helium, not much else

Apparent Magnitudes

- Any sensible person would use luminosity and brightness to describe how bright something is and how bright it looks
  - But astronomers aren't sensible
- In ancient times, stars were given an *apparent magnitude* based on brightness
  - Brightest stars were called 1st magnitude
  - Dimmest were 6th magnitude
  - Denoted by *m*

*Why I hate astronomers #3

- Note that the bigger *m* is, the smaller the brightness *

$$F \propto 100^{-m/5}$$

- It was later realized that it is approximately a logarithmic scale
  - Each five magnitudes is a factor of 100 brightness
  - Each unit increase is $100^{1/5} = 2.512$ times dimmer

$$F = 2.518\times10^{-8}\,\mathrm{W/m^2}\left(10^{-\frac{2}{5}m}\right)$$

$$10^{\frac{2}{5}m} = \frac{2.518\times10^{-8}\,\mathrm{W/m^2}}{F}$$

$$\tfrac{2}{5}m = \log_{10}\left[\left(2.518\times10^{-8}\,\mathrm{W/m^2}\right)/F\right]$$

$$m = 2.5\log\left[\left(2.518\times10^{-8}\,\mathrm{W/m^2}\right)/F\right]$$

Comments on Luminosity and Brightness

- We can simply measure the brightness of a star

$$L = 4\pi d^2 F$$

- If we know the distance, we can get the Luminosity
  - Much, much more on this later
- If we know the luminosity, we can get the distance
  - Much, much more on this later

Complications: (don't worry about these)

- Dust and other obscurations complicate this
  - A problem we will ignore, mostly
- Filters: How bright something looks depends partly on the filter you use
  - This formula must be modified for the type of filter you use
  - Hence there are *many* apparent magnitudes, m_V, m_B, m_R, etc.
- We'll deal with only the total brightness (unfiltered)
  - Called bolometric magnitude, denoted m_{bol}

Sample Problem 1.4

$$F = 2.518\times10^{-8}\ \mathrm{W/m^2}\left(10^{-\frac{2}{5}m}\right)$$

Star *X* is 1000 times brighter than star *Y*. How do their apparent magnitudes compare? What if it is *N* times brighter?

$$F_X \propto 10^{-\frac{2}{5}m_X} \qquad F_Y \propto 10^{-\frac{2}{5}m_Y}$$

$$\frac{F_X}{F_Y} = \frac{10^{-2m_X/5}}{10^{-2m_Y/5}} = 10^{2(m_Y - m_X)/5}$$

$$\tfrac{2}{5}\left(m_Y - m_X\right) = \log\left(F_X / F_Y\right)$$

$$m_Y - m_X = \tfrac{5}{2}\log 1000 = 2.5\times 3 = 7.5$$

$$m_Y - m_X = \tfrac{5}{2}\log\left(F_X / F_Y\right) = \tfrac{5}{2}\log N$$

Absolute Magnitudes

- How bright a star *really is* depends on how bright it looks and how far away it is
- Define: *Absolute Magnitude*: The apparent magnitude of the star *if* it were 10 pc away
  - Why 10 pc? I don't know – but it's a typical distance for a nearby star
  - Denoted by M
- We can then determine a nice formula relating distance, apparent magnitude, and absolute magnitude:

$$F = \frac{L}{4\pi d^2} = \frac{L}{4\pi(10\text{ pc})^2}\left(\frac{10\text{ pc}}{d}\right)^2 = F_{10}\left(\frac{10\text{ pc}}{d}\right)^2$$

$$\frac{F_{10}}{F} = \left(\frac{d}{10\,\text{pc}}\right)^2$$

$$m - m_{10} = \frac{5}{2}\log\left(F_{10}/F\right)$$

$$m - M = \frac{5}{2}\log\left[\left(\frac{d}{10\text{ pc}}\right)^2\right] = 5\log\left(\frac{d}{10}\right) = 5\log d - 5$$

$$m - M = 5\log d - 5$$

$$d = 10^{1+\frac{m-M}{5}}\text{ pc}$$

The Hertzsprung-Russell Diagram

- Stars have lots of properties which we could use to characterize them:
  - Brightness
  - Distance
  - Velocity
  - Mass
  - Age
  - Composition
  - Color
  - Radius
  - Luminosity
  - Temperature

Not intrinsic (Brightness, Distance, Velocity)

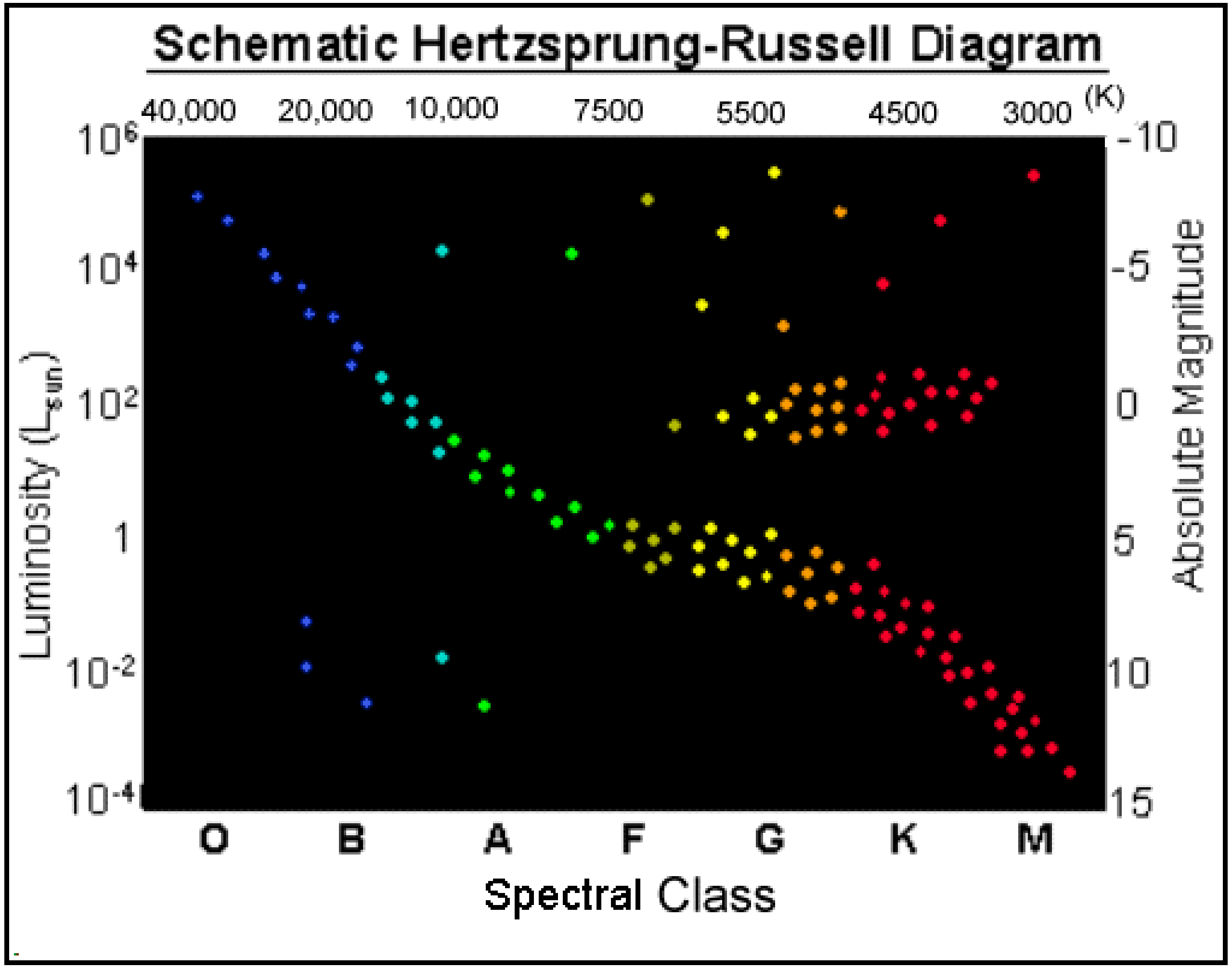
Hard to Measure (Mass, Age)

Uniform (Composition)

Derivable (Color, Radius)

- The Hertzsprung-Russell (HR) diagram is a plot of spectral class vs. luminosity
  - Hot on the left*

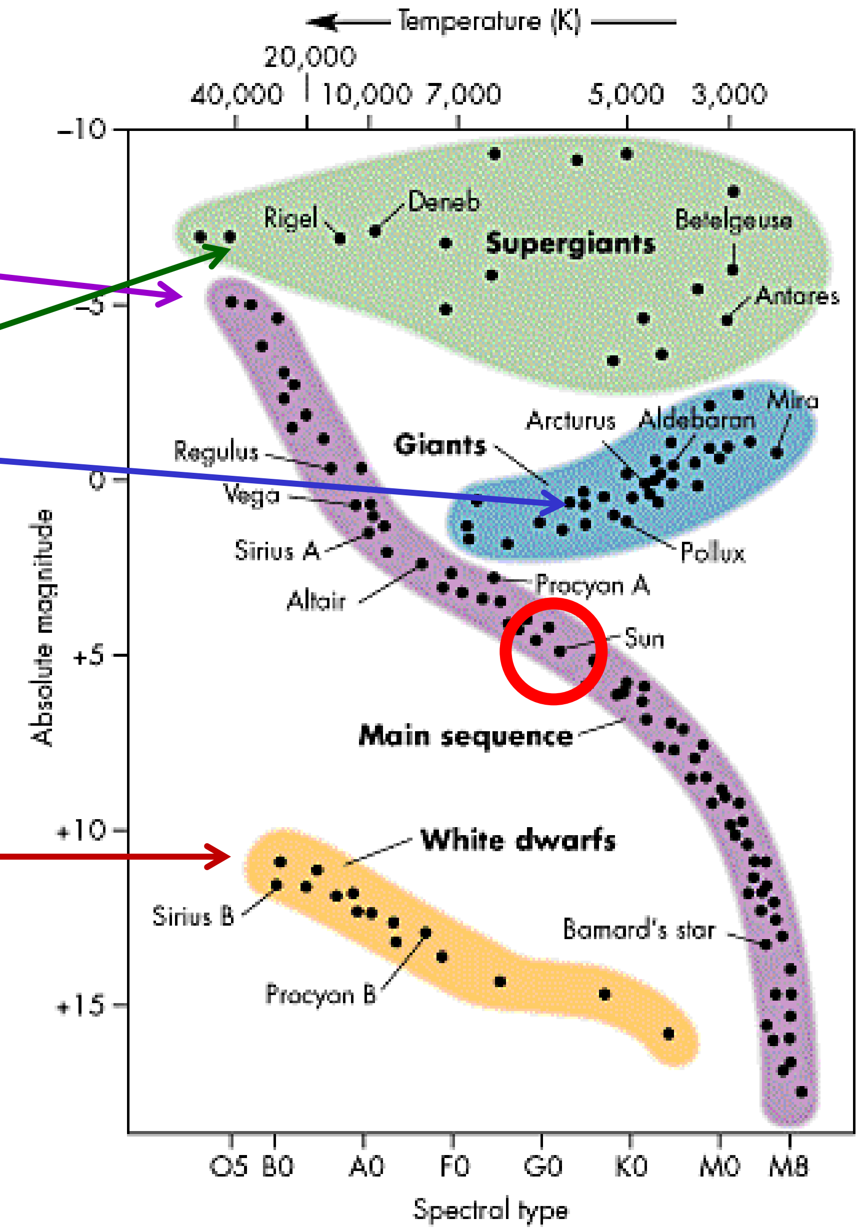
*Why I hate astronomers #4



The H-R Diagram: Patterns

The stars are *not* distributed randomly on the H-R diagram:

- *Main sequence* stars
  - 90+% of living stars
- Giants
  - Rare
  - Easy to see because they are bright
- Supergiants
  - Very rare
- White dwarfs
  - Dead stars, very dim



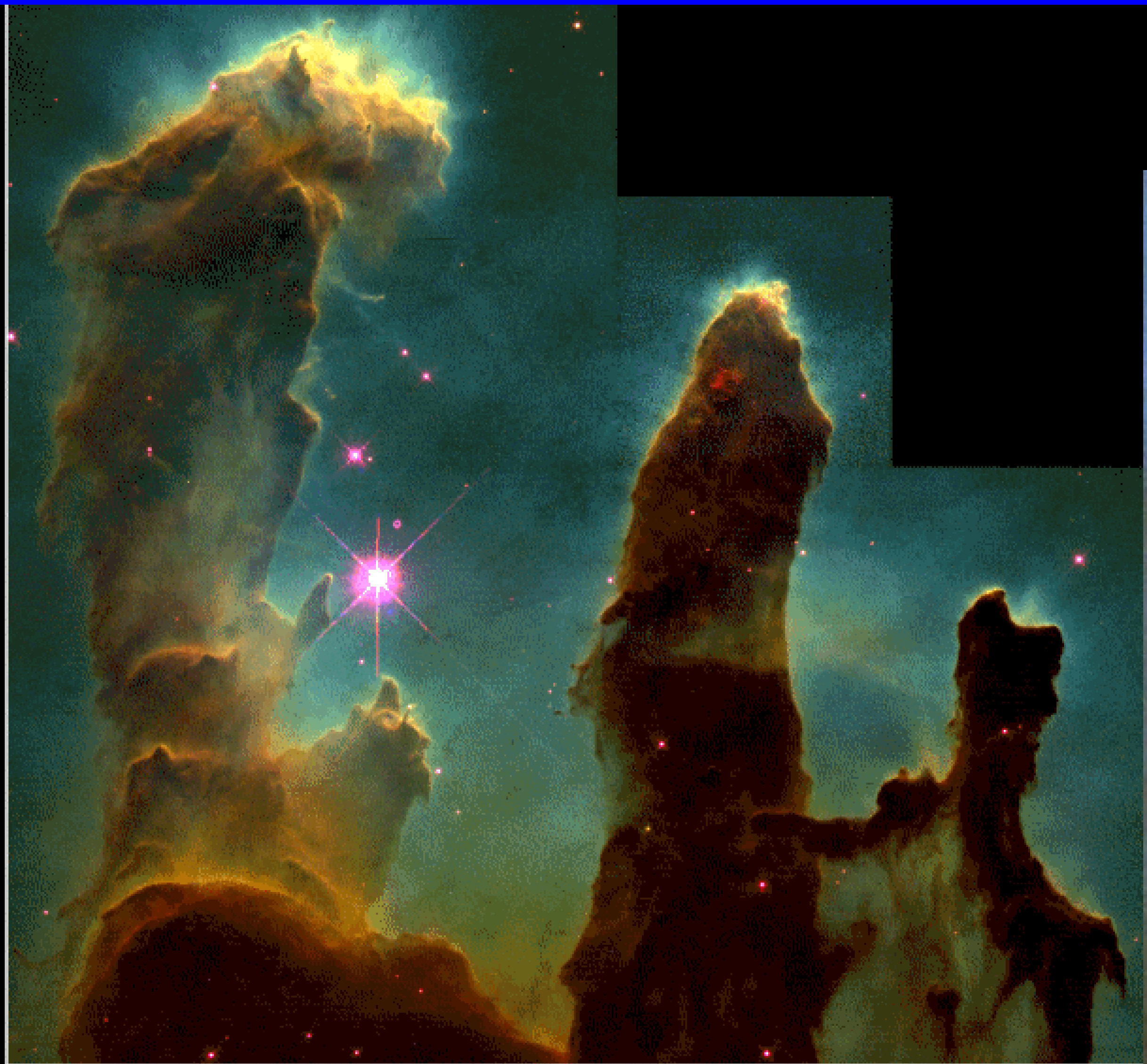
Basics of Stellar Evolution

Stars form from cool clouds of gas called *molecular clouds*

- Gravity overcomes pressure, and several stars begin to form
- Usually get multiple stars in the same region, about the same age
  - Called clusters
- Initially, the stars are all moving together at the same speed
- Lots of stars with low mass, few with large mass
  - Lowest mass: about 0.08 $M_\odot$
  - Highest mass: about 200 $M_\odot$

- The life history of a star depends primarily on its mass
  - A little bit on its metallicity (Z)
  - Sometimes influenced by nearby stars
- Low mass stars ($M < 8M_\odot$) live a long life and die slowly
- High mass stars ($M > 8M_\odot$) live fast and die violently
- The more massive a star is, the faster it does everything

Molecular Clouds



Gaseous Pillars · M16 HST · WFP
PRC95-44a · ST ScI OPO · November 2, 1995
J. Hester and P. Scowen (AZ State Univ.), NASA

Outline of Stellar Evolution

Molecular Clouds

Low Mass Stars
$M < 8M_\odot$

- Protostar
- Main Sequence Star
- Red Giant star
- Horizontal Branch
- Asymptotic Giant Branch
- Planetary Nebula
- White Dwarf

High Mass Stars
$M > 8M_\odot$

- Protostar
- Main Sequence Star
- Supergiant Stages
- Type II Supernova
- Neutron Star or Black Hole

