Cosmology in the Unknown

Particle Physics Inspired Ideas

Limits of Experimental Physics

- We have very limited experience about particle physics above the electroweak scale

- For example, the most powerful collider in the world is the Large Hadron Collider
  - Protons hitting protons at 7 TeV + 7 TeV
- Protons are not fundamental
  - They contain a combination of three quarks plus some gluons
  - Optimistically, the quarks might carry ~ 2 TeV of energy each
- As they collide, they can produce particles of energy up to ~ 4 TeV
  - This corresponds to temperatures of $k_B T$ up to ~ 1.3 TeV

- Cosmic rays hit the Earth at much higher energy
- But these are rare and difficult to study directly

- Anything above temperatures of about 1.5 TeV have not been tested by experiment

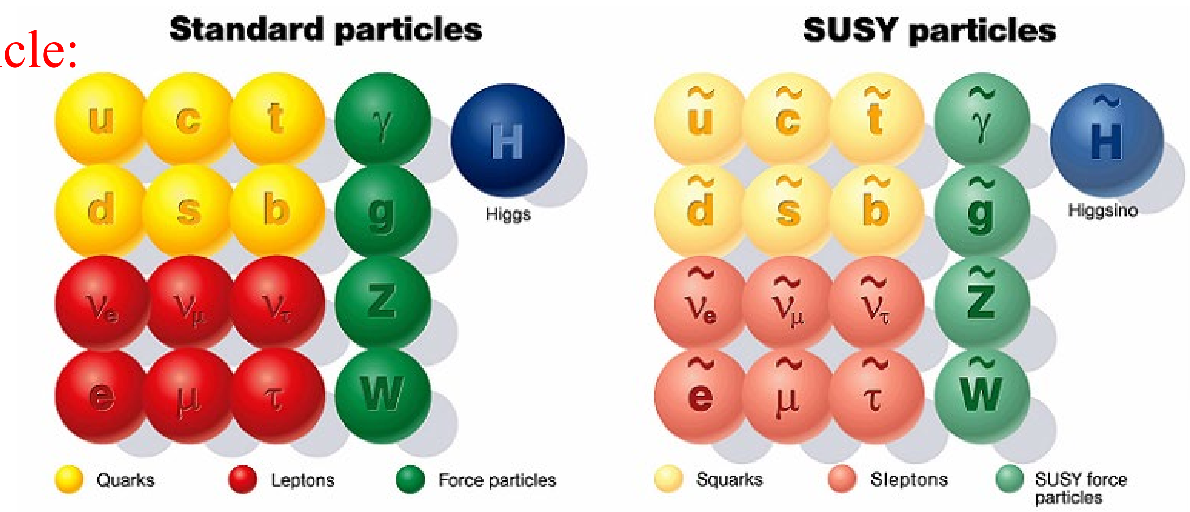
Sources of Inspiration

- No one thinks the standard model is the final answer on particle physics
- Many promising ideas have developed because of developments in particle physics
- Sometimes, these developments *should* have cosmological consequences
  - In some cases, we can learn about particle physics by studying these consequences
- The early universe can reach much higher energies than we can ever achieve in the laboratory

- There are also some unsolved problems in cosmology
- Sometimes, we can use the ideas *already* developed in particle physics to see if they can solve our cosmology problem
  - Sometimes, this seems very forced
- Other times, new ideas in particle physics are thought up in hopes of solving cosmological problems

Supersymmetry

- In conventional particle physics, fermions and bosons are fundamentally different
  - And never the twain shall meet
- In a hypothesis called *supersymmetry*, fermions and bosons are interrelated
- There must be a *superpartner* for every particle:



- Supersymmetry also helps solve a problem called the *hierarchy problem*
  - But only if it doesn't happen at too high an energy
- If supersymmetry is right, then scale of supersymmetry breaking probably around ~~k_BT = 500 GeV or so.~~ k_BT = 1.5 TeV or so.
- If this is right, the LHC should discover it
- If supersymmetry is right, then we must revise upwards our estimate of where it happens

Complete list of superpartners discovered at the Large Hadron Collider:

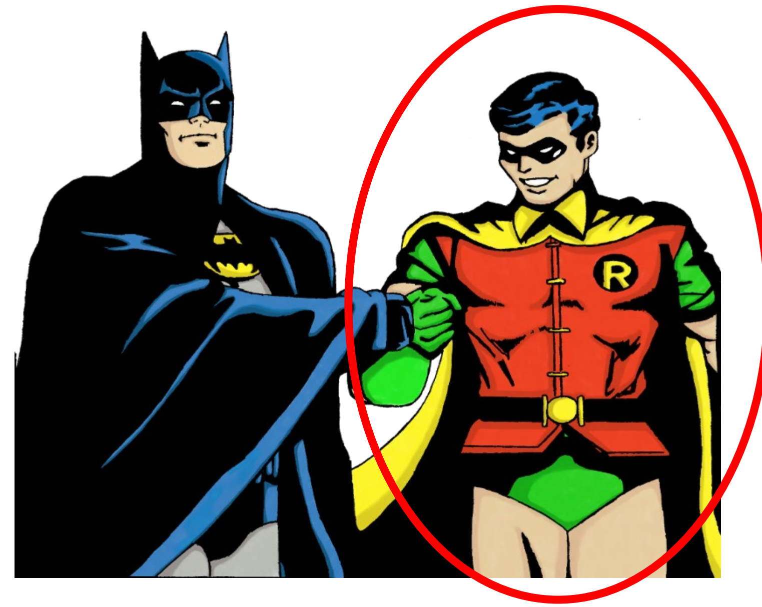
1.

The Lightest Superpartner

- When would supersymmetry become relevant?
  - Assume supersymmetry breaking scale around 1.5 TeV

$$t = \frac{2.42\ \text{s}}{\sqrt{g_{\text{eff}}}}\left(\frac{\text{MeV}}{k_B T}\right)^2 = \frac{2.42\ \text{s}}{\sqrt{106.75}}\left(\frac{\text{MeV}}{1.5\times 10^6\ \text{MeV}}\right)^2 \approx 10^{-13}\ \text{s}$$

Time	T or $k_B T$	Events
10^{-13} s	1.5 TeV	Supersymmetry Breaking
10^{-11} s	160 GeV	Electroweak Symmetry Breaking



$$e^- + e^+ \leftrightarrow \tilde{\gamma} + \tilde{\gamma}$$

- Most variations of supersymmetry assume that supersymmetric particles can only be created or destroyed in pairs
  - Guaranteed by something called *R*-parity

