So, What's on the Test?

History of the Universe Stuff

- There are a lot of things that happened, and a lot of times and temperatures were named
  - Some of them were no better than approximations
- However, the order in which they occurred is pretty well known
- I don't expect you to remember all the details of when they happened and at what temperature
- I do expect you to remember the order in which they happened
- And you should know a few approximate dates and temperatures

Outline of History of Universe

Time	T or $k_B T$	Events
10^{-43} s	10^{18} GeV	Planck Era; time becomes meaningless?
10^{-39} s	10^{16} GeV	Inflation begins; forces unified
10^{-35} s	10^{15} GeV	Inflation ends; reheating; forces separate; baryosynthesis (?)
10^{-13} s	1500 GeV	Supersymmetry breaking, LSP (dark matter)
10^{-11} s	160 GeV	Electroweak symmetry breaking
14 μs	150 MeV	Quark Confinement
0.4 s	1.5 MeV	Neutrino Decoupling
1.5 s	0.7 MeV	Neutron/Proton freezeout
20 s	170 keV	Electron/Positron annihilation
200 s	80 keV	Nucleosynthesis
57 ky	0.76 eV	Matter-Radiation equality
370 ky	0.26 eV	Recombination
600 My	30 K	First Structure/First Stars
13.8 Gy	2.725 K	Today

Which Ones Should I Have *t* or *T* Memorized?

- Now:
  - $t \sim 14$ Gyr, $T \sim 2.7$ K
  - Structure formation in the recent past

- Recombination and matter/radiation equality:
  - $t \sim 10^5$ yr, $k_B T \sim$ eV

- Neutrino decoupling, n/p freezeout, e^+e^- annihilation, nucleosynthesis
  - $t \sim$ s, $k_B T \sim$ MeV

- Everything else:
  - $t \sim$ tiny, $k_B T \sim$ hot

Scaling

- You should know that the scale factor of the universe (a), and all distances and wavelengths scale the same way.

- All these factors are related to the red-shift z by

- And the scale factor is also roughly inversely proportional to the temperature

$$\frac{a}{a_0} = \frac{d}{d_0} = \frac{\lambda}{\lambda_0} = \frac{1}{1+z} \approx \frac{T_0}{T}$$

- Why is this last relationship approximate?
  - Whenever a particle annihilates, its energy raises the temperature of everything else

- Still, this formula works so well it is still good for approximations

Scaling And Densities

$$\frac{a}{a_0} = \frac{d}{d_0} = \frac{\lambda_{\text{then}}}{\lambda_{\text{now}}} = \frac{1}{1+z} \approx \frac{T_0}{T}$$

- You should know how all components of the universe scale as the universe expands
- Matter: $\rho \sim a^{-3}$
  - Includes both ordinary (baryonic) matter and dark matter
- Radiation: $\rho \sim a^{-4}$
  - Includes any relativistic particles
- Cosmological constant: $\rho \sim a^{0}$
  - Does not scale
- This allows us to relate densities at any time to densities today

$$\rho_b = \rho_{b0}(1+z)^3,$$
$$\rho_d = \rho_{d0}(1+z)^3,$$
$$\rho_r = \rho_{r0}(1+z)^4,$$
$$\rho_\Lambda = \rho_{\Lambda 0}.$$

Radiation Densities

- You will probably be given

$$\rho_r = g_{\text{eff}} \frac{\pi^2 (k_B T)^4}{30 (\hbar c)^3}$$

- You will also be given g_{eff} today
  - This applies all the way back to electron positron annihilation

$$g_{\text{eff},0} = 3.36$$

- However, you may have to calculate g_{eff} yourself

$$g_{\text{eff}} = g_b + \tfrac{7}{8} g_f$$

- And most important, know which particles to include in g_{eff}:
  - At temperature T, typical particles have energy:
  - If $3k_BT > mc^2$, particle is present and treat it as massless
  - If $3k_BT < mc^2$, particle will have annihilated

$$\bar{E} = 3k_BT$$

Time-Temperature Relations

- You will probably be given two or three of the following time-temperature relations:

$$t = \frac{17.1\ \text{Gyr}}{(1+z)^{3/2}} = \frac{62.2\ \text{kyr}}{(k_B T/\text{eV})^{3/2}}$$

$$t = \frac{2.42\,\text{s}}{\sqrt{g_{\text{eff}}}}\left(\frac{\text{MeV}}{k_B T}\right)^2$$

- The two on the left are for matter dominated only, right is for radiation dominated only

- You should know when to use which:
  - Matter for $z < 3000$, $k_B T < 1$ eV
  - Radiation for $z > 3000$, $k_B T > 1$ eV

- Don't use either of them today
  - We are cosmological constant dominated
  - $t_0 \approx 14$ Gyr

Cosmological Numbers to Memorize

- Here are some important cosmological parameters you should probably memorize:

$$\Omega = 1$$
$$\Omega_\Lambda \approx 0.69$$
$$\Omega_d \approx 0.26$$
$$\Omega_b \approx 0.05$$
$$H_0 \approx 68 \text{ km/s/Mpc}$$
$$T_0 \approx 2.7 \text{ K}$$
$$t_0 \approx 13.8 \text{ Gyr}$$

- Well, not really memorize, but:
- You should know $\rho_\Lambda > \rho_d > \rho_b >> \rho_\nu >> \rho_\gamma$
- You should know that $\Omega = 1$
- You should know Hubble's constant is around 70 km/s/Mpc
- You should know the universe is around 14 Gyr old
- You should know the temperature is around 3 K or so

Other Formulas to Memorize

Some formulas that you should know from the second half:

- Hubble's Law
  - Unreliable at large distances/red-shifts

$$v = H_0 d \approx zc$$

- "Horizon size": The largest distance on which things can have any influence, based on age of universe

$$d = ct$$

- Rates for particles to collide:

$$\Gamma = n\sigma(\Delta v)$$

- n is the number density of particles (units m^{-3})
- σ is the cross-section of the particles (units m^2)
  - You probably can't calculate it without help
- Δv is the relative velocity (units m/s)
  - Often just estimate it as the velocity of one particle
- Γ is the rate (units s^{-1})
  - Often multiply by age of the universe to find out if something happens or not
  - Happens if $\Gamma t > 1$, otherwise not

End of Material for Cosmology