Stars are powered by *nuclear fusion*

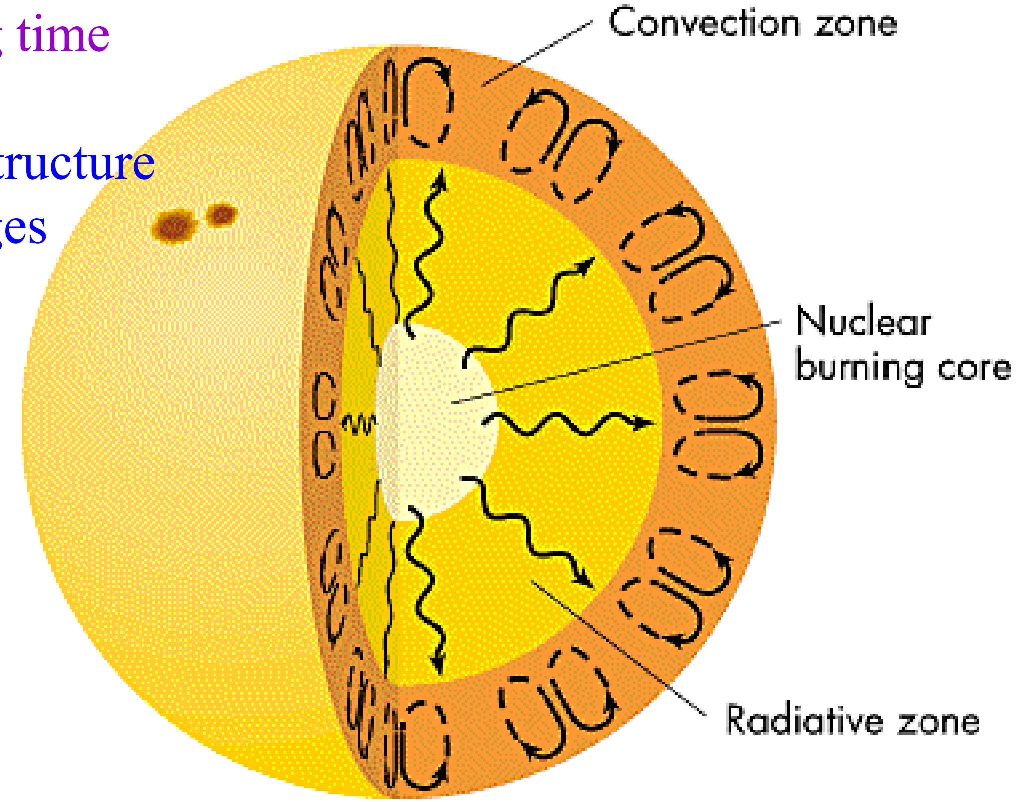
- The combining of simple nuclei to make more complex ones
- Stages are defined by what is going on at the center

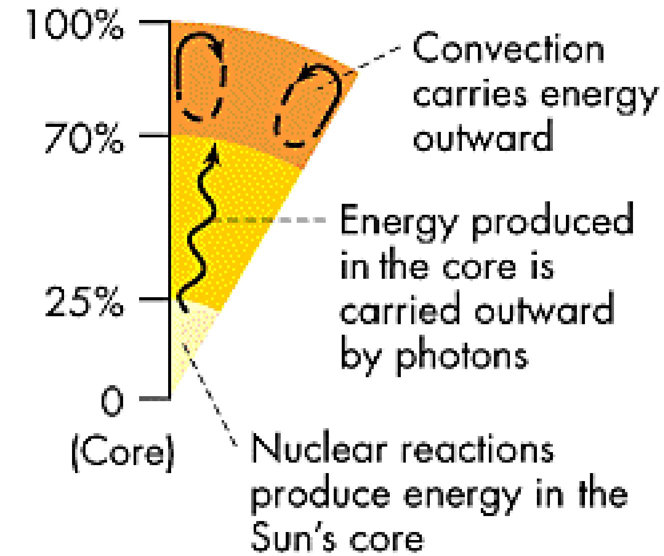
Main Sequence Stars: Introduction

A *Main Sequence Star* is a star that is burning hydrogen to helium at its center

$4\ ^1H \rightarrow ^4He$

- This is *nuclear* burning, not combustion
  - No oxygen
  - We don't care about the details
- This process is *extremely* efficient
  - It can go for a long time
- During this stage, the structure of the star hardly changes
  - *Small* increase in luminosity
  - Spectral class stays almost the same
- *Small* motion upwards in the H-R diagram





Main Sequence Stars: Mass Dependance

Everything about the star depends on mass

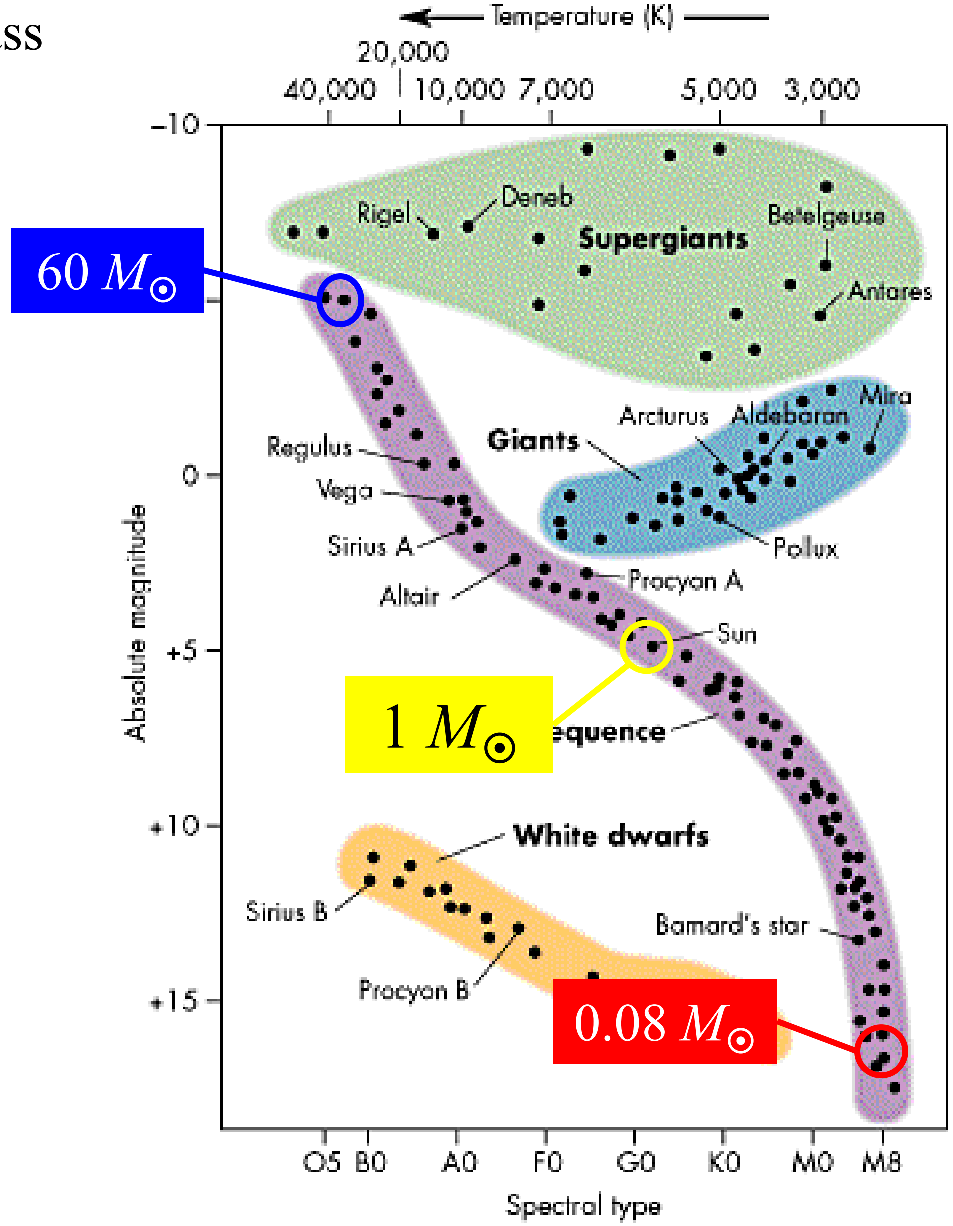
- Higher mass stars have:
  - Larger radius

    $R \sim M$
  - Somewhat higher temperature

    $T \sim M^{0.4}$
  - Much higher luminosity

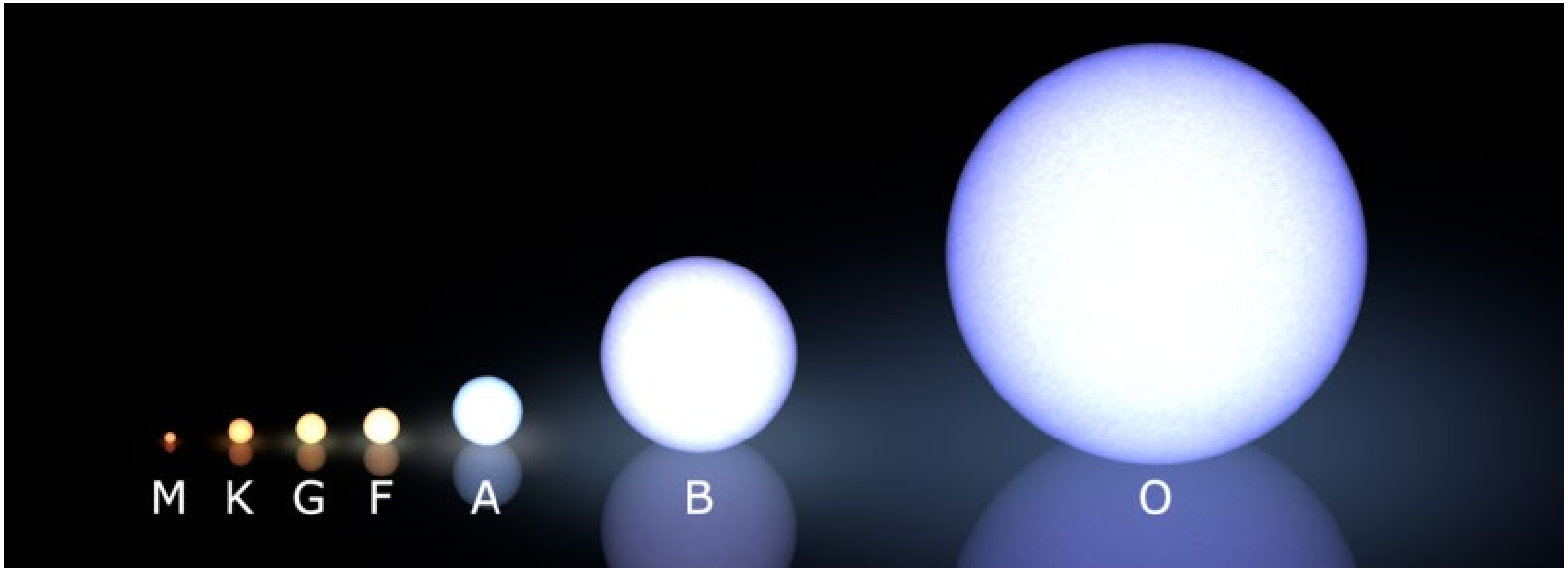
$$L \propto R^2 T^4 \quad \sim M^{3.5}$$

The main sequence is a *band* because
- Stars have variable metallicity
- Stars are different ages



Type	Mass
O5	60
B0	18
B5	5.9
A0	2.9
A5	2.0
F0	1.6
F5	1.3
G0	1.05
G5	.92
K0	.85
K5	.74
M0	.51
M5	.21
M8	.06

Main Sequence Stars: Lifetime



A star stays as a main sequence star until it runs out of hydrogen

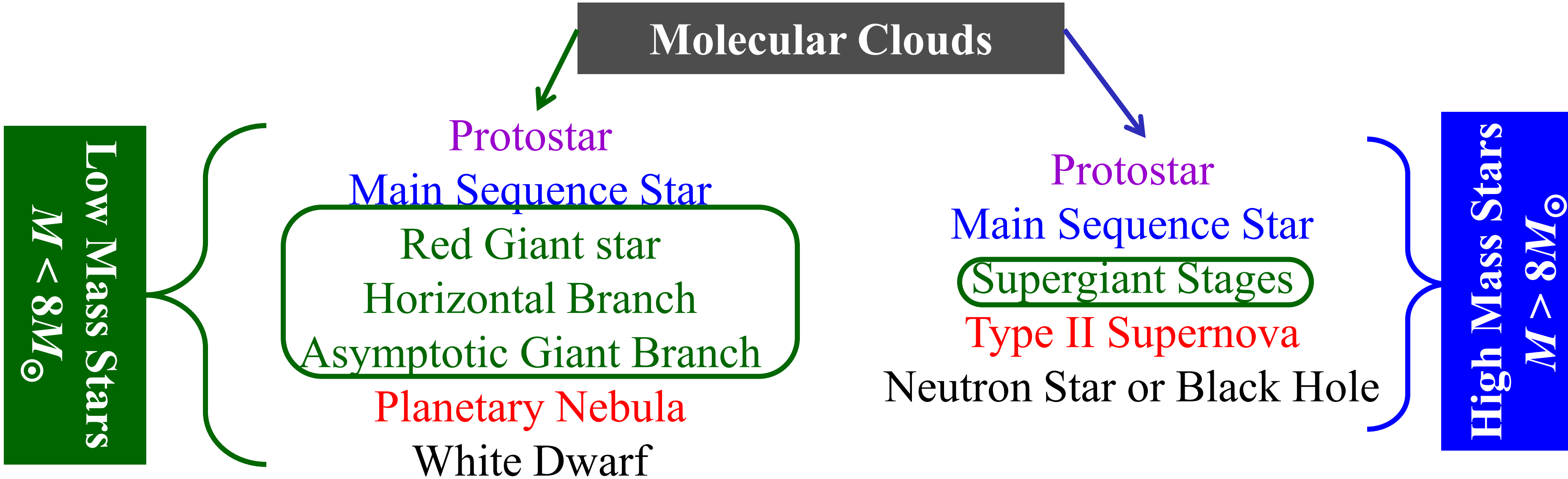
- The amount of fuel is proportional to its mass: $F \sim M$
- The rate it consumes fuel depends on its mass: $L \sim M^{3.5}$
- How long it lasts depends on mass: $T \sim \frac{F}{L} \sim \frac{M}{M^{3.5}} \sim M^{-2.5}$

Big Stars Die Fast

The Sun lasts about 10 Gyr on main sequence

$$T_{MS} \sim (10\ \text{Gyr})\left(\frac{M}{M_{\odot}}\right)^{-2.5}$$

Giant Stars



The stars run out of hydrogen to burn to helium

- Low mass stars burn helium to produce carbon and oxygen ($Z = 6, 8$)
- High mass stars also produce elements through iron ($Z = 26$)
- These produce *much* less energy than hydrogen
    - The fuel is used faster and runs out faster
- All giant stages together last about 10% of the previous stages

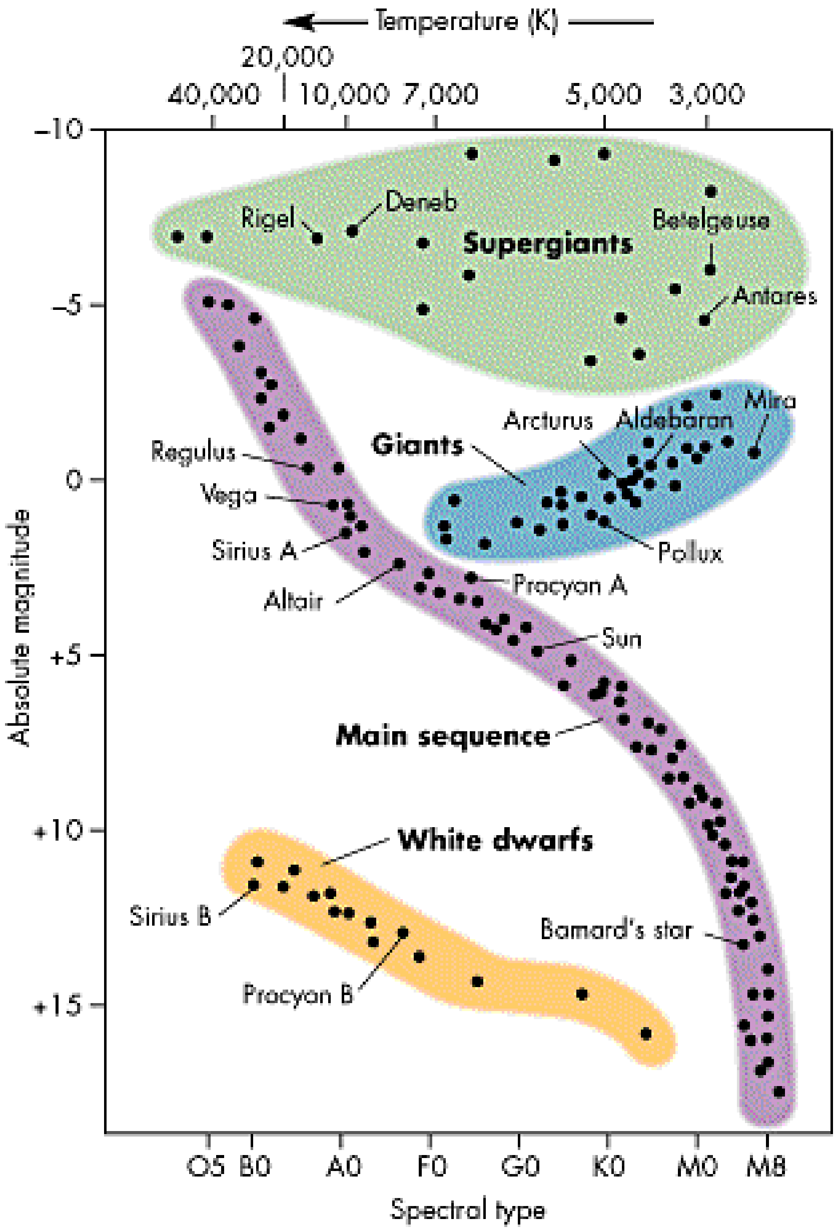
Giant Stars: Movement on HR-diagram

- Low mass stars get cooler and more luminous
  - Up and right on the HR diagram

- High mass stars get cooler
  - Right on the HR diagram

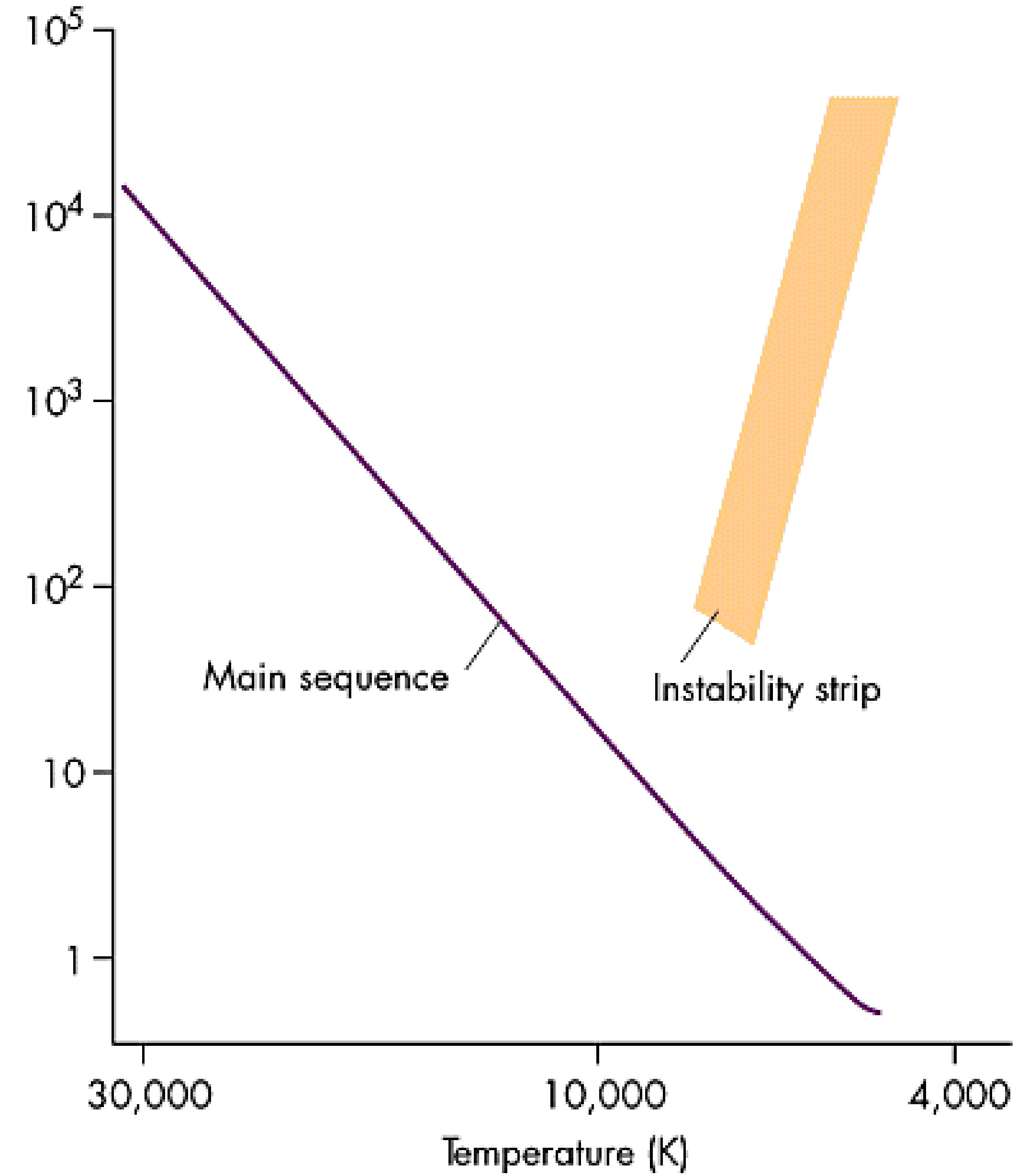
- The high mass stars move off from the main sequence first