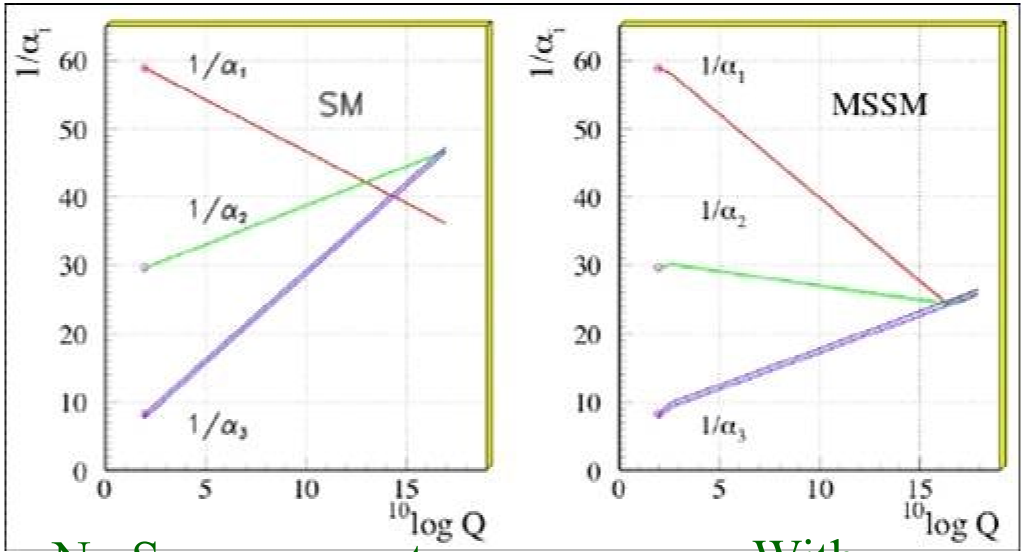
- If this is true, then the lightest superpartner (LSP) must automatically be stable

- Such a particle could be the dark matter
  - More on this later

Grand Unification Theories (GUT's)

- In the standard model, there are three fundamental forces, and three corresponding coupling constants
- These have rather different values
- But their strength changes as you change the energy of the experiment, theoretically

- How much they change depends on whether supersymmetry is right or not
- If supersymmetry is right, then at an energy of about 10^{16} GeV, the three forces are equal in strength
- At $k_B T = 10^{16}$ GeV, there will be another phase transition – the Grand Unification transition



No Supersymmetry

With Supersymmetry

$$t = \frac{2.42\ \text{s}}{\sqrt{210}}\left(\frac{\text{MeV}}{10^{19}\ \text{MeV}}\right)^2 \approx 10^{-39}\ \text{s}$$

Baryogenesis might occur at this scale

Scale could be right for inflation

Inflation

How Flat Was the Universe?

- The universe today is very nearly flat:
- How flat was it in the past?
- Recall the Friedman equation:

$$\Omega = 0.9993(37)$$

$$H^2 = \frac{8\pi}{3} G\rho - \frac{k}{a^2}$$

- The radiation fraction Ω_r is defined as

$$H^2 \Omega_r = \frac{8\pi}{3} G\rho_r$$

- The curvature (the last term) is given by

$$\frac{k}{a^2} = \frac{8\pi}{3} G\rho - H^2 = H^2 (\Omega - 1)$$

- Taking the ratio of these equations, we have
- Recall that radiation scales as a^{-4}
- So it all scales approximately, as T^{-2}
  - Not exactly, due to reheating

$$\frac{\Omega - 1}{\Omega_r} = \frac{3k}{8\pi G \rho_r a^2} \propto a^2 \quad \propto T^{-2}$$

The Flatness Problem

- When you include all the corrections for reheatings, you get $\dfrac{\Omega-1}{\Omega_r} \propto \dfrac{1}{T^2}$

$$\frac{\Omega-1}{\Omega_r} = \left(\frac{3.91}{g_{\text{eff}}}\right)^{2/3} \left(\frac{k_B T_0}{k_B T}\right)^2 \left(\frac{\Omega-1}{\Omega_r}\right)_0$$

- For definiteness, pick g_{eff} =200 and $k_B T = 3\times10^{15}$ GeV
- Current values
- In the early universe expect $\Omega_r = 1$

$$\Omega_0 = 0.9993(37)$$
$$\Omega_{r0} = 9\times10^{-5}$$
$$T_0 = 2.7255\ \text{K}$$

- So we find

$$\Omega = 0.999999999999999999999999999999999999999999999999999999997(18)$$

- You should memorize this value to about 55 digits or so
- How did it happen to be so close to 1?

The Horizon Problem

- The universe looks pretty uniform today
- The universe used to be smaller
    - Could it have been smoothed out in the past?

- Let's look at the universe at the time of Grand Unified scale ($k_B T = 10^{16}$ GeV):
- At the time the universe was about 10^{-39} seconds old
- Universe then *could be* smooth on scales of about

$$d = 2ct \quad = 2\left(3\times10^{8}\ \text{m/s}\right)\left(10^{-39}\ \text{s}\right) = 6\times10^{-31}\ \text{m}$$

- Since then, the universe has grown *immensely* in size
- Assume aT is roughly constant (not exactly)

$$\frac{a_0}{a} = \frac{k_B T}{k_B T_0}\left(\frac{g_{\text{eff}}}{3.91}\right)^{1/3} = \frac{10^{25}\ \text{eV}}{\left(8.617\times10^{-5}\ \text{eV/K}\right)\left(2.725\ \text{K}\right)}\left(\frac{200}{3.91}\right)^{1/3} \approx 1.6\times10^{29}$$

- So universe today could be smooth on scale of:

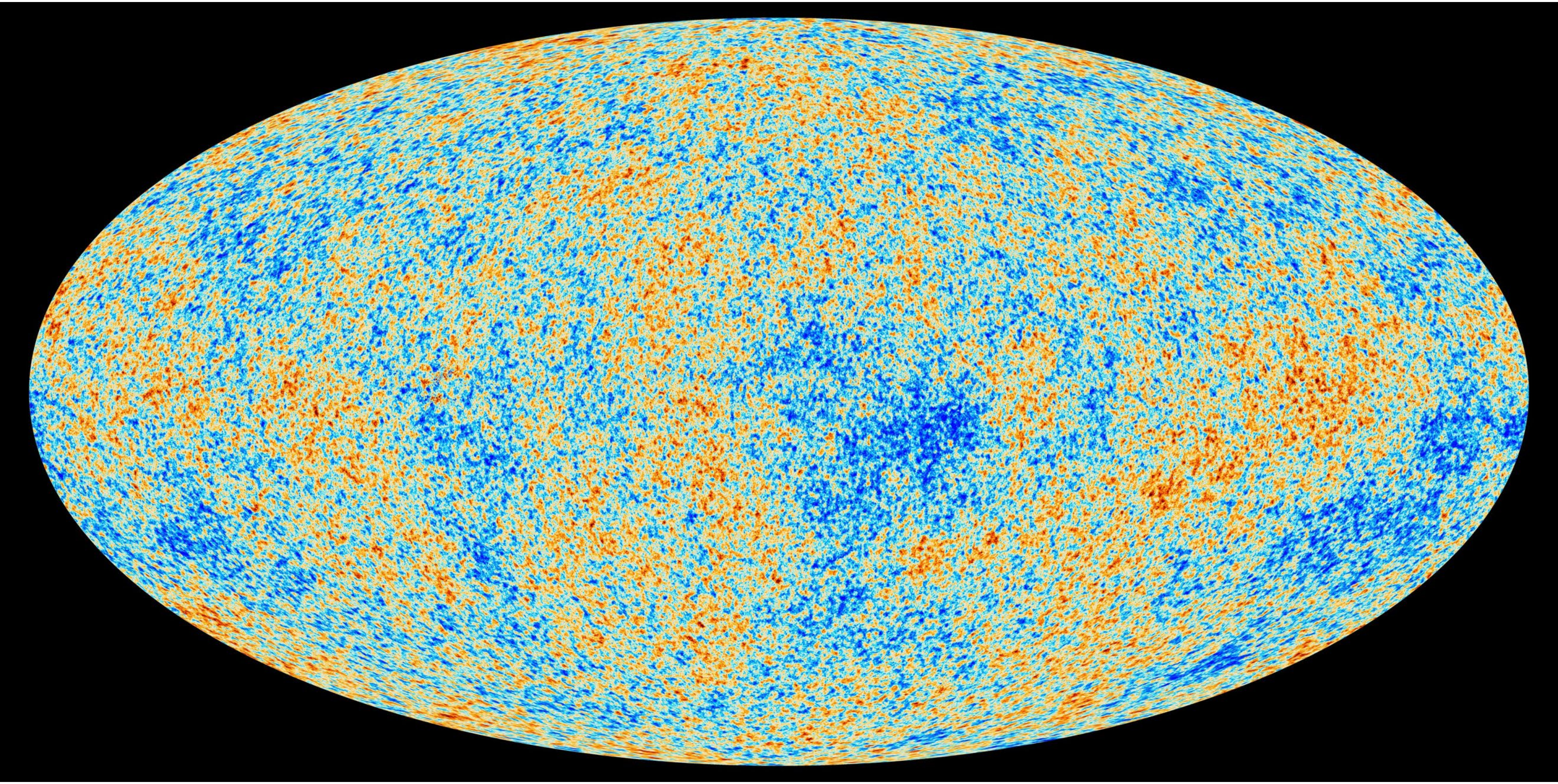
$$d_0 = \frac{a_0 d}{a} \approx 0.1\ \text{m}$$

- Actual universe today smooth on scale of:

$$d \approx 3.3ct_0 \approx 4\times10^{26}\ \text{m}$$

The Origin of Structure Problem

- Where did all this variation come from?



The Solution: Inflation

- How can we get Ω to be close to one at some point in the past?
- Recall,

$$\frac{\Omega - 1}{\Omega} \sim \frac{k/a^2}{\rho}$$

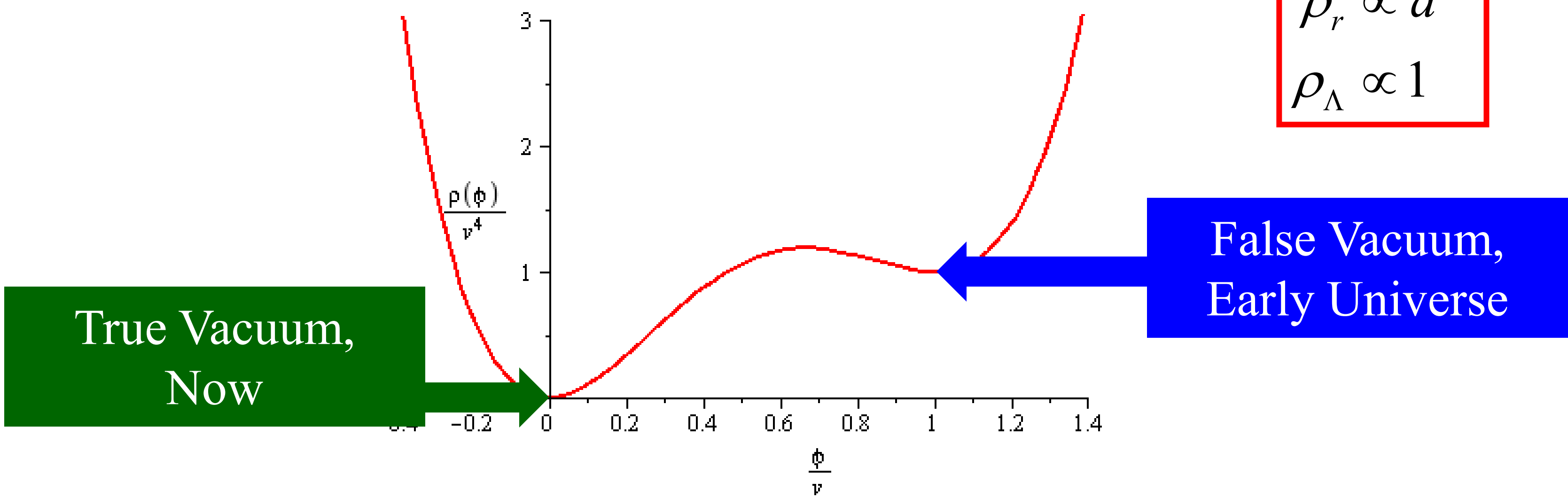
- Since ρ falls faster than $1/a^2$, as universe expands, Ω is driven away from 1
- If ρ fell slower than $1/a^2$, as universe expands, Ω would be driven *towards* 1
- Could there have been a period when universe dominated by vacuum in the past?
- Need some field that is stuck in "wrong place" – "false vacuum"

$$\rho_m \propto a^{-3}$$
$$\rho_r \propto a^{-4}$$
$$\rho_\Lambda \propto 1$$

- Friedmann Equation:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho_\Lambda \equiv H_\Lambda^2$$

$$a(t) \propto e^{H_\Lambda t}$$



The Inflationary Era

$$H_\Lambda^2 = \tfrac{8}{3}\pi G \rho_\Lambda$$

$$a(t) \propto e^{H_\Lambda t}$$

- Assume inflation occurs around the Grand unified scale, $v = 10^{16}$ GeV

$$\rho_\Lambda \sim \frac{v^4}{10(\hbar c)^3 c^2} = \frac{\left(10^{25}\ \text{eV}\right)^4 \left(1.602\times10^{-19}\ \text{J/eV}\right)}{10\left(1.97\times10^{-7}\ \text{eV}\cdot\text{m}\right)^3 \left(3\times10^8\,\text{m/s}\right)^2} = 2.33\times10^{83}\ \text{kg/m}^3$$

- During inflation, Hubble's constant is:

$$H_\Lambda = \sqrt{\tfrac{8}{3}\pi G \rho_\Lambda} = \sqrt{\tfrac{8}{3}\pi\left(6.67\times10^{-11}\ \text{m}^3/\text{kg/s}^2\right)\left(2.33\times10^{83}\ \text{kg/m}^3\right)} = 1.1\times10^{37}\ \text{s}^{-1}$$

- Universe increases by factor of e every 10^{-37} s or so

- We are <u>not</u> radiation dominated, so we can't trust radiation era formulas

$$t \neq \frac{2.42\ \text{s}}{\sqrt{g_{\text{eff}}}}\left(\frac{\text{MeV}}{k_B T}\right)^2$$

How Much Inflation Do We Need?