- You can estimate the age of a cluster by which stars have left the main sequence
  - The turn off point
  - More about this later

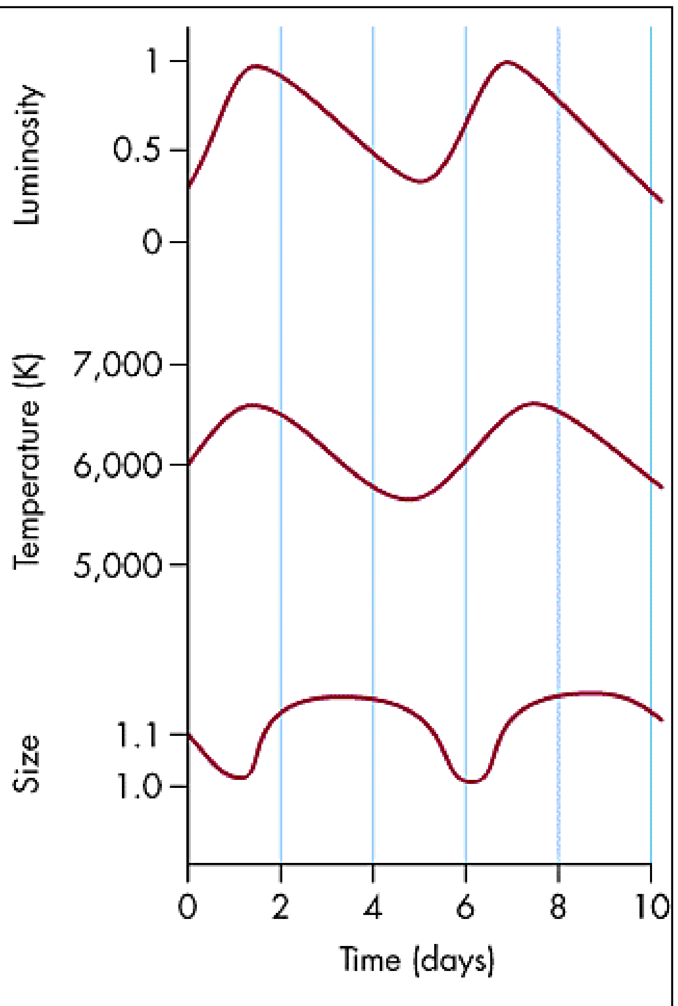


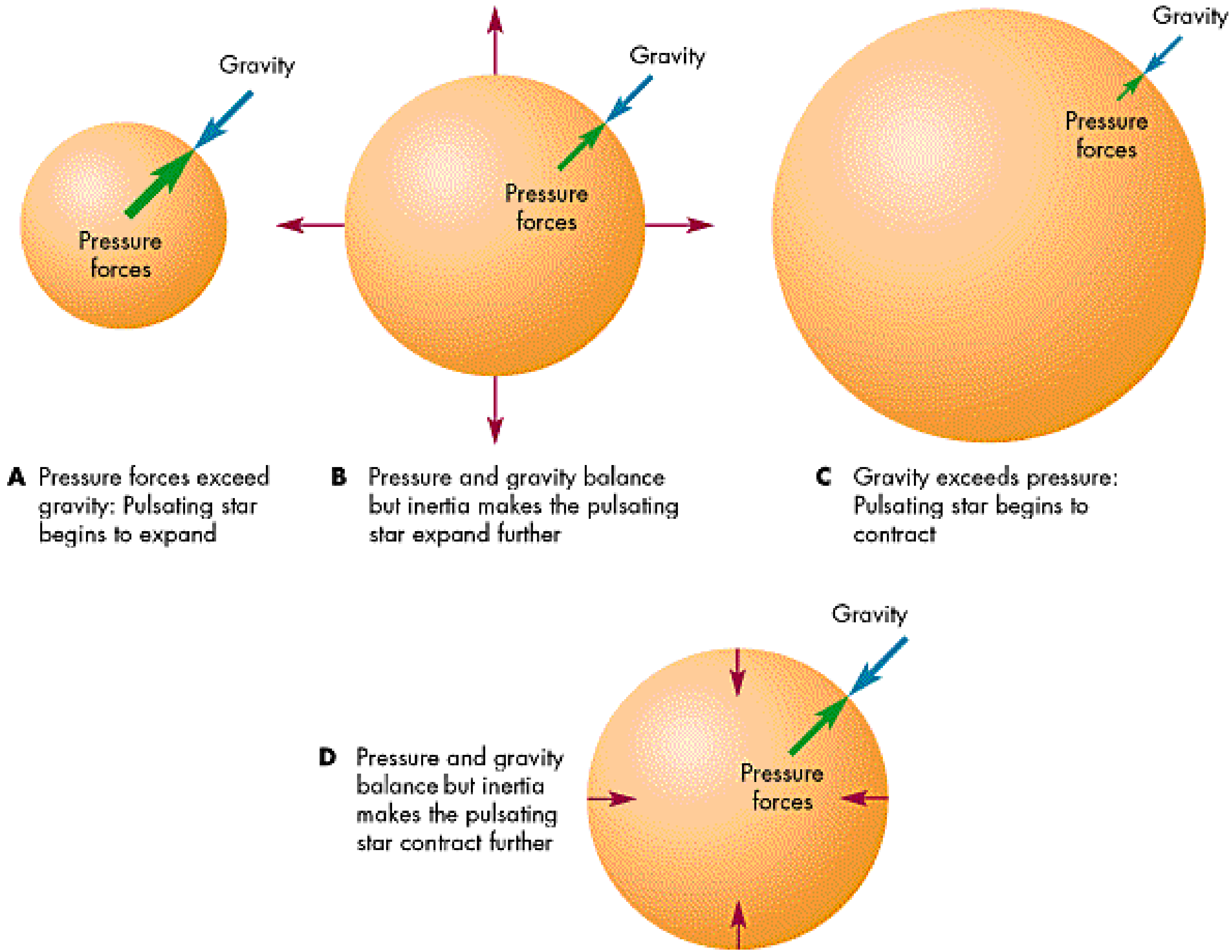
Cepheid Variable Stars (1)

- Not all stars are constant luminosity
- There is a region of the HR diagram where stars pulsate, called the *instability strip*
  - Not Main Sequence stars
- The temperature, size, and luminosity all vary periodically
- Many Cepheids are *extremely bright* - much more luminous than typical main sequence stars
- We can see them far away, even in nearby galaxies
- One of the biggest motivations for the Hubble telescope was to study Cepheids in galaxies a few Mpc away



Cepheid Variable Stars (2)





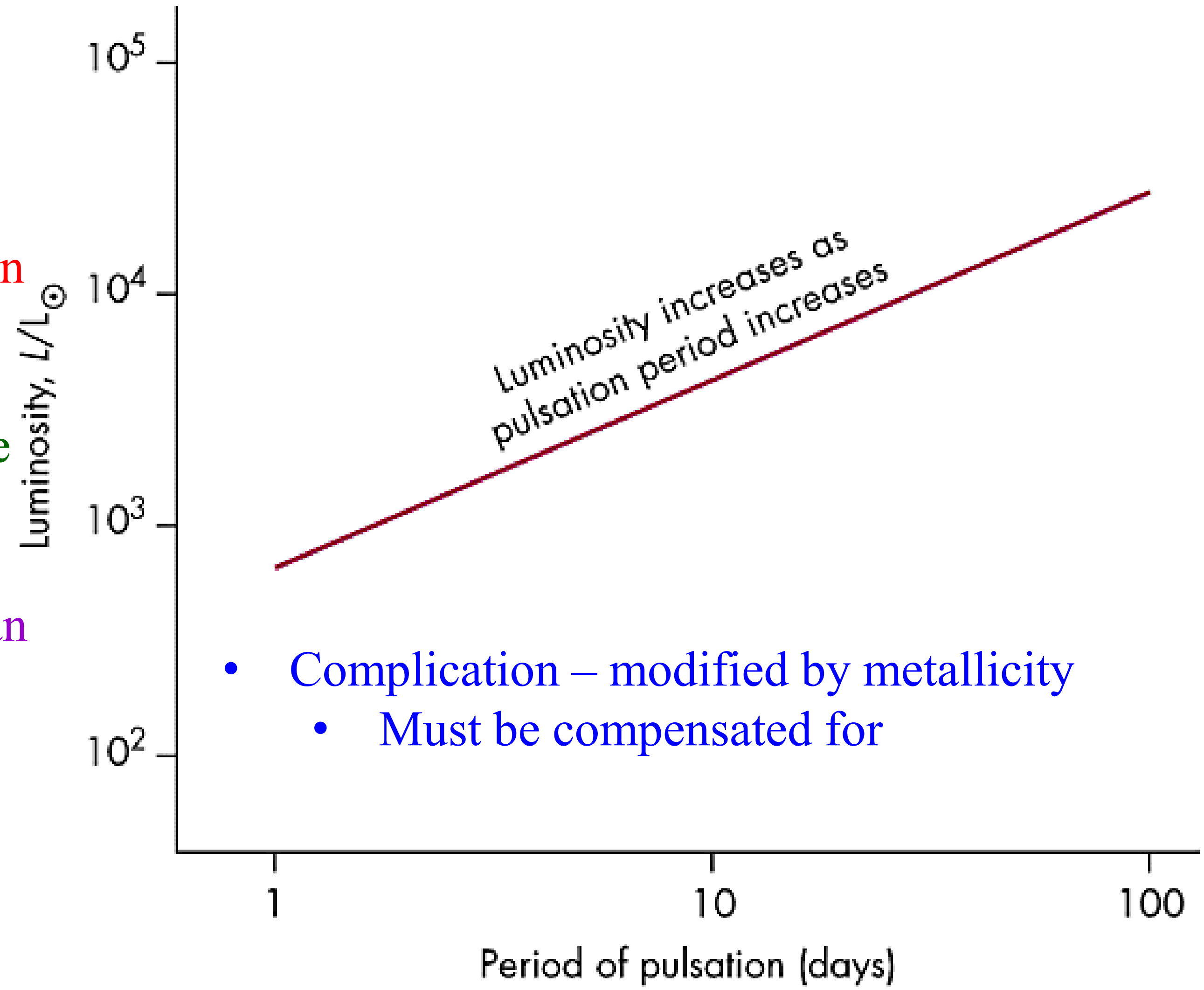
Cepheid Variable Stars (3)

- Bigger stars pulsate more slowly
- Bigger stars are more luminous
- There is a simple relationship between the period and the luminosity
- If you know the period, you know the luminosity
- If you measure the brightness, you can then get the distance

$$M = -2.67\log_{10}(P) - 1.29$$

P is period in days

- In this formula, *M* is the average visible luminosity M_V



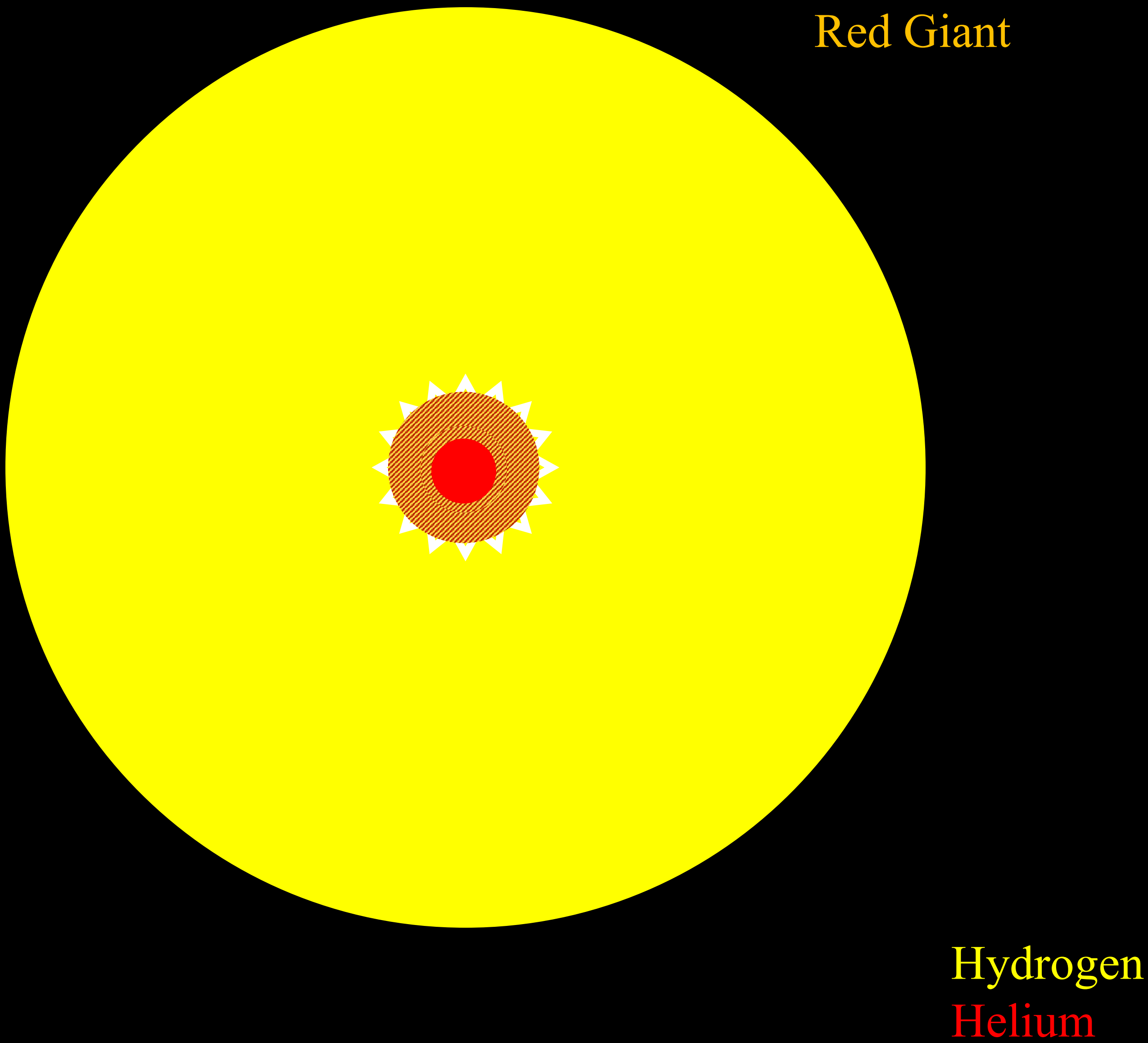
- Complication – modified by metallicity
  - Must be compensated for