- During inflation, Ω is driven towards one
- $\Omega - 1$ at GUT scale is less than 10^{-55}
- We need *a* to increase by about 10^{27}

$$\frac{\Omega - 1}{\Omega} \sim \frac{k/a^2}{\rho} \propto a^{-2}$$

- At GUT scale, universe was smooth on scale that becomes *at most* 0.1 m today
- Today universe is smooth on scale of *at least* 10^{27} m
- We need *a* to increase by *at least* 10^{28}

$$a(t) \propto e^{H_\Lambda t} \qquad e^{H_\Lambda t_I} > 10^{28} \qquad H_\Lambda t_I > \ln\left(10^{28}\right) = 65$$

- Probably want 100 or so *e*-foldings of growth

$$t_I > 100 H_\Lambda^{-1} = 10^{-35}\ \text{s}$$

- If it is any number significantly bigger than the minimum required, then
  - $\Omega = 1$ to *many* more digits than we need at GUT scale
  - $\Omega = 1$ today to *many* digits
  - Universe is smooth on scales *much* bigger than we can see

More Comments on Inflation

- During inflation, the universe expands by at least factor of 10^{28}

- If there is anything *besides* vacuum, it is getting red shifted like crazy

$$T \propto a^{-1} \qquad T_{\text{end}} < T_{\text{begin}} 10^{-28} \qquad k_B T_{\text{end}} < 10^{-28} \cdot 10^{16}\ \text{GeV} \qquad T_{\text{end}} < 10\ \text{K}$$

- Completely ignore radiation

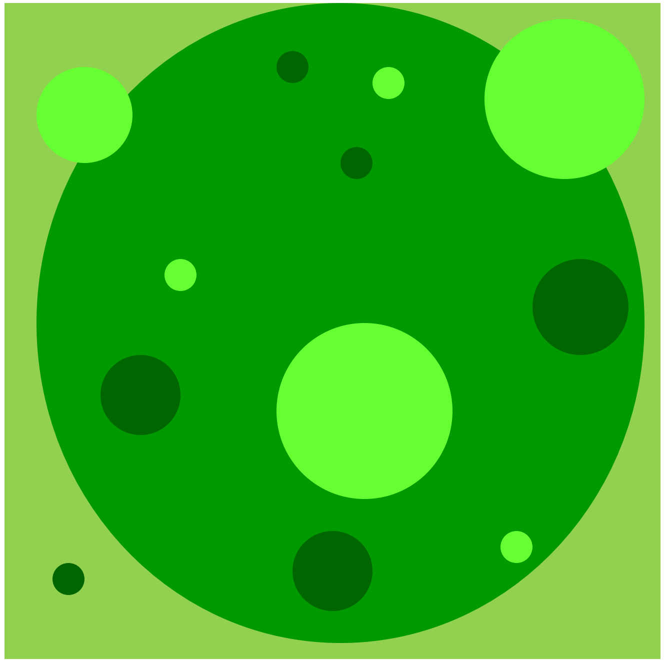
- Anything else in the universe also is diluted and disappears

$$n \propto a^{-3} \qquad n_{\text{end}} < 10^{-84} n_{\text{begin}}$$

- By the time inflation is over, any record of previous universe is effectively erased

Density Fluctuations from Inflation

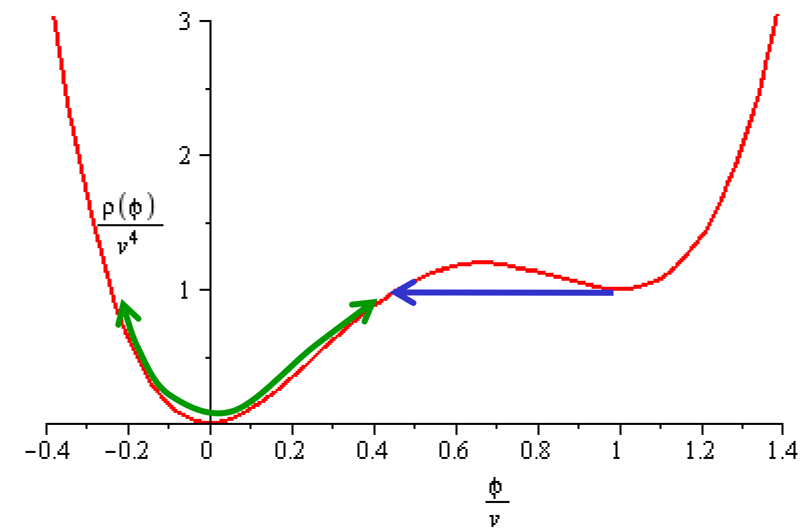
- During inflation, you could think universe is very smooth
- On microscopic scales, universe will have tiny density fluctuations, thanks to quantum mechanics
- The size and magnitude of these fluctuations can be calculated
  - Gaussian, distribution, for example



- In size, these are so small they would be irrelevant
  - Except the universe is inflating!
- Little fluctuations grow up to be big fluctuations
- Constant new fluctuations are appearing at all scales
- During inflation: predict “scale invariant” density perturbations
  - “Spectral index” $n = 1$

Exiting Inflation and Reheating

- Universe must make transition to true vacuum
  Old inflation picture:
- It quantum tunnels from false vacuum to true
- It oscillates back and forth around true
- Oscillations generate all kinds of particles
  - Reheating



$$\rho_\Lambda \to \rho_r$$

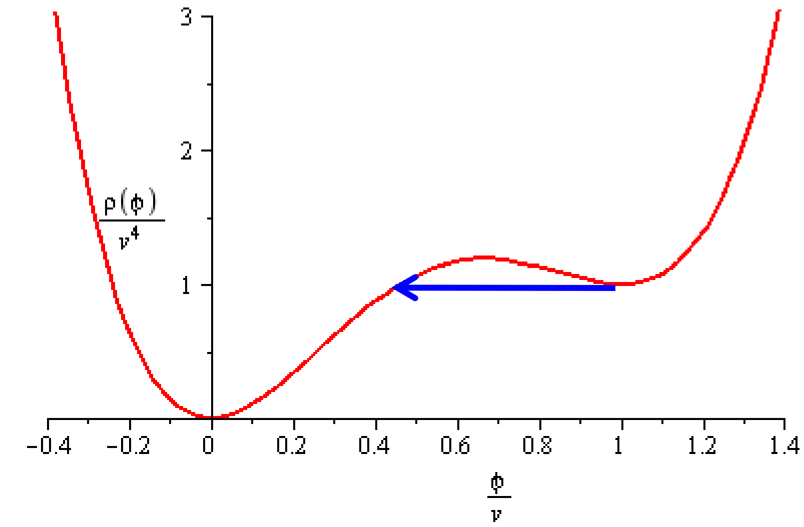
$$\frac{\pi^2}{30} g_{\text{eff}} \frac{(k_B T)^4}{(\hbar c)^3 c^2} = \frac{v^4}{10(\hbar c)^3 c^2} \qquad k_B T_r = v\left(\frac{10}{\pi^2 g_{\text{eff}}}\right)^{1/4} \qquad k_B T_r \approx \tfrac{1}{4} v$$

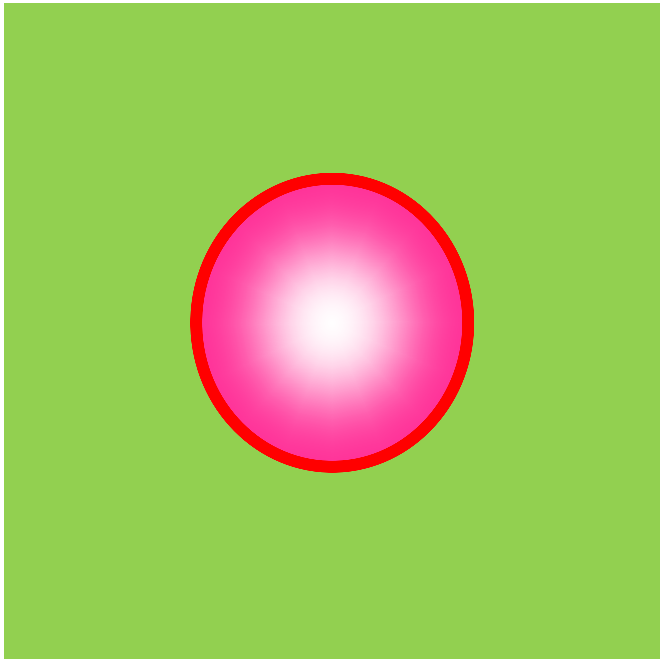
- Baryogenesis must occur <u>after</u> inflation ends
  - Otherwise any baryons would just inflate away
- We re-enter radiation era, but with *t* a little increased because of inflationary era

Event	$k_B T$ or T	Time
Inflation begins	10^{16} GeV	10^{-39} s
Inflation ends, baryons created, etc.	10^{15} GeV	10^{-35} s

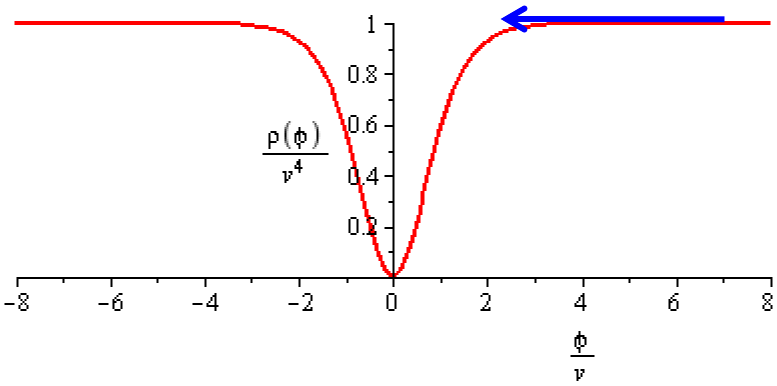
Inflation is Dead. Long Live Inflation!

- The problem – it's hard to quantum tunnel
- It will only occur *very rarely*
- It will occur at one place
  - Bubble nucleation
- All the energy is on the walls of the expanding bubble
- If bubbles collided, we could get reheating
  - But universe expands so fast, bubbles never find each other





- The solution: change the potential
- No quantum tunneling needed
- "Slow roll" potentials
- Whole universe moves to minimum at same time

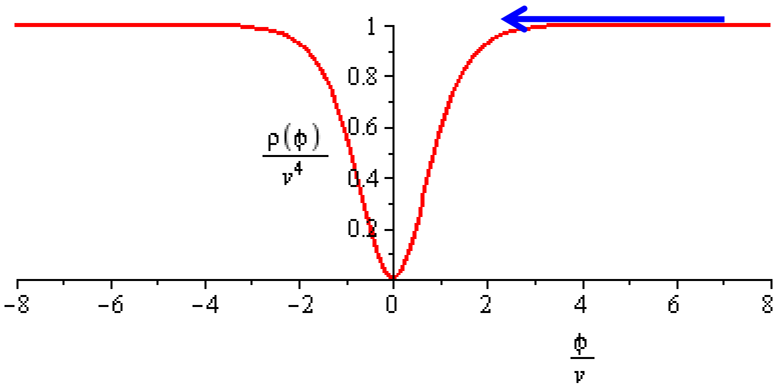


Too Many Kinds of Inflation

- Many *many* variations on inflation have since been explored
  - Slow Roll inflation
  - Chaotic inflation
  - Designer inflation
- We don't know which (if any) of these are correct

Generic features of modern inflation:

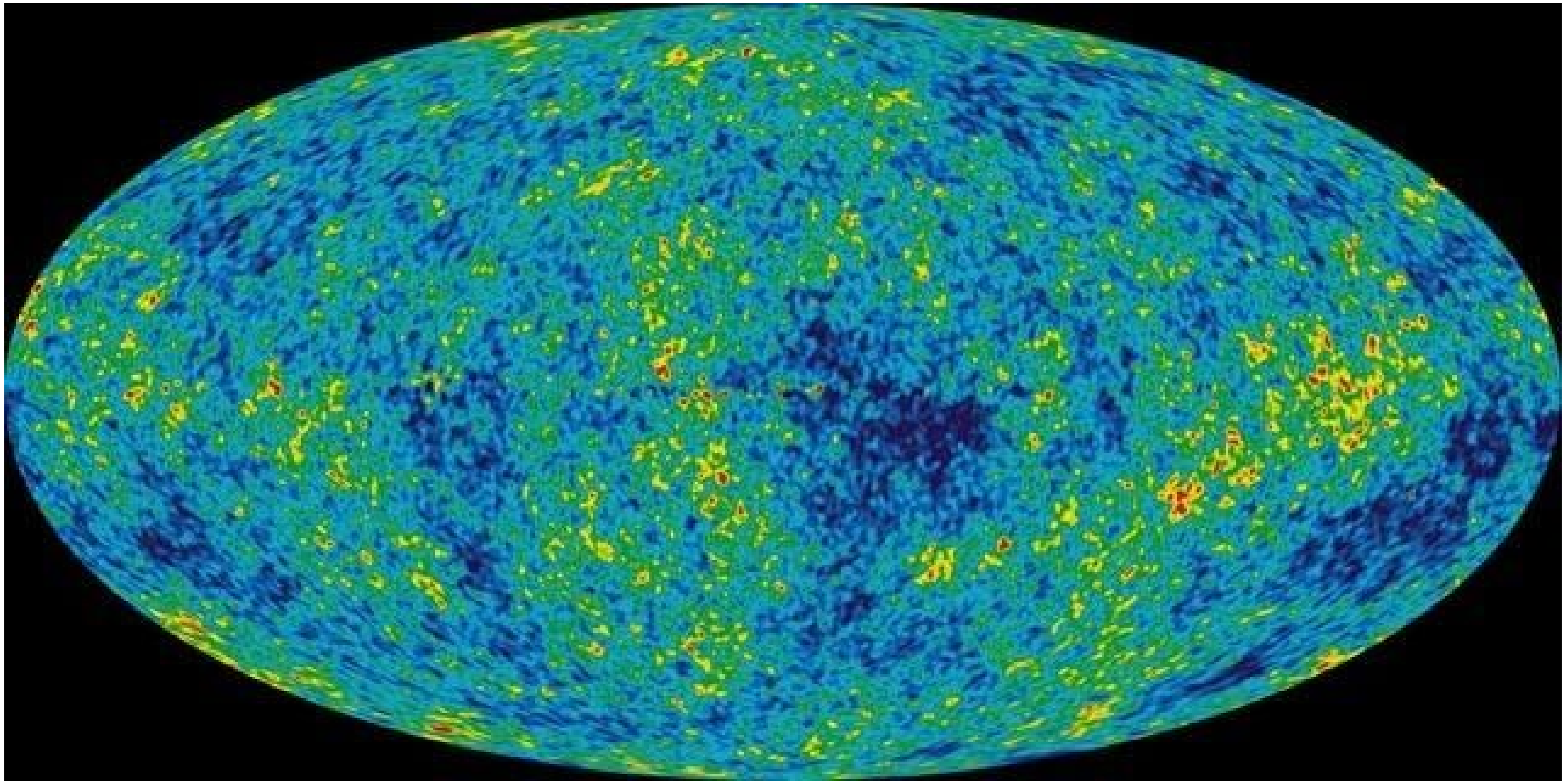
- Universe "slows down" just before exiting inflation
- Expansion not so fast at the end
- Small fluctuations don't grow quite so big
- Fluctuation spectrum has more power at smaller scales
  - Not quite scale invariant
  - Spectral index just under 1



Density Fluctuations

Why We Need Them

- If the universe were *perfectly* uniform, then how come the microwave background isn't uniform?
- Where did all the structure (galaxies, clusters, etc.) come from?
- Can we understand where these variations come from?
- Can we explain how these variations led to the structures we see?



Review: Spherical Harmonics

- The temperature is a function of angle only
- The *spherical harmonics* are a series of functions of angle only
  - Taught about them in PHY 215 and other courses
- There are an infinite number of them
  - $l = 0, 1, 2, 3, \ldots$
  - $m = -l, -l+1, \ldots, l$

$$T(\theta,\phi)$$

$$Y_{lm}(\theta,\phi)$$

- The smaller indices represent functions that vary slowly with angle, the larger ones vary more quickly
- For example, the ϕ dependence depends on m
- The angular scale at which the function changes sign is, crudely,
  - This could be based on changes in θ or ϕ

$$Y_{lm}(\theta,\phi) \propto e^{im\phi} = \cos(m\phi) + i\sin(m\phi)$$

$$\theta \approx \frac{\pi}{l}$$

- Any function that depends only on angle can be written as a linear combination of spherical harmonics
- The coefficients C_{lm} contain the same information as the function $T(\theta,\phi)$

$$T(\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} C_{lm} Y_{lm}(\theta,\phi)$$

How to Describe the Fluctuations

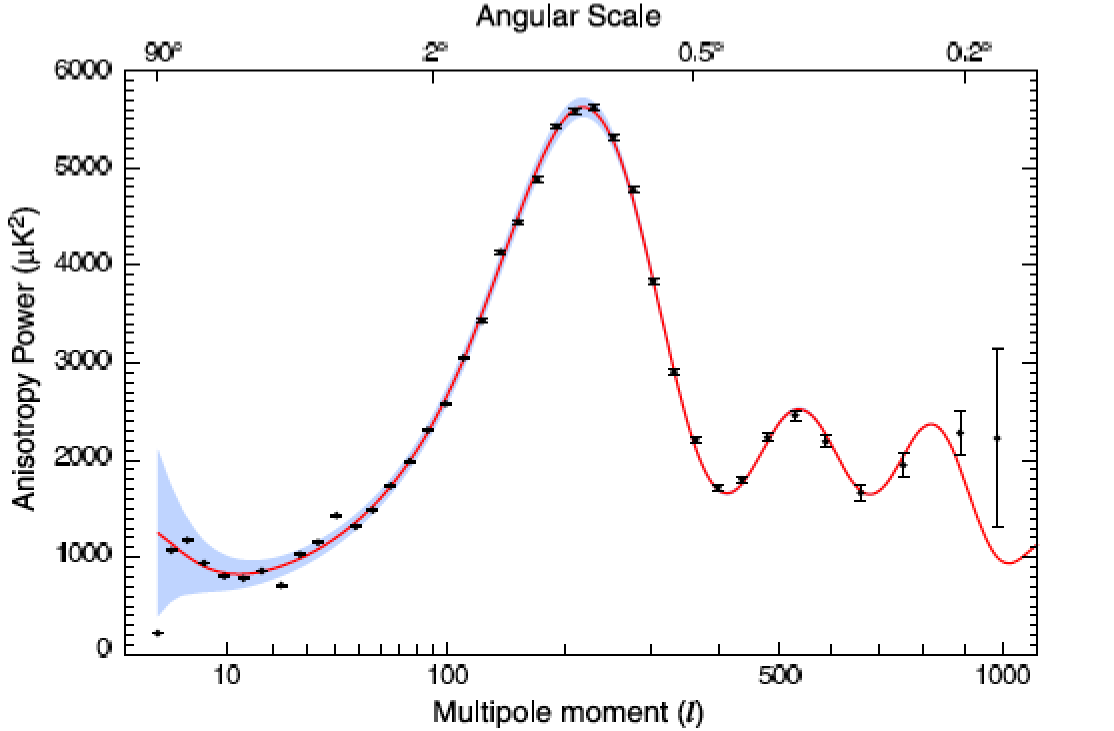
- When you rotate the coordinates, you change Y_{lm} spherical harmonics into others with the same value of l but different values of m

$$T(\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} C_{lm} Y_{lm}(\theta,\phi)$$

- The coefficients C_{lm} get mixed within different values of m when you rotate
  - Exact values of C_{lm} depends on choice of axes
- But, if you sum up the C_{lm}'s over fixed l, the total is independent of m, depends only on l
- So the information about how much variation on each angular "scale" is contained in

$$\langle C_l^2 \rangle = \frac{1}{2l+1} \sum_{m=-l}^{l} |C_{lm}|^2$$

- Plot the result as a function of l
  - Dots are the data, solid is the fit
- How do we explain all these features?