$$d = 10^{1+\frac{m-M}{5}}\ \text{pc}$$

Red Giant

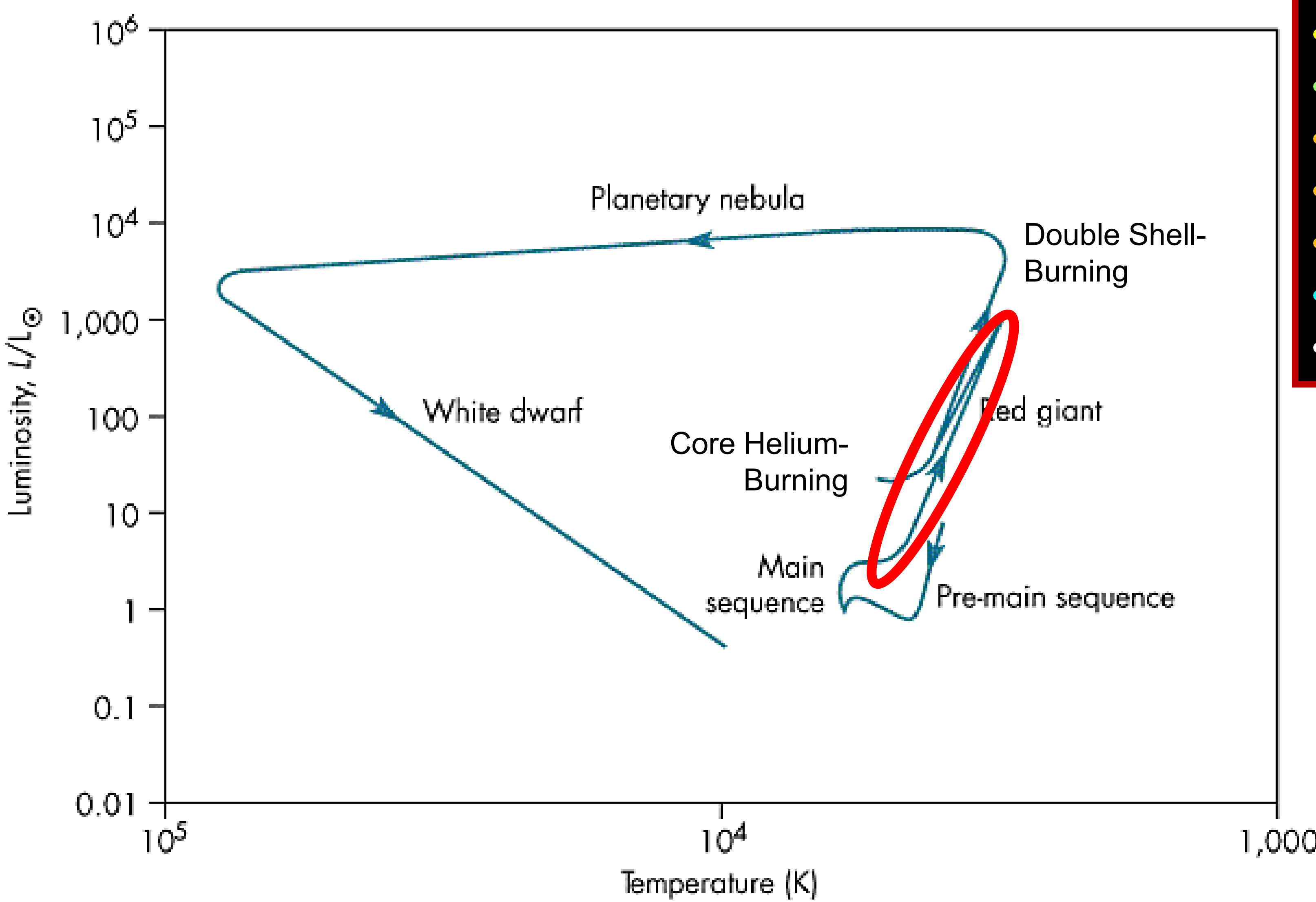
- Stars that have run out of hydrogen; they now have pure helium “ash” in their centers
- The gravity from their inert cores pulls in the next layer out, causing it to become very hot
- This causes intense burning of hydrogen, now in a thin shell
- The heat causes the star to grow immensely, and get very luminous
  - A cool (red) giant star
- Eventually, the star gets so hot, that helium can begin burning to carbon and then oxygen
  - The helium flash
- There is a maximum luminosity for the star before it transitions to this stage
- This is called the tip of the red giant branch (TRGB)

Main Sequence → Red Giant



- Molecular Cloud
- Protostar
- Main Sequence
- Red Giant
- Core Helium-Burning
- Double Shell-Burning
- Planetary Nebula
- White Dwarf

Red Giant



- Molecular Cloud
- Protostar
- Main Sequence
- Red Giant
- Core Helium-Burning
- Double Shell-Burning
- Planetary Nebula
- White Dwarf

- Star moves up and right on H-R diagram

Planetary Nebulas (1)

Molecular Clouds

Low Mass Stars $M < 8M_\odot$

- Protostar
- Main Sequence Star
- Red Giant star
- Horizontal Branch
- Asymptotic Giant Branch
- Planetary Nebula
- White Dwarf

High Mass Stars $M > 8M_\odot$

- Protostar
- Main Sequence Star
- Supergiant Stages
- Type II Supernova
- Neutron Star or Black Hole

Low Mass stars end their lives as *planetary nebulas*

- Outer layer is expelled from the star
  - This mixes carbon/oxygen/helium back into interstellar space
- Inner super-hot layer gradually revealed
- This star is now radiating in the ultraviolet – visible luminosity is low
- But the ultraviolet light excites the atoms in the gas that has been expelled

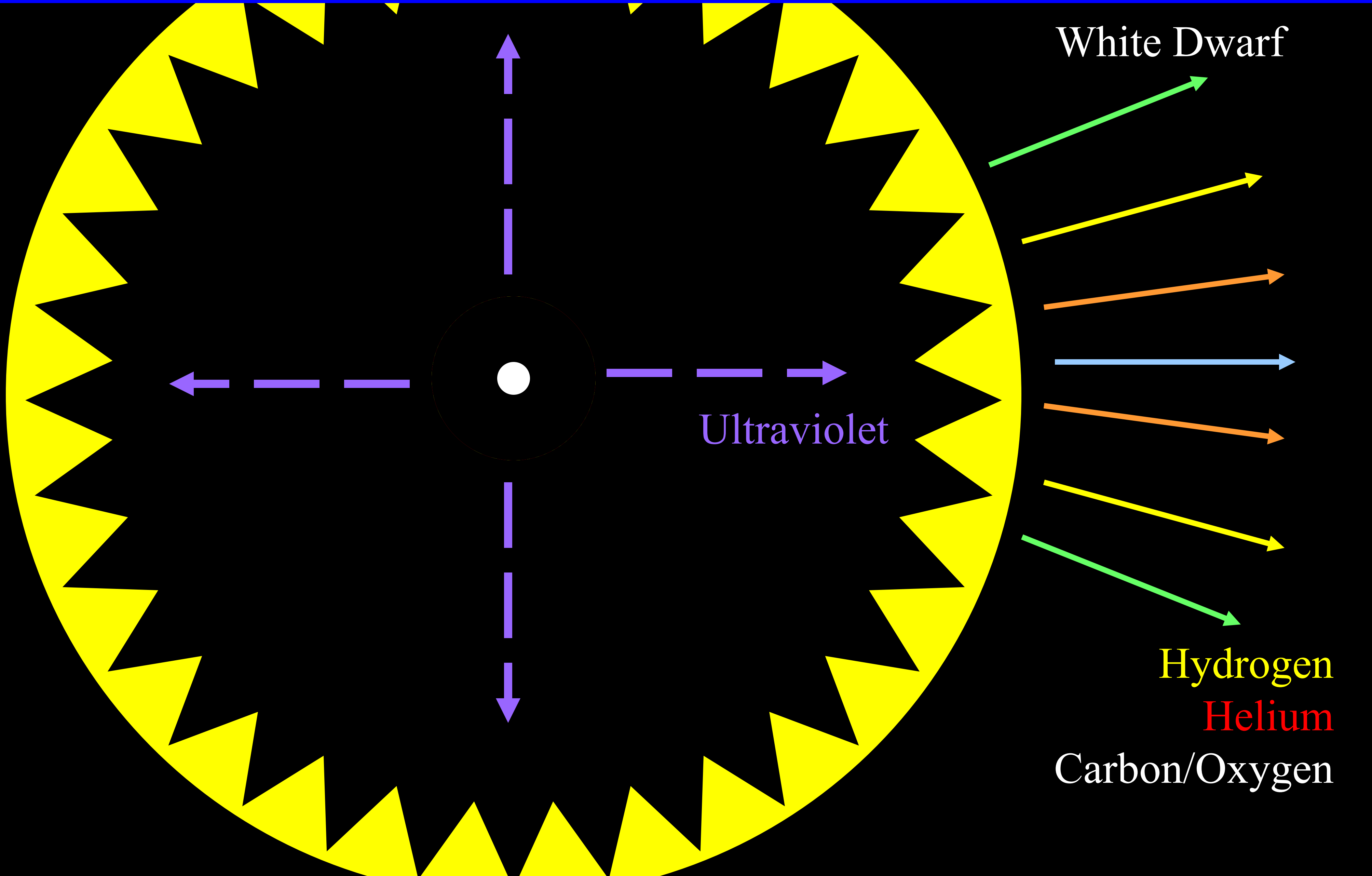
Planetary Nebulas (2)

- The planetary nebula glows brightly in visible light
  - An emission-line spectrum
- How bright any given planetary nebula is is difficult to predict
- *Statistically*, there are more dim ones than bright ones
- However, there is an approximate maximum luminosity:

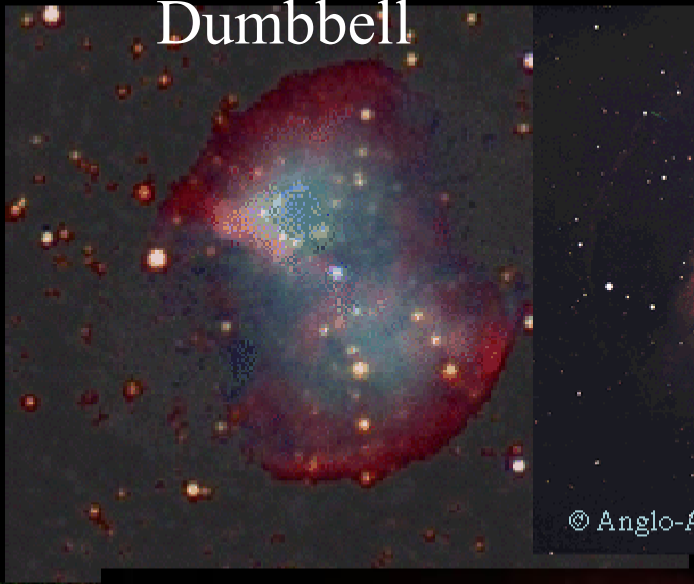
$$M^* = -4.47 \pm 0.05$$

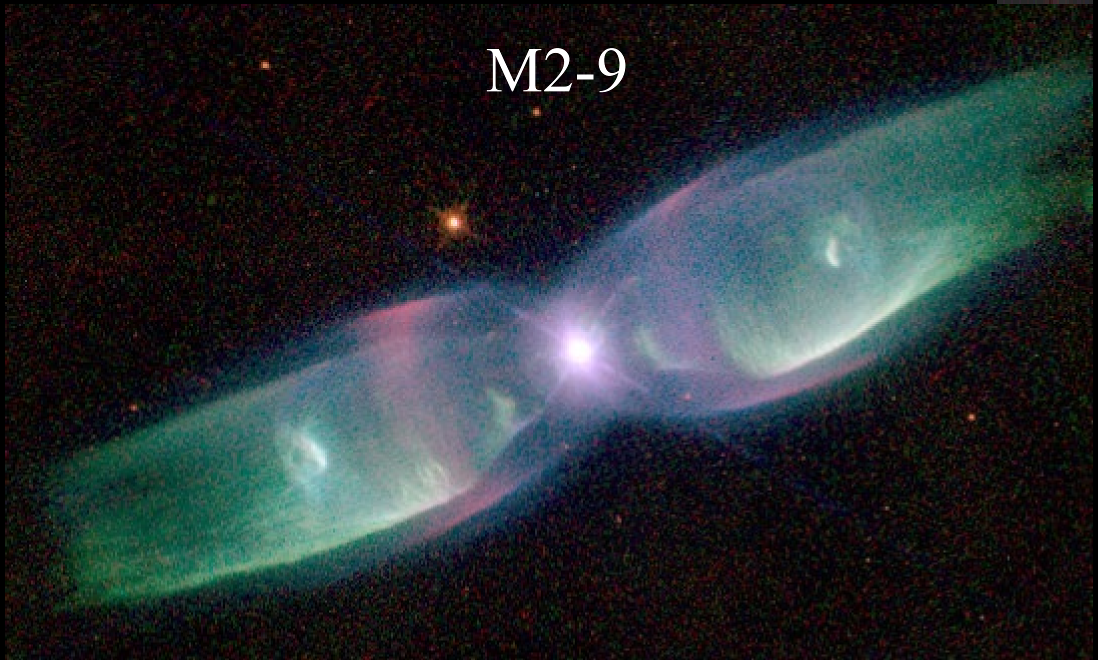
- Almost independent of metallicity
- Can be used to measure distance to objects containing *many* planetary nebulae
  - Such as a galaxy

Asymptotic Giant → … → White Dwarf

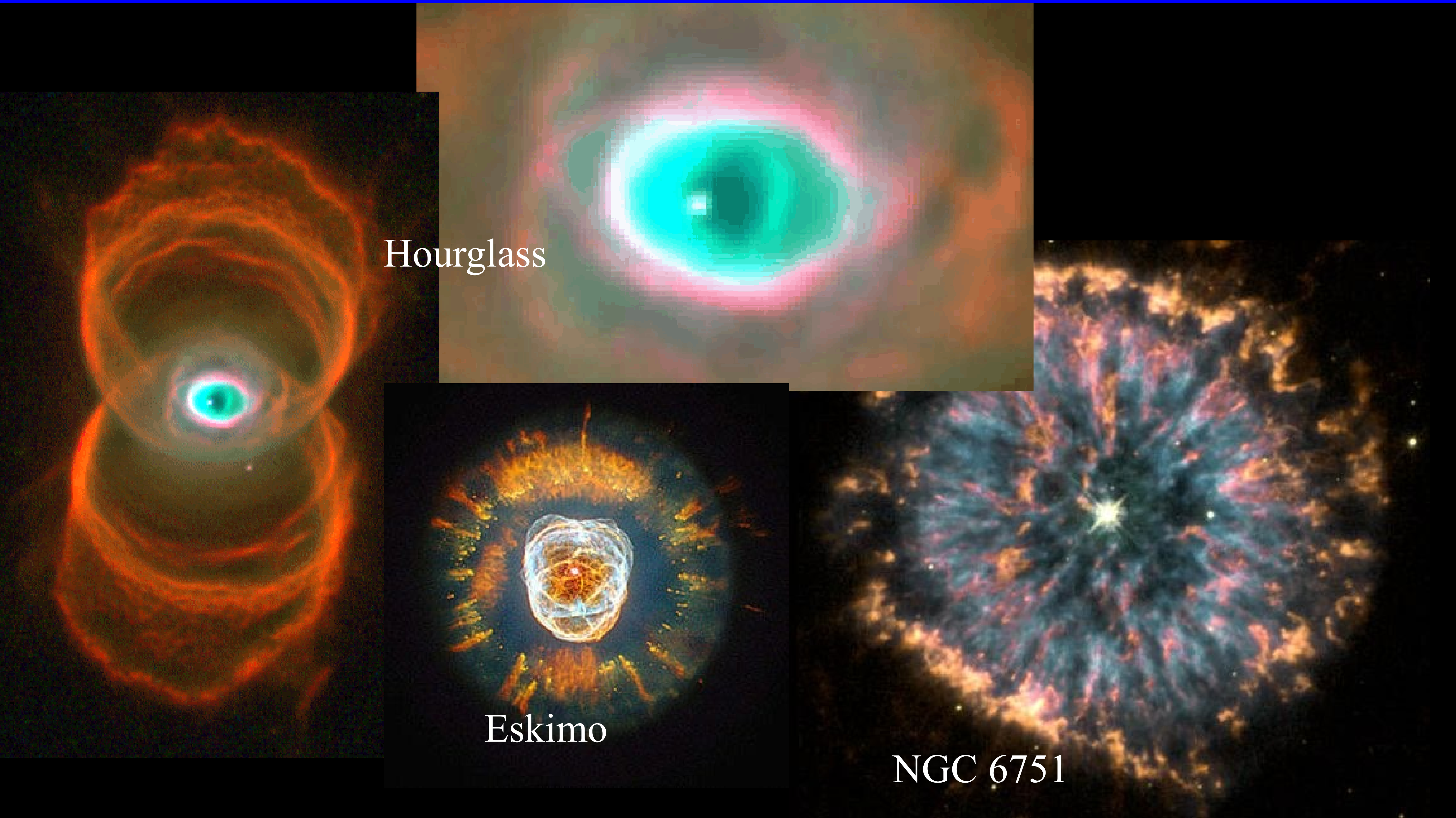


Planetary Nebula Pictures (1)



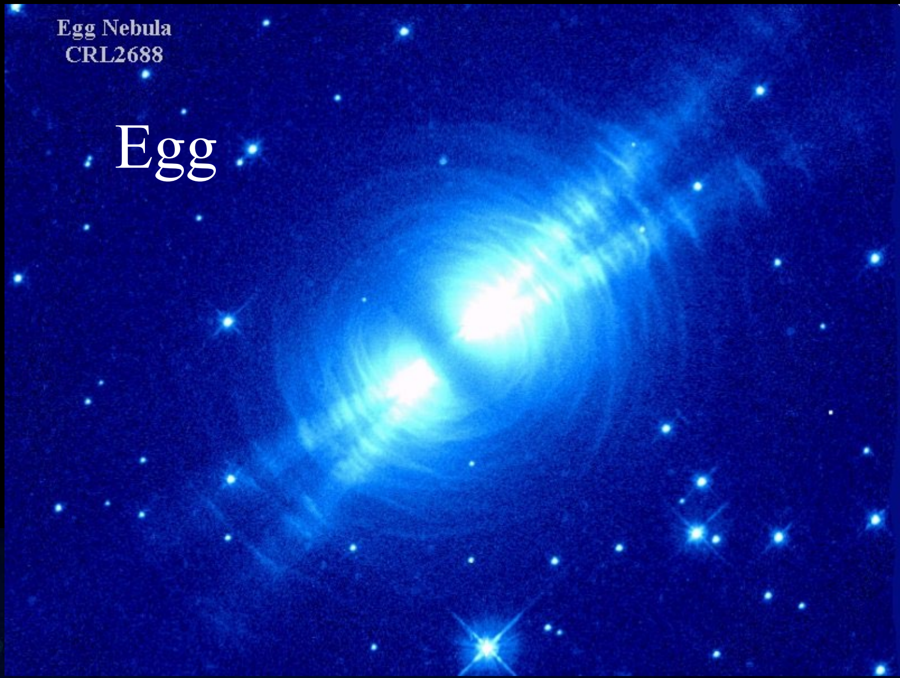


Planetary Nebula Pictures (2)



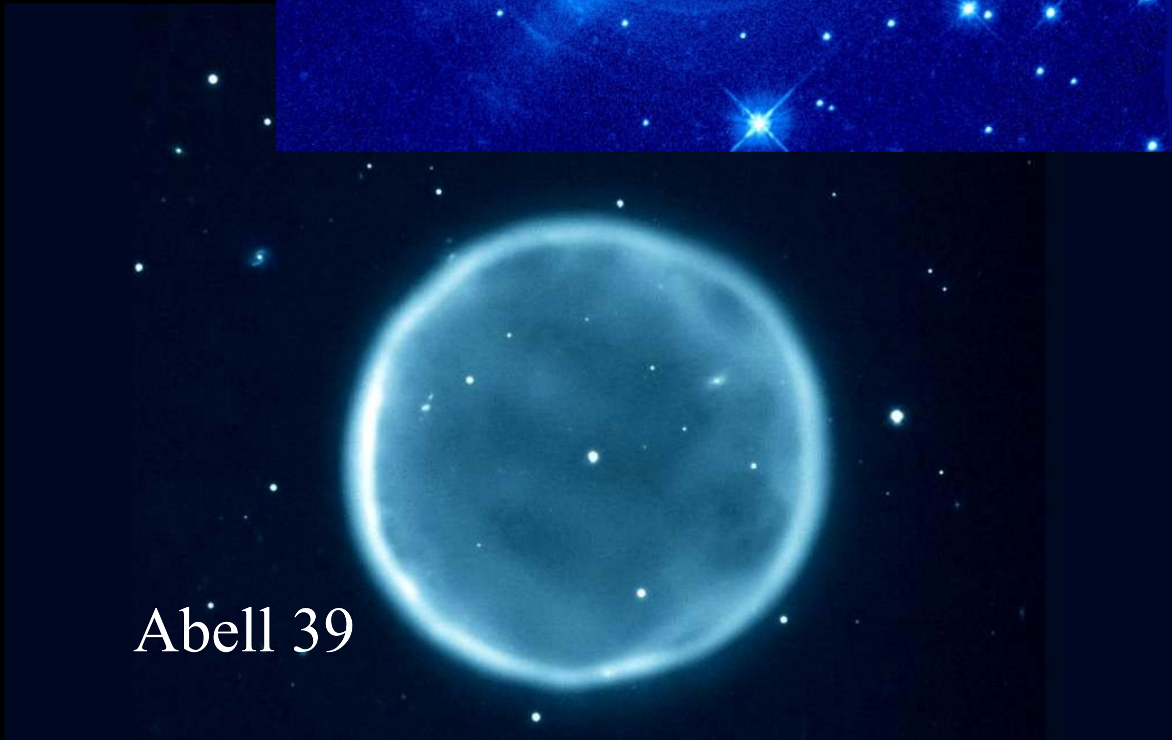
Planetary Nebula Pictures (3)

NGC 2440



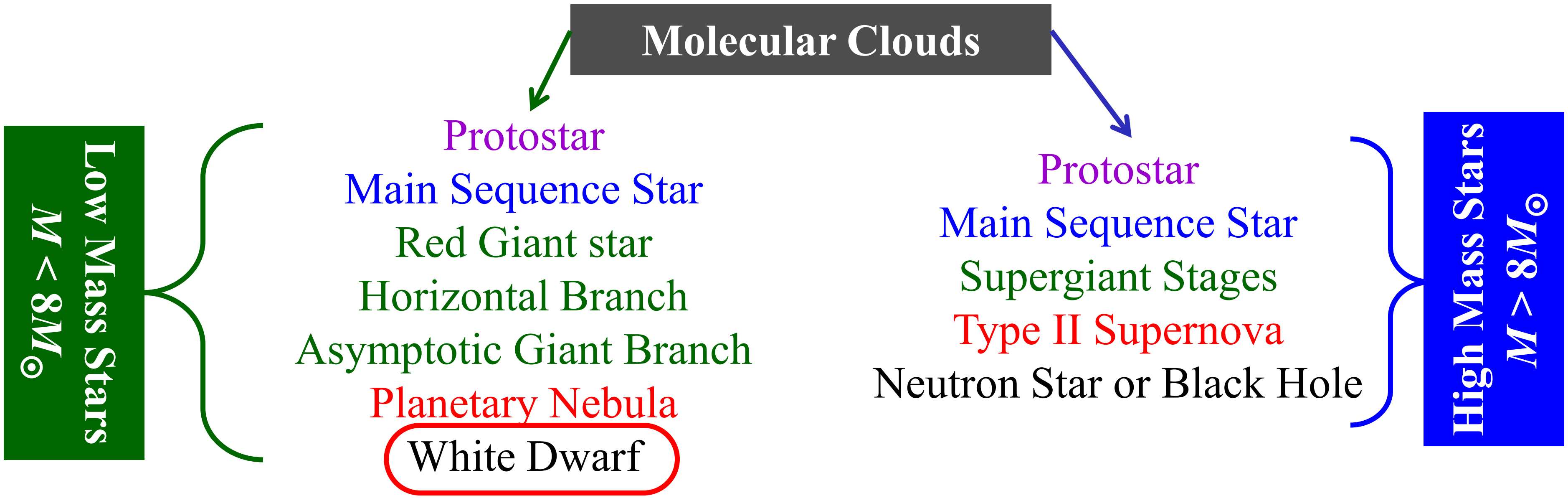
Egg

Spirograph



Abell 39

White Dwarf



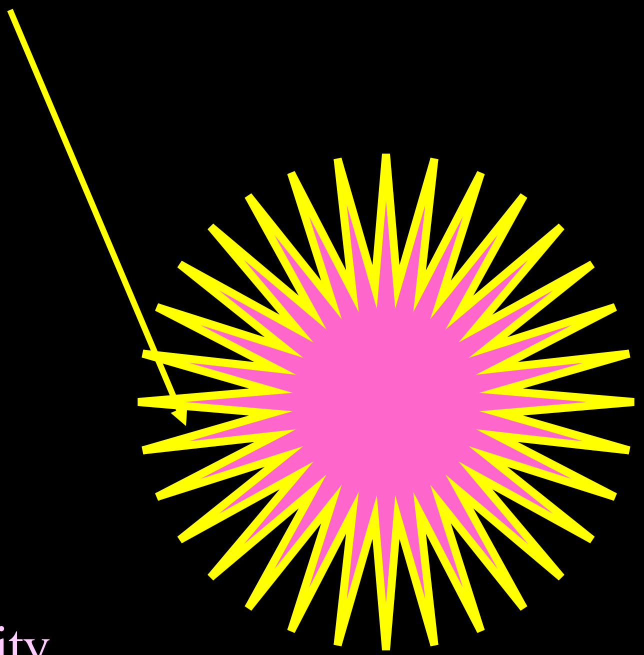
A white dwarf is a burned-out star consisting of carbon and oxygen

- Dead – dim and getting dimmer
- Mass of Sun, size of Earth
- Gravity is opposed by degeneracy pressure
  - Quantum mechanical effect due to Pauli Exclusion principle
- There is a maximum mass – called the Chandrasekhar mass: about 1.42 M_{Sun}

Type Ia Supernova

A white dwarf with a giant companion can gain mass from its companion

- As it gains mass, gravity increases – it shrinks
- When it reaches Chandrasekhar mass it collapses, catastrophically
- Temperature increases drastically
- Fusion begins again in the core
- The *entire star* explodes, all at once
- Star is super bright – as bright as an entire galaxy
  - We can see them most of the way across the universe



- All type Ia supernovae have nearly identical precursors
- They should blow up almost exactly the same way
- They should have almost uniform luminosity
- For reasons that aren't well understood, they are not
  - Some are more efficient than others
  - Amount of time they take to get bright seems to predict luminosity

Other Types of Supernovae

Molecular Clouds

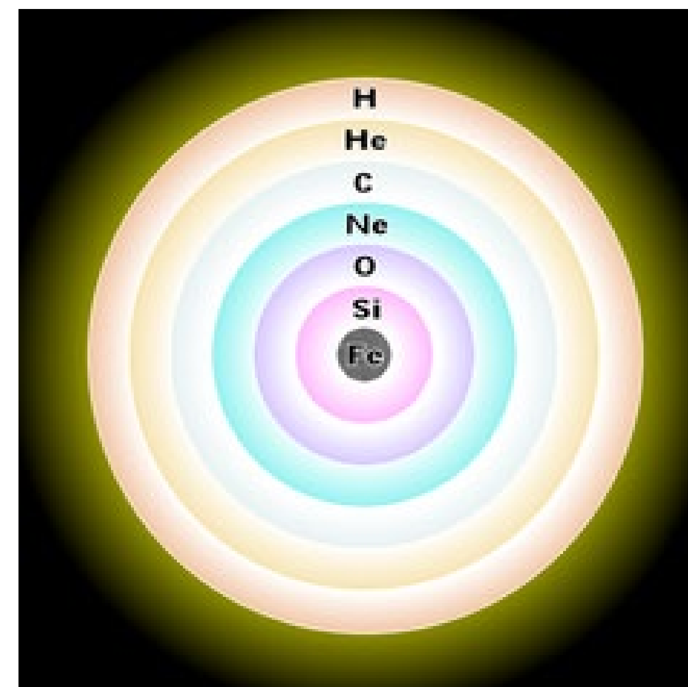
Low Mass Stars $M < 8M_\odot$

- Protostar
- Main Sequence Star
- Red Giant star
- Horizontal Branch
- Asymptotic Giant Branch
- Planetary Nebula
- White Dwarf

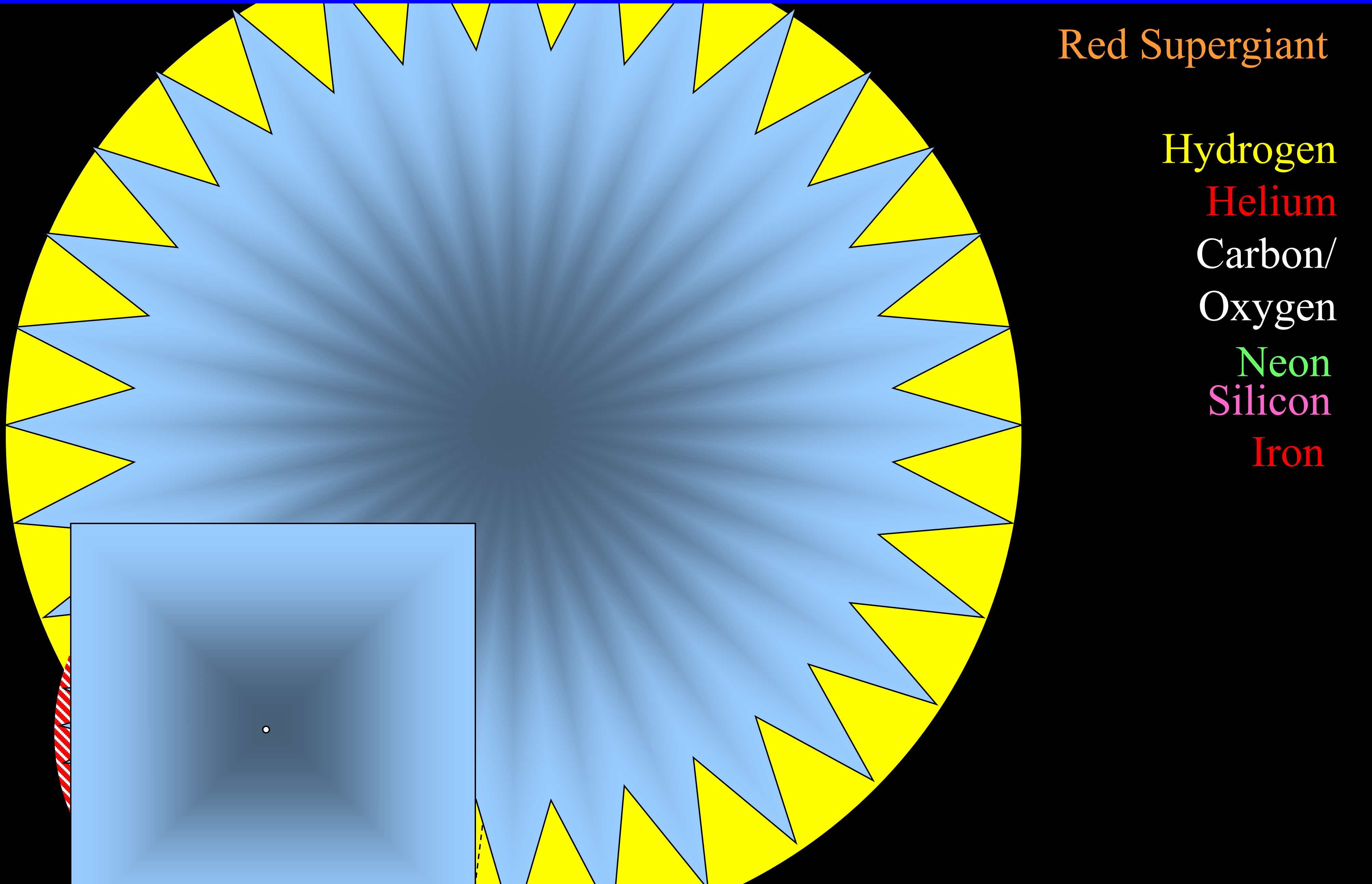
High Mass Stars $M > 8M_\odot$

- Protostar
- Main Sequence Star
- Supergiant Stages
- Type II Supernova
- Neutron Star or Black Hole

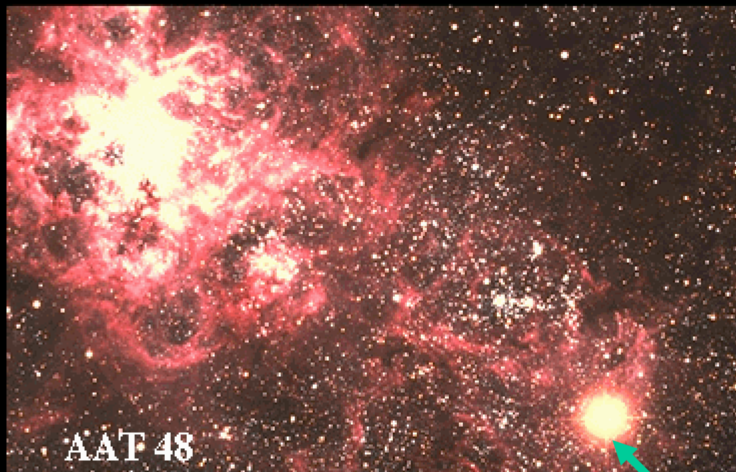
- High mass stars end their life as Type II supernovae
  - Or occasionally, type Ib or Ic
- These stars are very complex, containing numerous elements
- All supernovae mix "metals" back into inter-stellar medium
  - They already contain many elements
  - More are made during the explosions



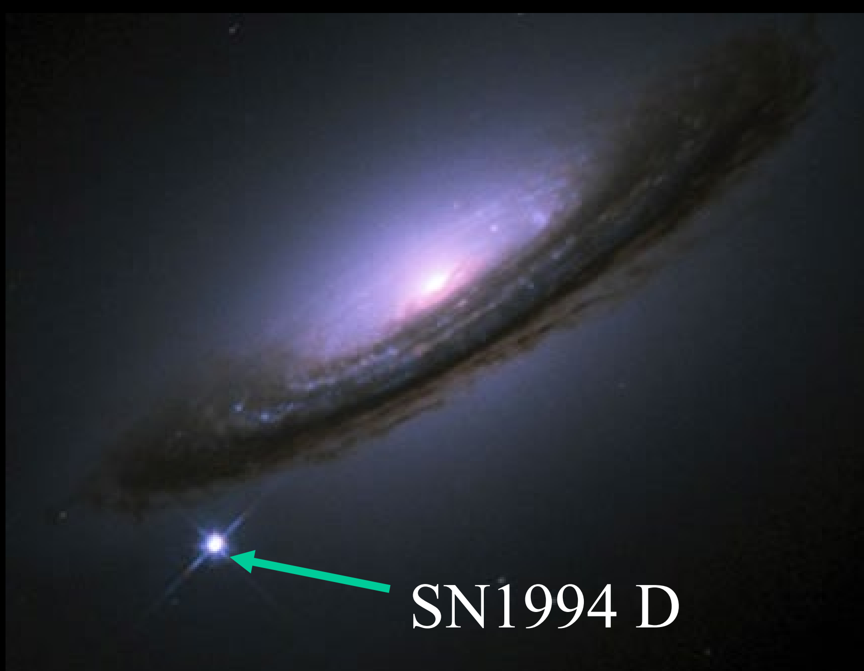
Type II Supernovae

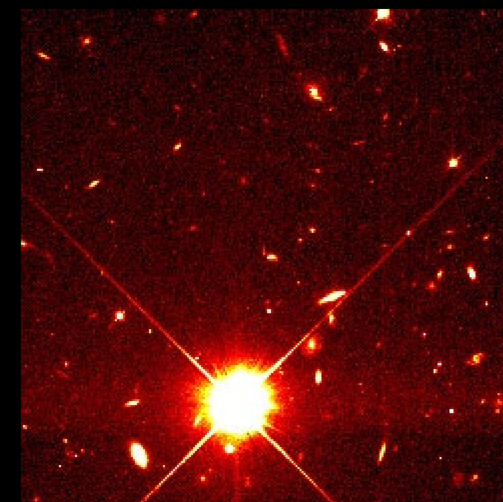


Supernovae

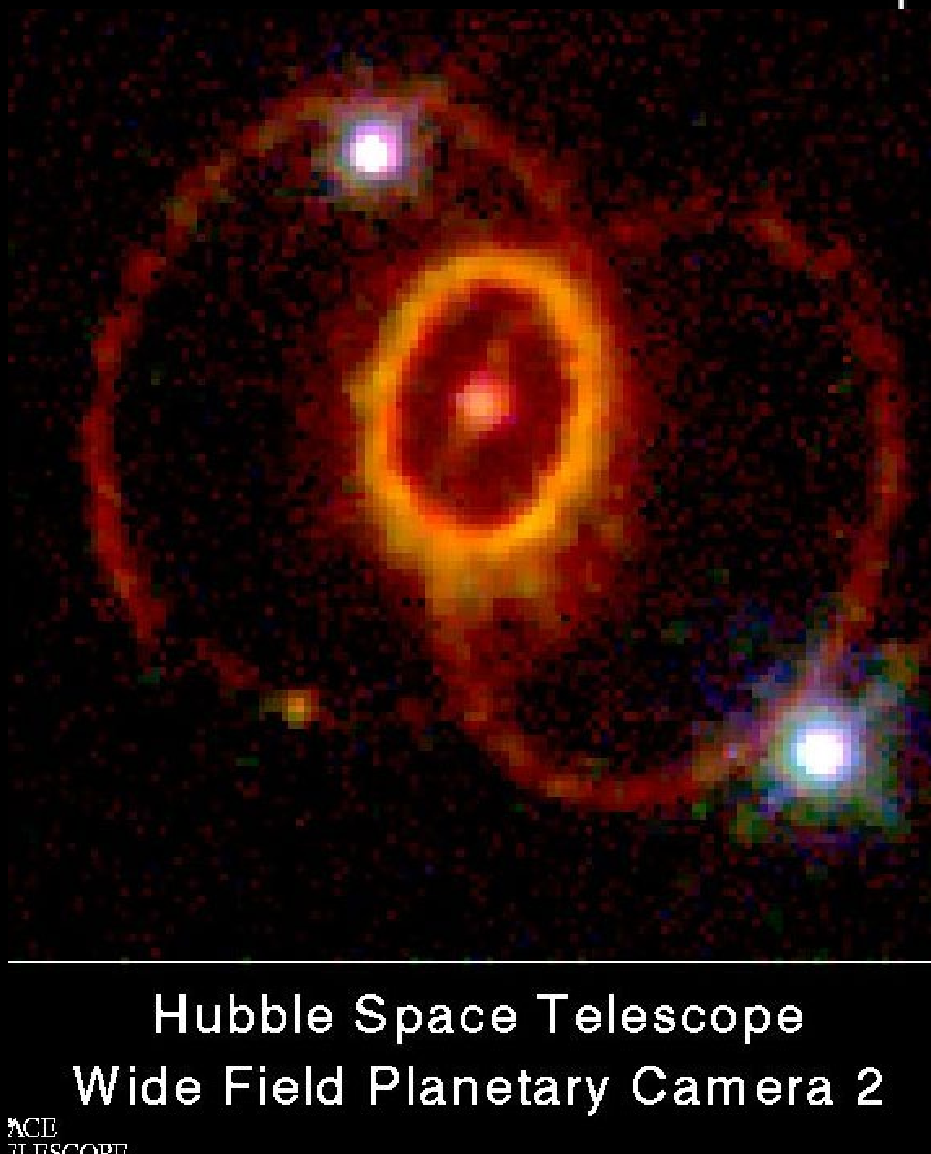


SN1987A

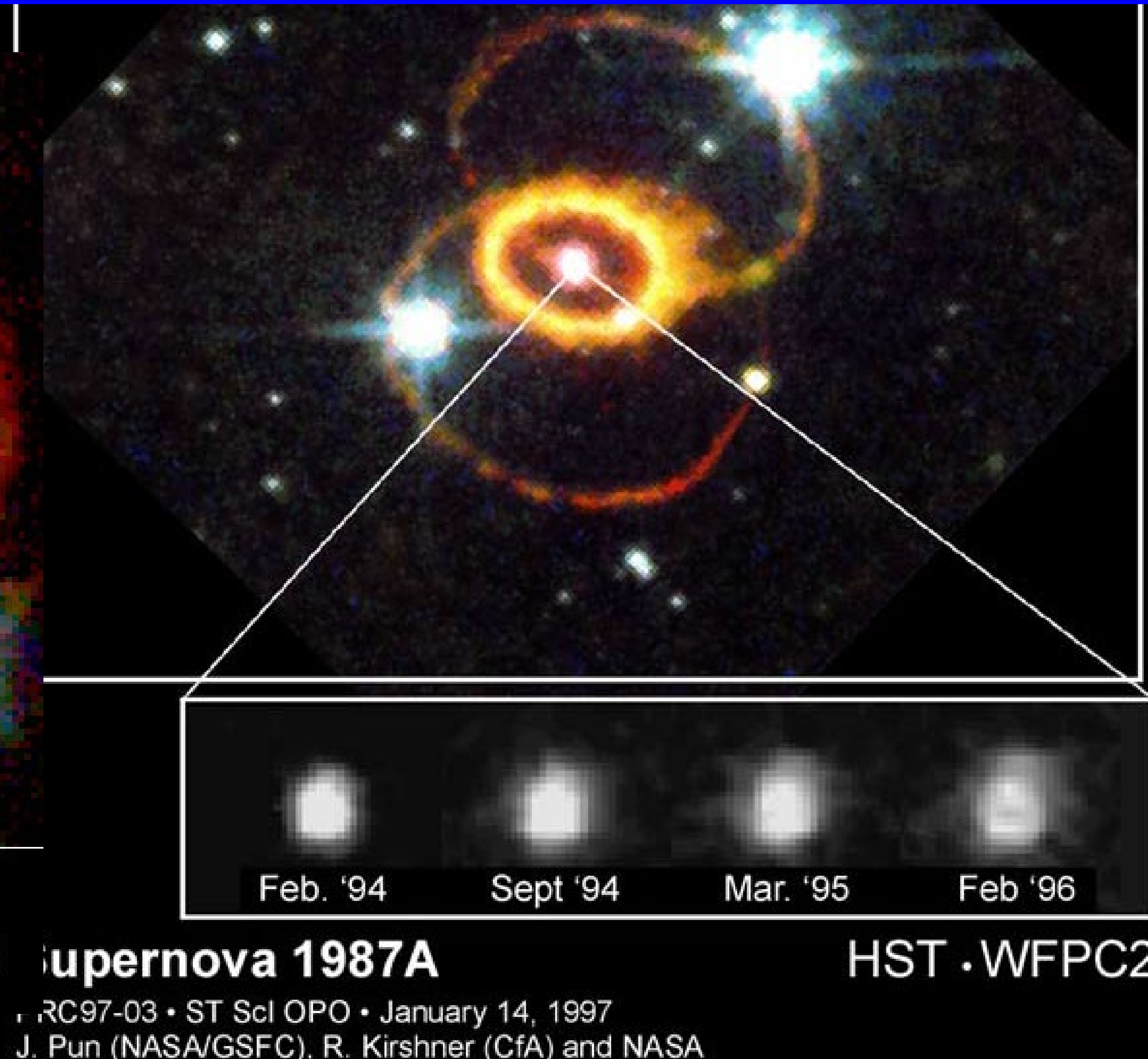




SN 1987a

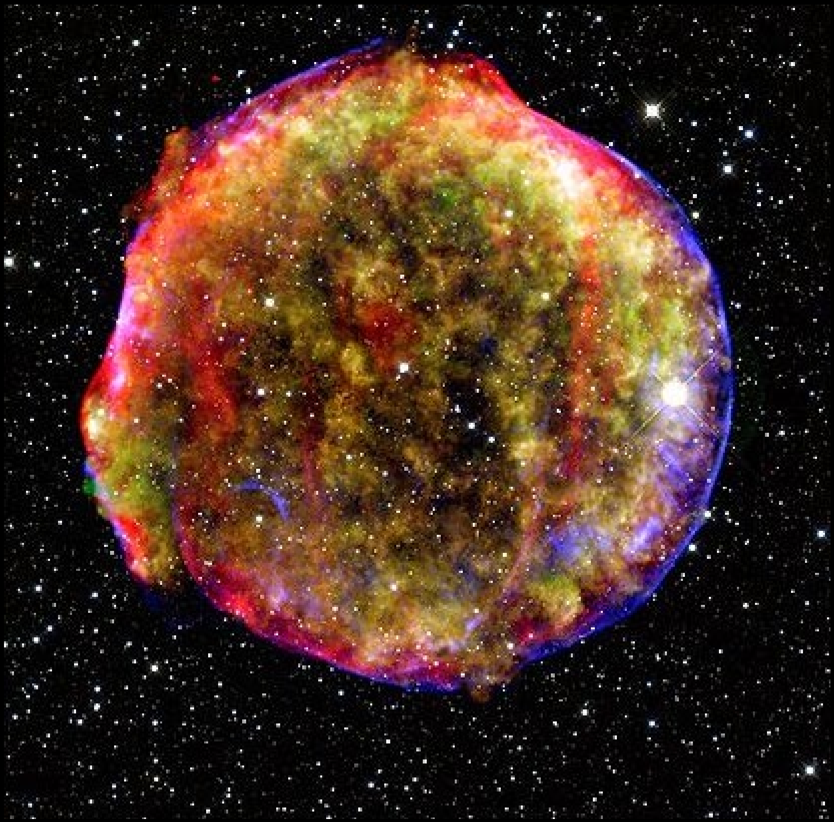


Hubble Space Telescope
Wide Field Planetary Camera 2



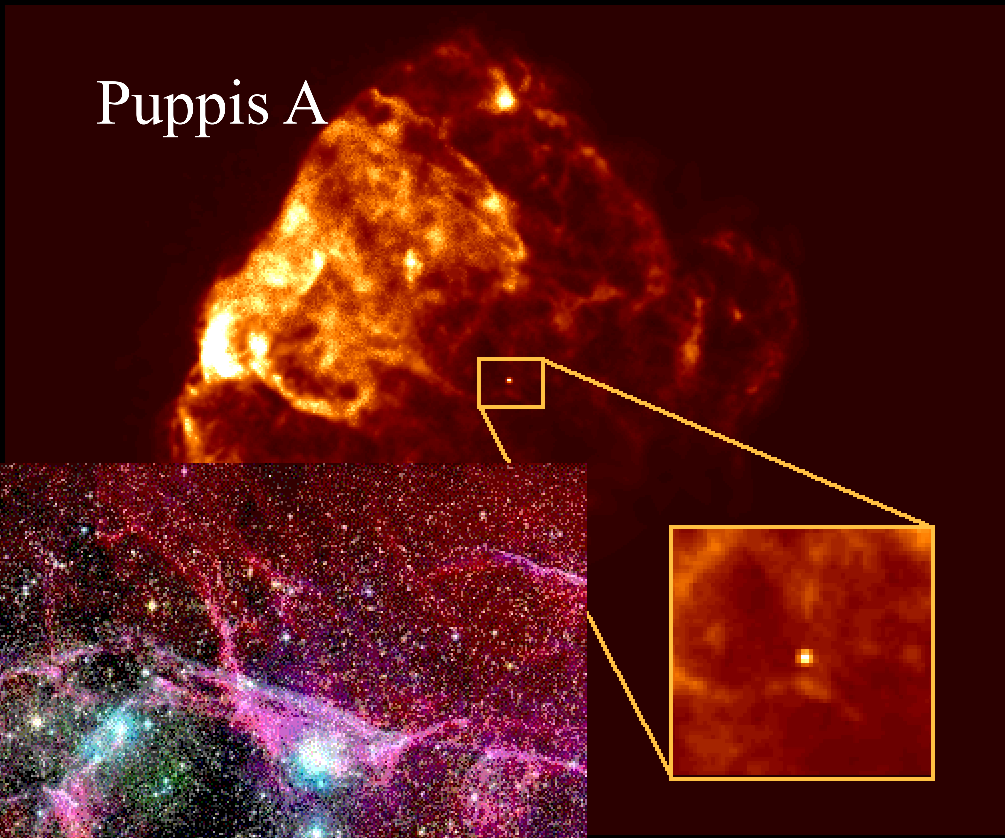
Feb. '94
Sept '94
Mar. '95
Feb '96
Supernova 1987A
HST • WFPC2
PRC97-03 • ST ScI OPO • January 14, 1997
J. Pun (NASA/GSFC), R. Kirshner (CfA) and NASA

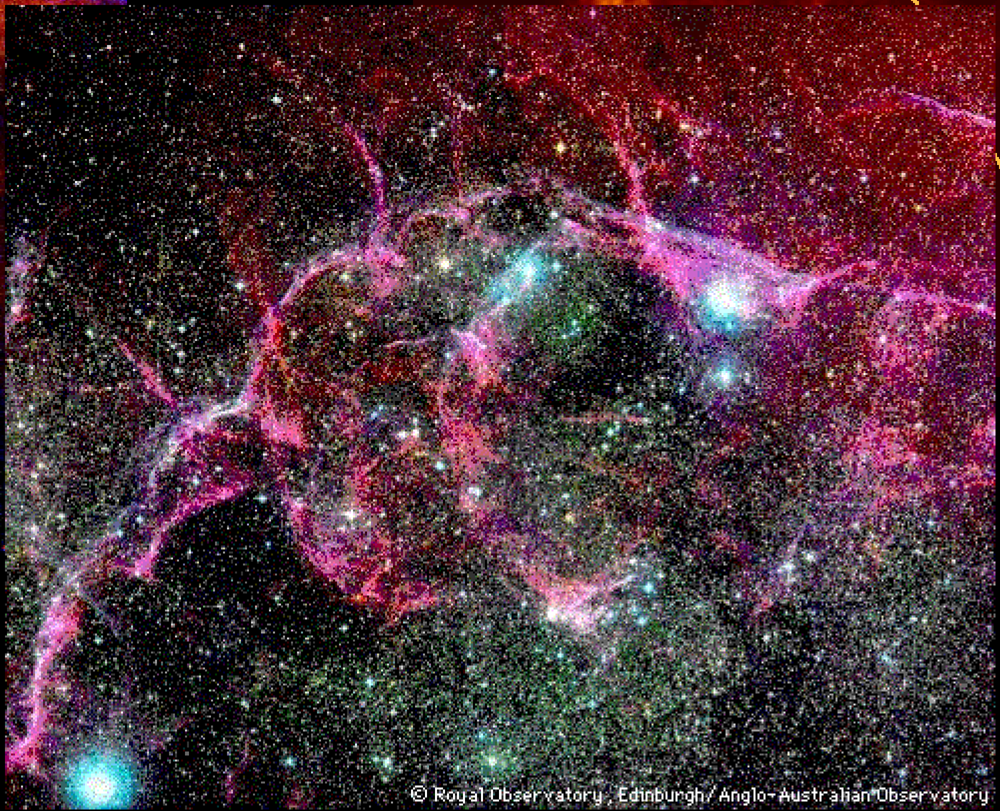
Supernova Remnants (1)



Tycho's Supernova Remnant

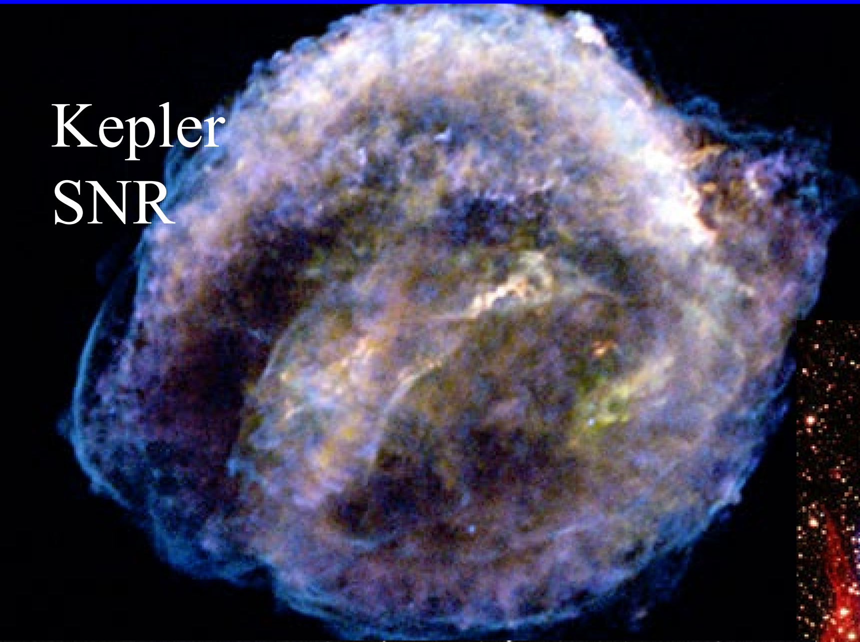
Supernova Remnants (2)

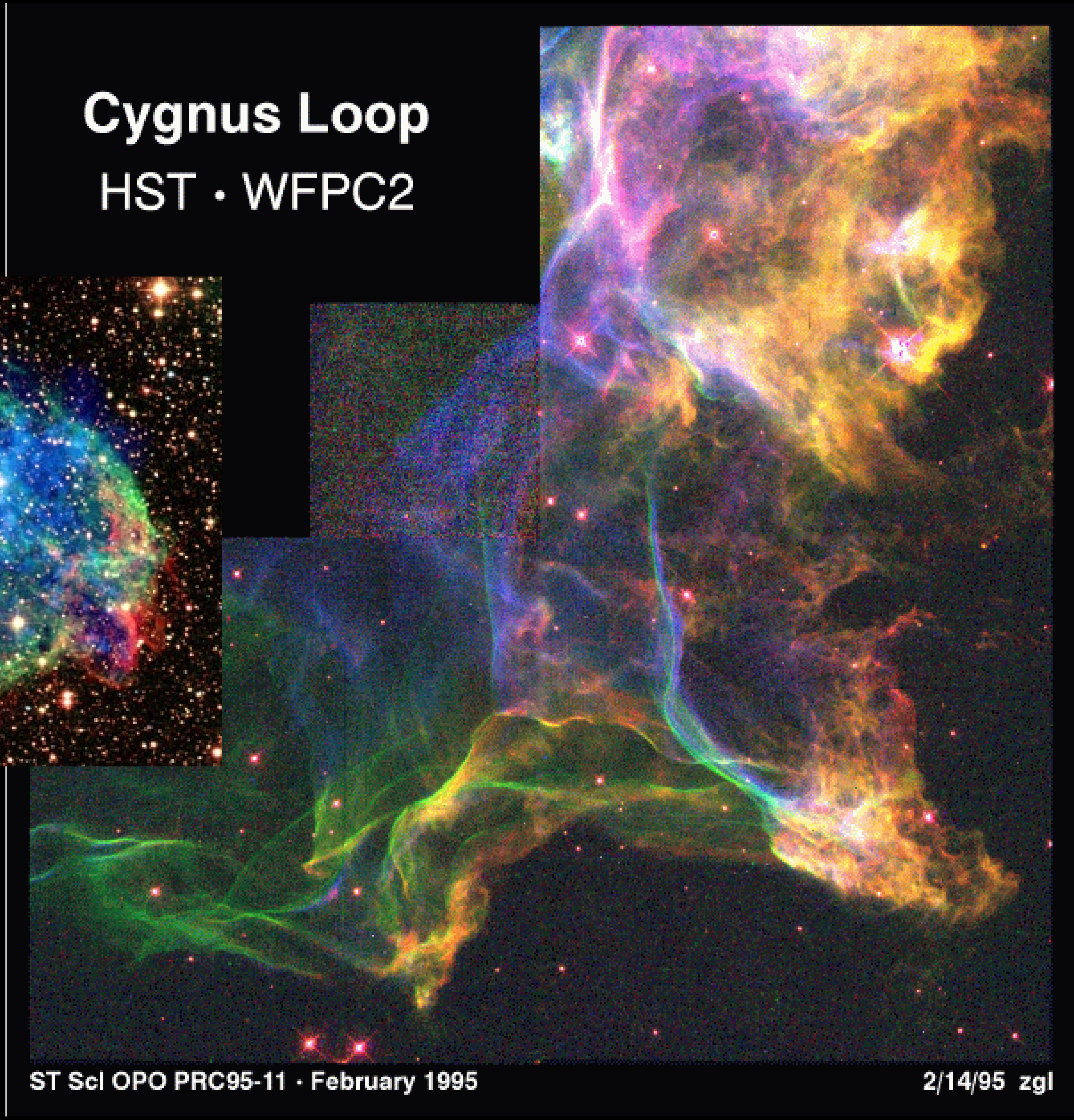




Vela Nebula

Supernova Remnants (3)





Black Holes

Molecular Clouds

Low Mass Stars $M < 8M_{\odot}$

- Protostar
- Main Sequence Star
- Red Giant star
- Horizontal Branch
- Asymptotic Giant Branch
- Planetary Nebula
- White Dwarf

High Mass Stars $M > 8M_{\odot}$

- Protostar
- Main Sequence Star
- Supergiant Stages
- Type II Supernova
- Neutron Star or Black Hole

A black hole is an object where gravity overcomes all other forces

- Mostly come from very heavy stars (> 25 M_{Sun})
- Gravity becomes so strong nothing can escape
- Relativity says nothing can go faster than light:
- If you get too close, the escape velocity equals speed of light
  - Called the Schwarzschild radius

$$v_{\text{esc}}^2 = \frac{2GM}{R}$$

$$c^2 = \frac{2GM}{R_S}$$

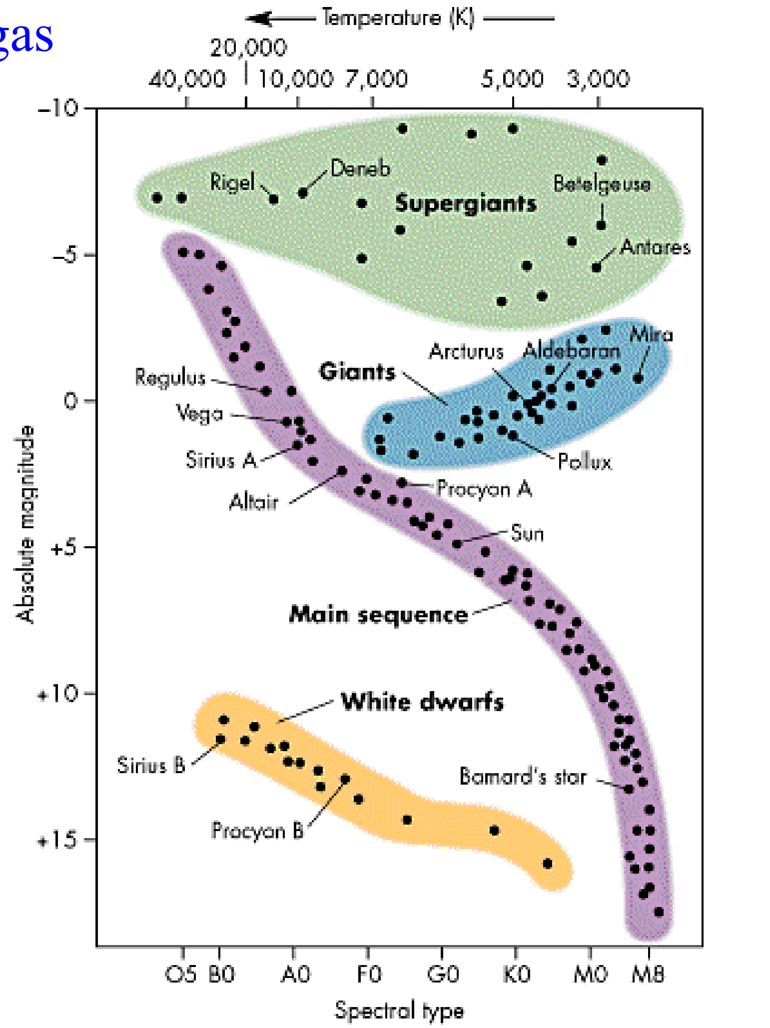
$$R_S = \frac{2GM}{c^2}$$

Stellar Clusters

General Comments on Clusters

- A *cluster* of stars is a group of stars born from a single cloud of gas
  - It appears as a group of closely spaced stars

- A *cluster diagram* is a Hertzsprung Russell diagram showing all the stars in a cluster



Recall:

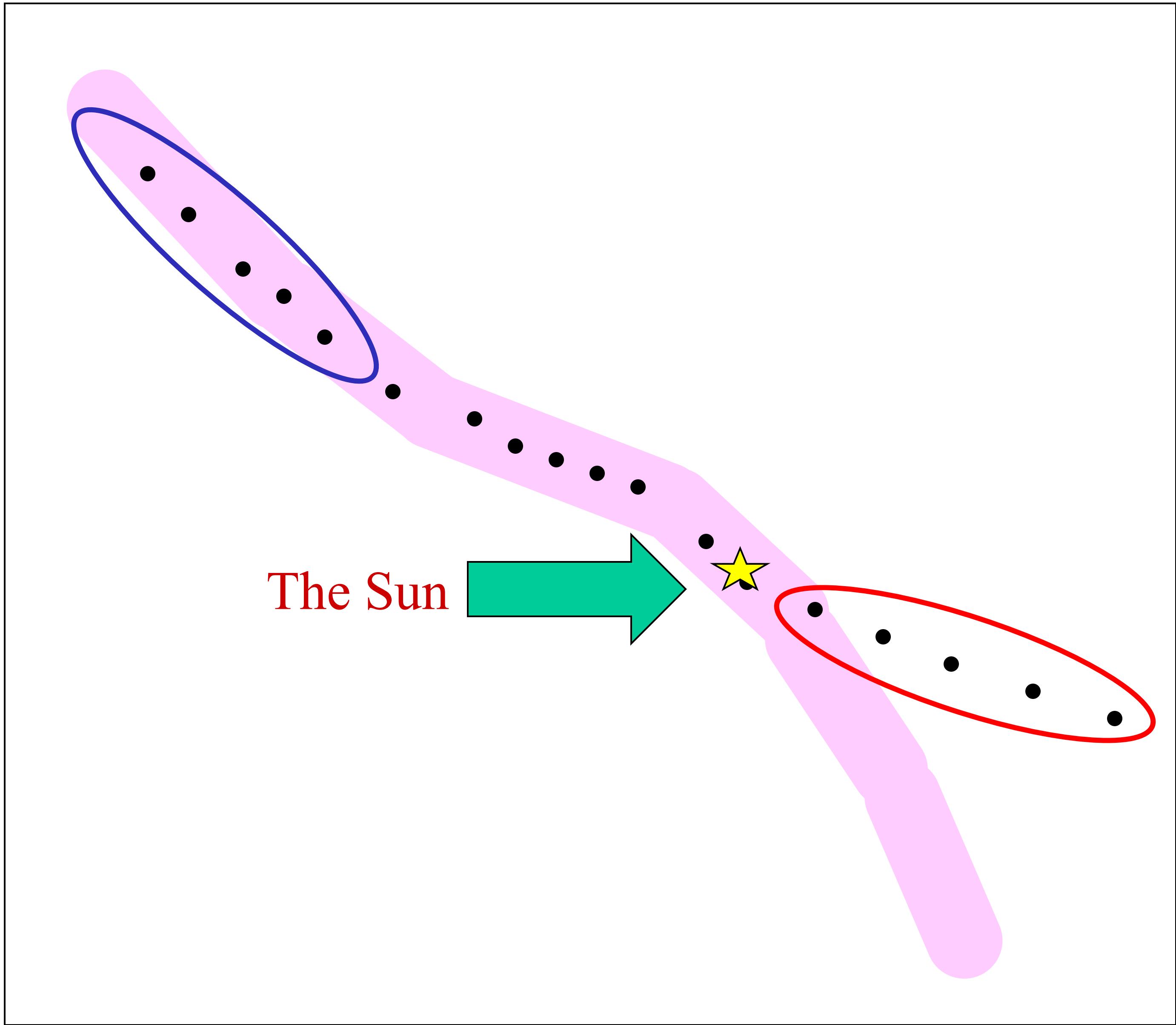
- Stars are "born" as Main Sequence Stars
- Massive stars are the hot luminous ones
- The most massive stars die first

Over time, the cluster diagram will change

1 Million Years Old

At 1 million years old:

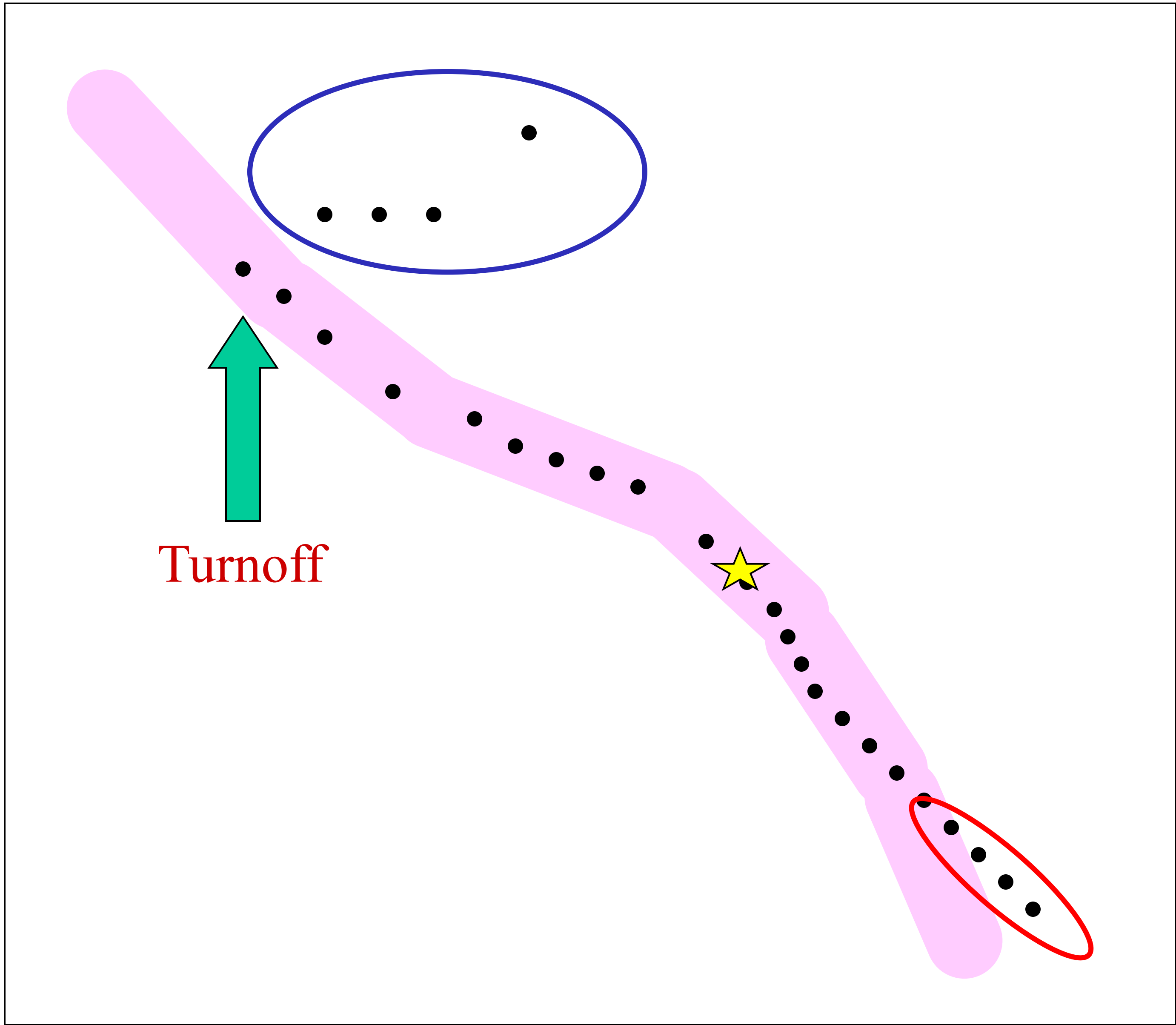
- Some stars aren’t even main sequence yet
- The brightest stars, though rare, dominate the light
- O and B stars
- Blueish tint to the cluster



10 Million Years Old

At 10 million years old:

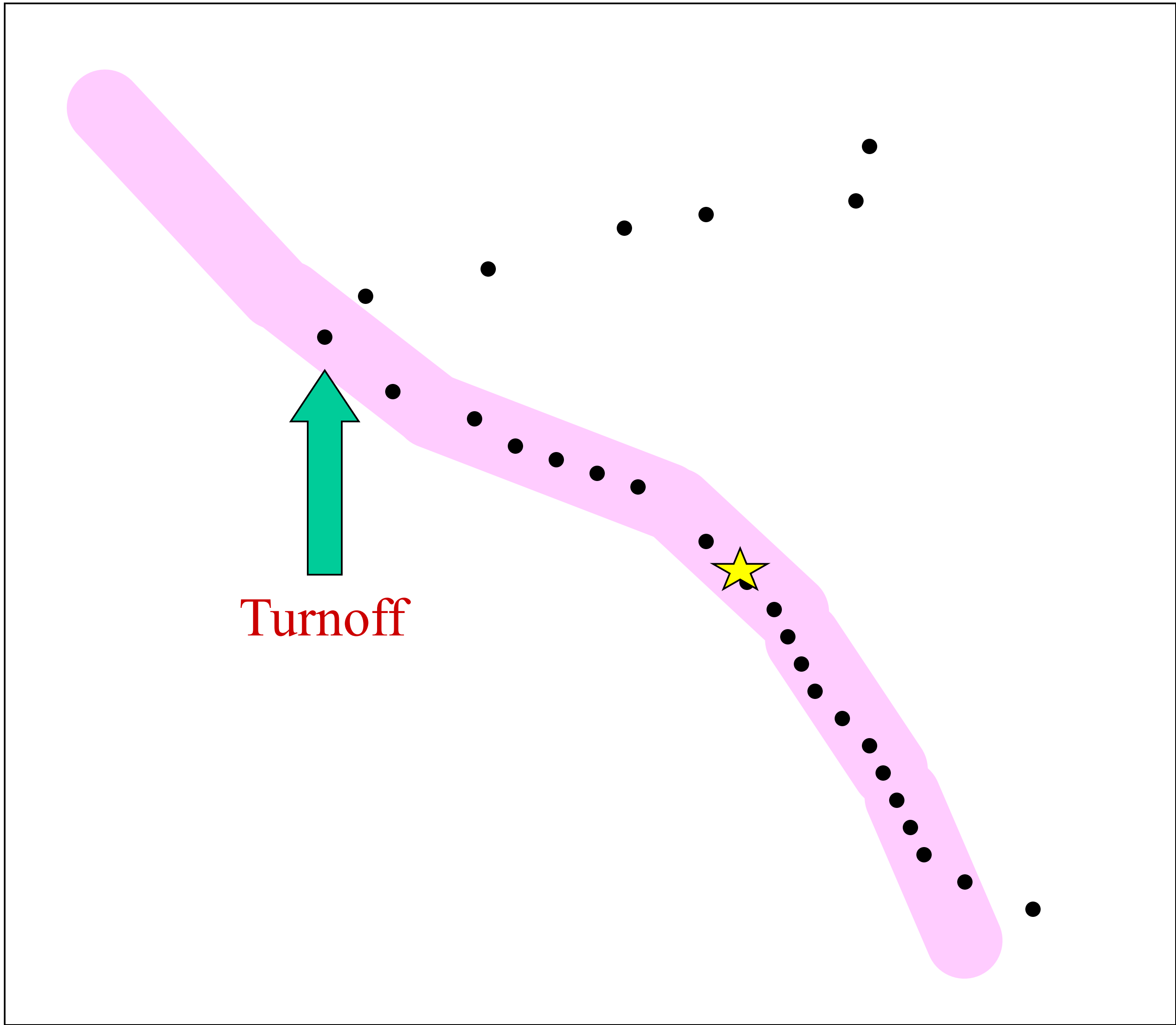
- Almost all stars are now main sequence
- Some of the heaviest are in their supergiant phases
    - The transition determines the turnoff point
- Some of them have died
- B and A stars dominate
- Blue/white tint to cluster



30 Million Years Old

At 30 million years old:

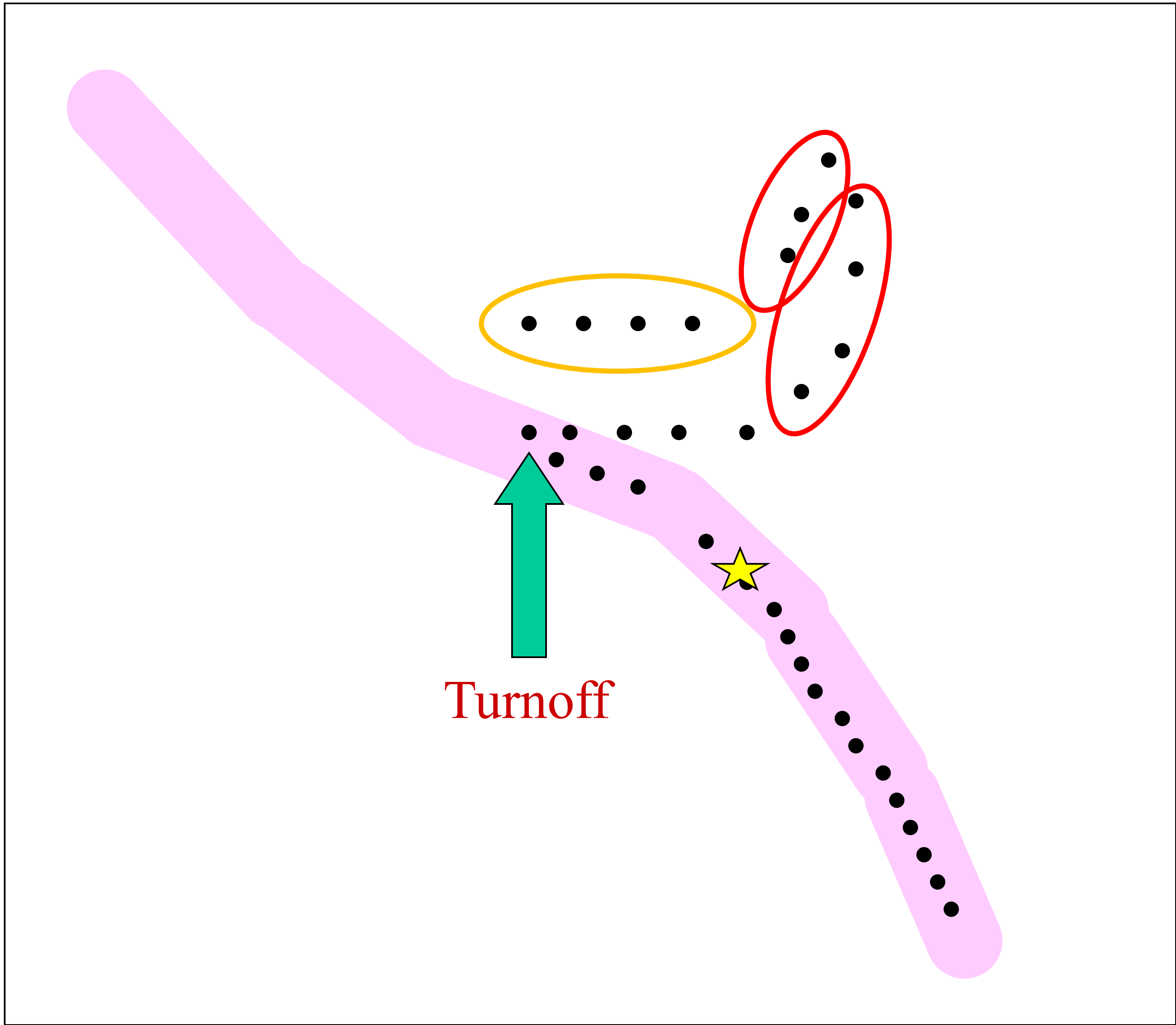
- More stars are supergiants
- Turnoff point has moved
- Mix of stars now
- White color to cluster



200 Million Years Old

At 200 million years old:

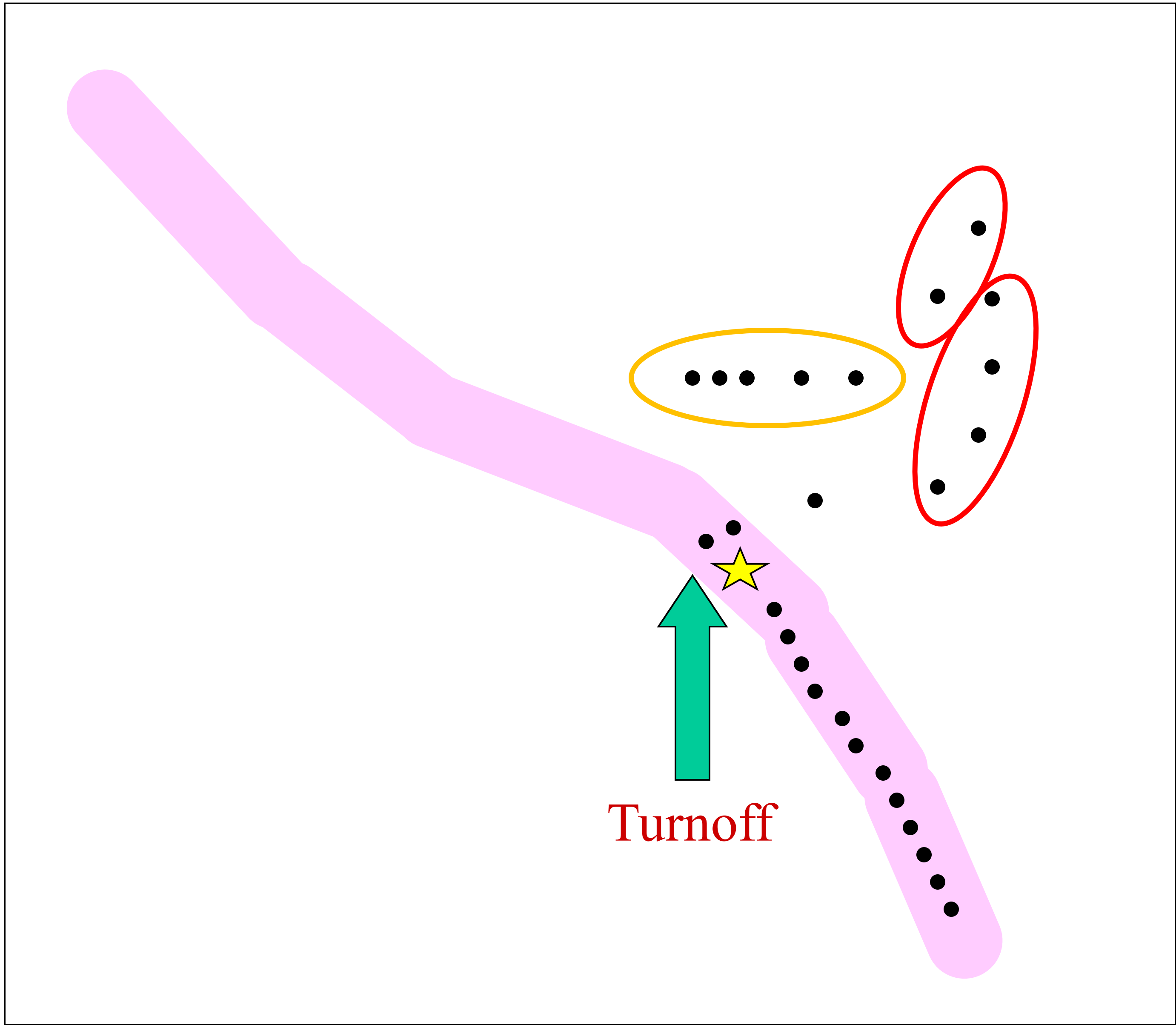
- Red giants, horizontal branch, and asymptotic giants
- Turnoff point moved farther
- Yellow tint to cluster



2 Billion Years Old

At 2 billion years old:

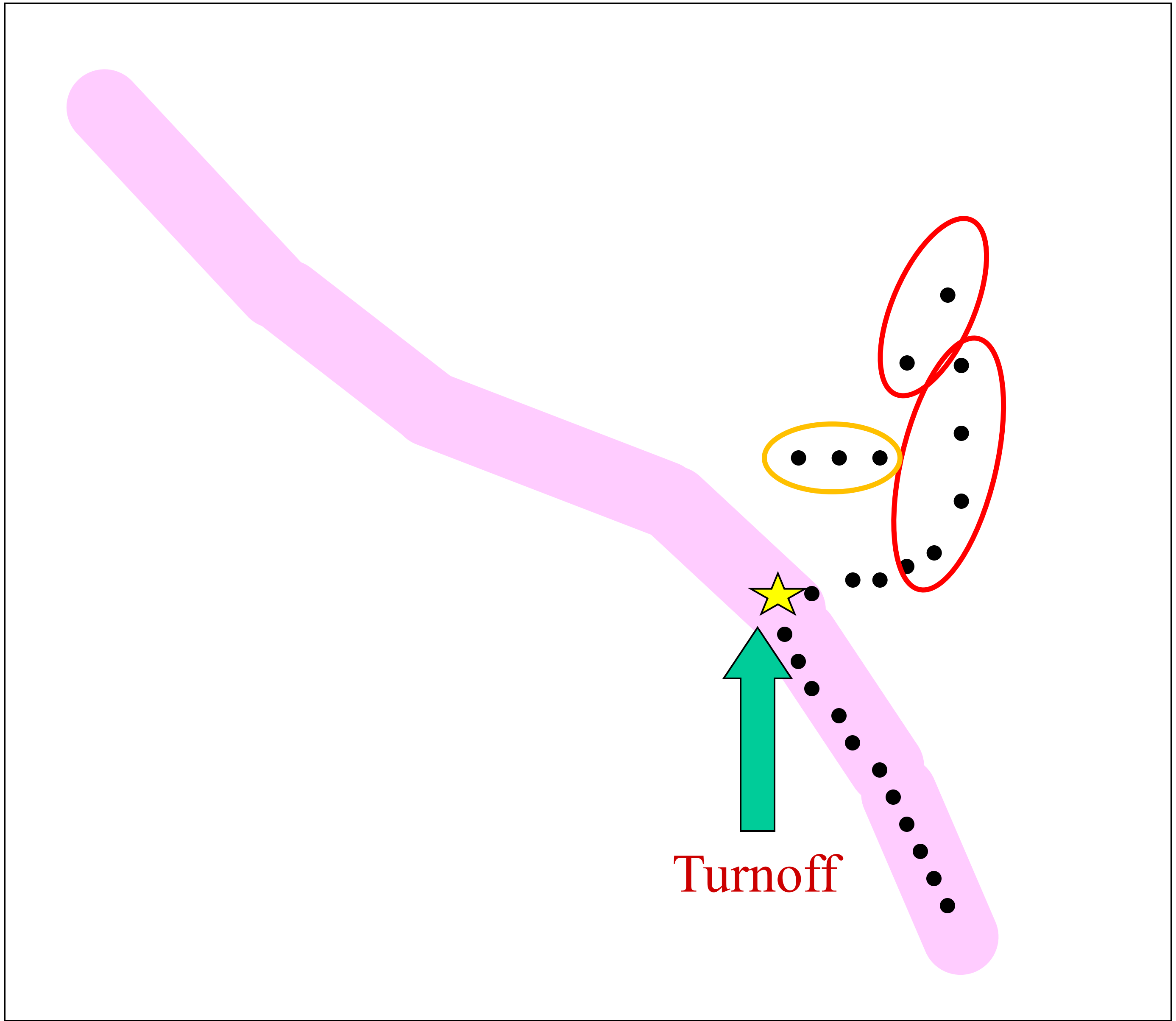
- G, K, M stars dominate
- Yellow/orange tint to cluster



10 Billion Years Old

At 10 billion years old:

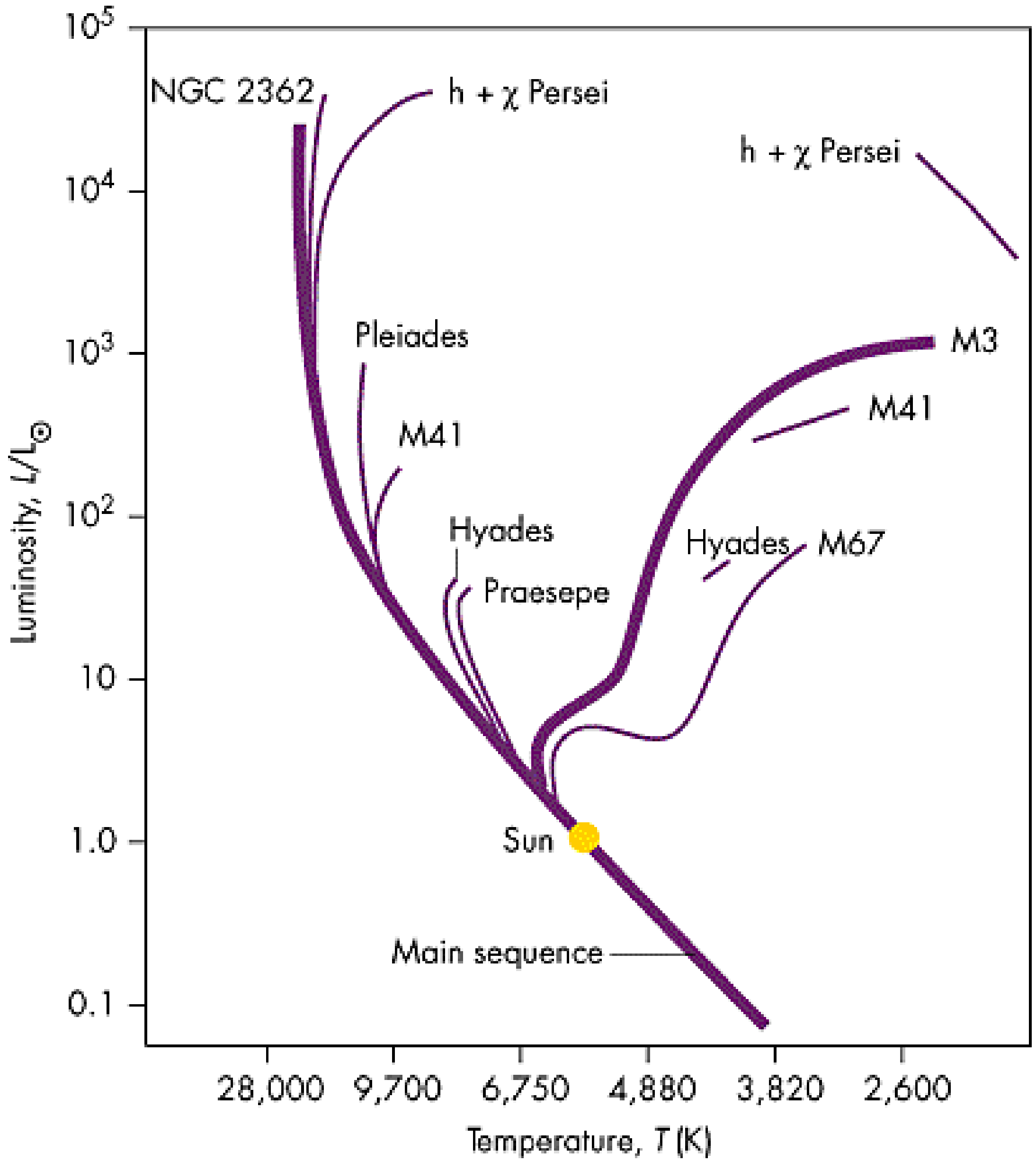
- K, M stars dominate
- Red tint to cluster
- Sun is about to turn off



The Turn Off Point

You can gauge the age of a collection of st
from the turn off point

- The color is also an indication
  - Blueish: young
  - Reddish: old



Doppler Effect

The General Formula

- Stars are often in motion, sometimes very fast
  - This causes a shift in spectral lines
- Due to waves getting "scrunched together" in front or "spread apart" in back
- Formula for shift in frequency
  - ν_0 is frequency now
  - ν is frequency then (emitted frequency)

In this class the subscript 0 always means "now"

- Objects normally move
  - Slowly (v^2/c^2 small), or
  - Straight away from us ($v = v\cos\theta = - v_r$)
- We therefore can simplify this formula:
  - v_r is the velocity away from us
- Normally, this is rewritten in terms of wavelength
  - Recall $\nu\lambda = c$.

$$\nu_0 = \nu \frac{\sqrt{1 - v^2/c^2}}{1 - v\cos\theta/c}$$

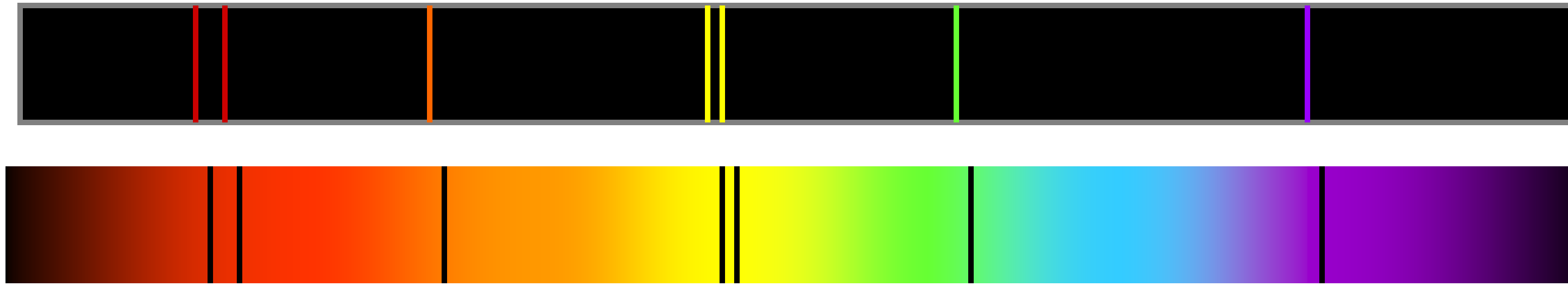
$$\nu_0 = \nu \frac{\sqrt{1 - v_r^2/c^2}}{1 + v_r/c} = \nu \sqrt{\frac{1 - v_r/c}{1 + v_r/c}}$$

$$\lambda_0 = \frac{c}{\nu_0} = \frac{c}{\nu}\sqrt{\frac{1 + v_r/c}{1 - v_r/c}} = \lambda \sqrt{\frac{1 + v_r/c}{1 - v_r/c}}$$

$$\frac{\lambda_0}{\lambda} = \sqrt{\frac{1 + v_r/c}{1 - v_r/c}}$$

Stars in Motion: Radial Velocities

- By studying the spectrum, we can measure the star's motion towards or away from us
  - Reference spectrum:
  - Star's spectrum



- Since spectral lines are shifted to shorter wavelengths, star is moving towards us
  - Called blue shift
- If the spectral lines are shifted towards longer wavelengths, star moving away
  - Called red shift
- The *red shift parameter*, denoted *z* is defined by*

$$1+z=\frac{\lambda_0}{\lambda}=\sqrt{\frac{1+v_r/c}{1-v_r/c}}$$

- For small velocities, we can see that:

$$1+z=\sqrt{\frac{1+v_r/c}{1-v_r/c}}=\frac{1+v_r/c}{\sqrt{1-v_r^2/c^2}}\approx 1+\frac{v_r}{c}$$

$$z\approx\frac{v_r}{c}$$

*Why I hate astronomers #5

- The red shift, and hence the radial velocity, can be measured for any object
  - Independent of distance