Growth of Density Pertubations

- Assume inflation, or some other source, produces density perturbations at all scales
  - Spectral index = 1, or slightly less than 1
  - A bit more "power" at small scales than at large

Now, we need to discuss what happens at all possible scales during four eras:

- Radiation dominated era: $z > 3000$
- Matter domination before recombination: $1100 < z < 3000$
- Post recombination: $10 < z < 1100$
- Structure formation: $z < 10$

The cosmic microwave background shows us the universe at recombination

Coupling in the Different Eras:

- The dark matter probably doesn't collide with anything
  - It will do its own thing
  - It is moving slowly and has little thermal pressure

- Neutrinos move nearly at the speed of light
  - As soon as they can, they simply erase any structure
  - Ignore them from now on

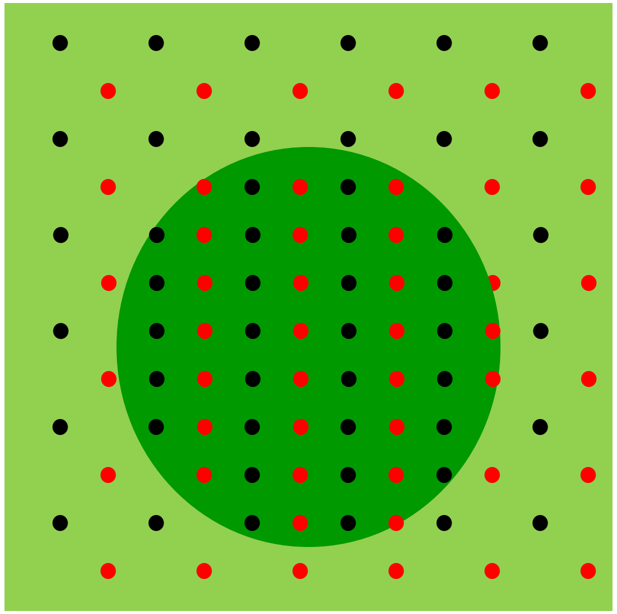
Everything else moves together before recombination:
- The baryons collide with electrons (Coulomb scattering)
- The photons collide with electrons (Thomson scattering)
- The baryons, photons, and electrons all move together
- The photons provide the lion's share of the pressure

After recombination, the atoms decouple from the photons
- The pressure drops effectively to zero

The Radiation Era: What Happens

- Suppose we have a perturbation (say, a high density region) during this era:
- Inflation predicts: every type of particle will have higher density in this region

Radiation
Dark Matter
Baryons/Atoms

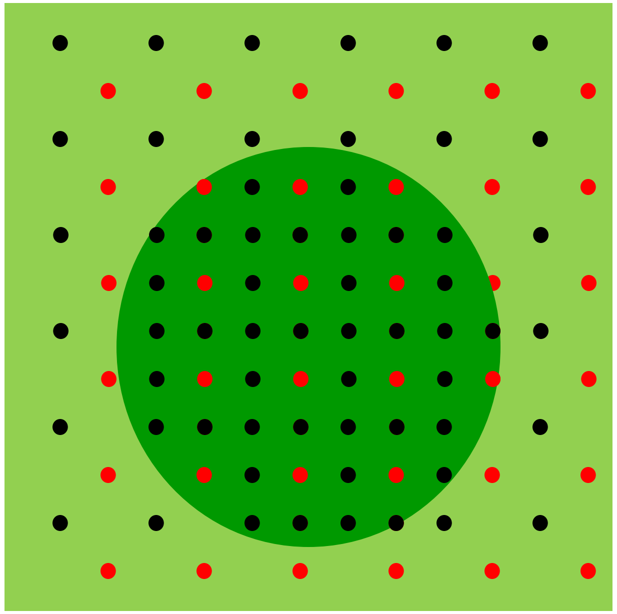


- Photons try to flow from high density to low density
- They move at speed of light
- When size of perturbation equals about *ct*, it gets wiped out
  - Actually, they oscillate, more on this later
- The baryons *also* get wiped out, because they are caught with the photons
- The dark matter stays behind, and gravity starts to gather it more
  - But very slowly, because radiation dominates

Matter Domination, Before Recombination

- As before, the photons and baryons flow from high to low density as soon as they are able
  - Actually, they oscillate, as I'll explain soon
- However, now the dark matter dominates
- Gravity causes high density regions to get higher density
- They start to form structures
- These structures will become galaxies, etc.
- We see the microwave background from the *end* of this era
  - We don't see large density perturbations from this era
- We are looking only at the photons (coupled to baryons)

Radiation
Dark Matter
Baryons/Atoms



Acoustic Modes

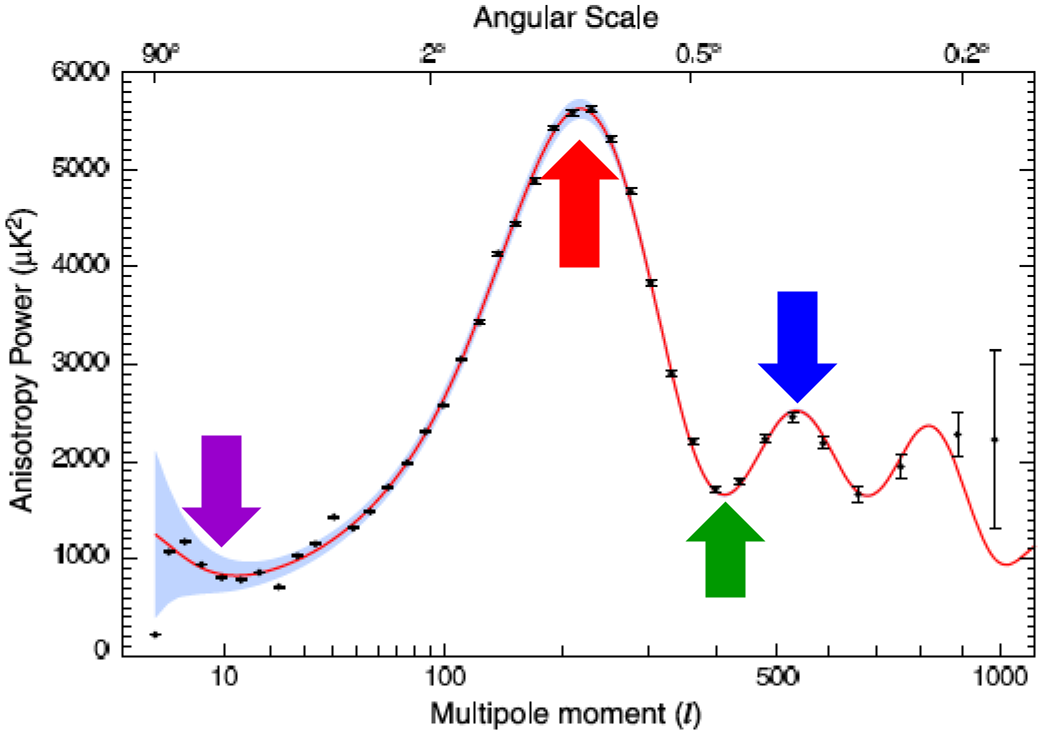
- Before recombination: The photons and baryons are all coupled together
- Consider regions of alternating high density/low density:

- High density regions try to flow to low density
- This creates a sound wave, just like an ordinary sound wave
- This process stops at recombination
  - We catch the wave, frozen at that moment of time

Acoustic Modes and Scales

Where will we catch the wave at recombination? It depends on wavelength

- Longest wavelength: It hasn't had time to even cycle at all
  - Relatively little "power" at largest scales
- Next longest: It will be right at its first peak in oscillation cycle
  - This will be sensitive to Ω
- Slightly longer: Wave is at a node, or minimum
- Longer still: The second peak
  - Wave has oscillated, it is now at anti-node
  - Baryons feel gravity of dark matter, and resist (baryon drag)
  - Sensitive to Ω_b



Where Will the First Peak Be? (1)

- This is a wave that has only gone through about a third of a cycle

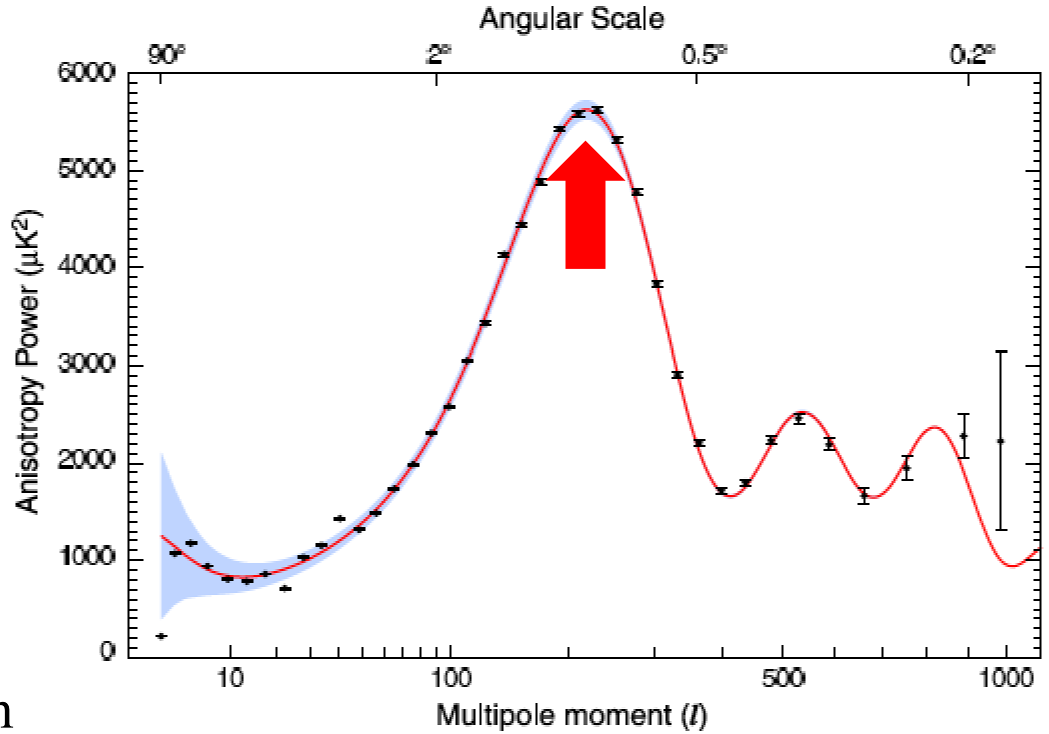
$$\tfrac{1}{3}\lambda \approx v_s t \qquad v_s = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{\frac{1}{3}u}{u/c^2}} = \frac{c}{\sqrt{3}}$$

$$\lambda = \sqrt{3}\left(3\times10^8 \text{ m/s}\right)\left(3.8\times10^5 \text{ y}\right)\left(3.16\times10^7 \text{ s/y}\right)$$

$$= 6.24\times10^{21} \text{ m}$$

- This scale has since grown to about

$$\lambda_0 = \lambda\left(1+z_*\right) = 1092\left(6.24\times10^{21} \text{ m}\right) = 6.81\times10^{24} \text{ m}$$



- The distance to this wave is now about
- The angular size of the wave is then

$$d \approx 3.3ct_0 \approx 4.4\times10^{26} \text{ m}$$

$$\theta = \frac{\lambda_0}{d} = \frac{6.81\times10^{24} \text{ m}}{4.4\times10^{26} \text{ m}} = 0.015 \qquad l = \frac{\pi}{\theta} \approx 210$$

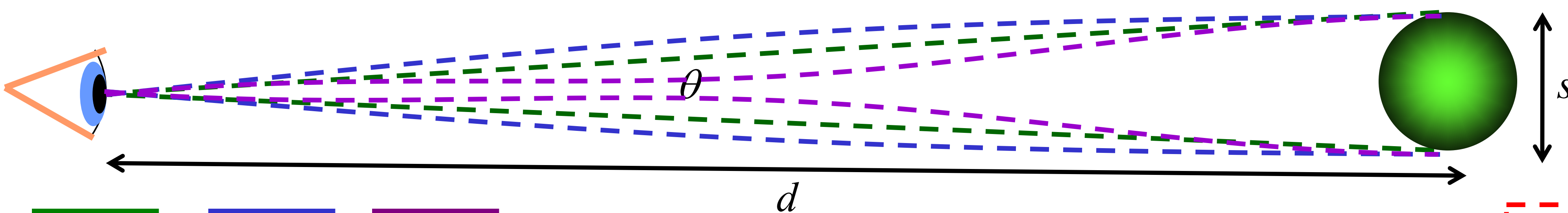
- More precise analysis says peak should be at $l = 220$

Where Will the First Peak Be? (2)

$l = 220$

- All calculations assumed standard cosmological values
  - Including $\Omega = 1$
- There are *many* effects that change this, most notably, the curvature of spacetime

- In flat space, $\Omega = 1$, we used the correct distance/angle relationship
- If $\Omega > 1$, then curvature of universe will make angular size appear bigger
- If $\Omega < 1$, then curvature of universe will make angular size appear smaller



$\theta = \frac{s}{d}$ $\theta > \frac{s}{d}$ $\theta < \frac{s}{d}$

- Actual peak occurs around: $\theta \approx 0.81^\circ \sqrt{\Omega}$

$l \approx \frac{220}{\sqrt{\Omega}}$

- By measuring actual position of peak, we can estimate Ω
  - *This* is the primary source of the information that $\Omega = 1.00$

Matter Domination, After Recombination

- Photons completely decouple, they become completely smooth
- Any dark matter high density regions will gather baryons
  - These regions quickly become high density again
- These regions continue to grow more and more massive
- Throughout all these periods (up to now) these regions are continuing to grow with the rest of the universe in physical size

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Dark Matter
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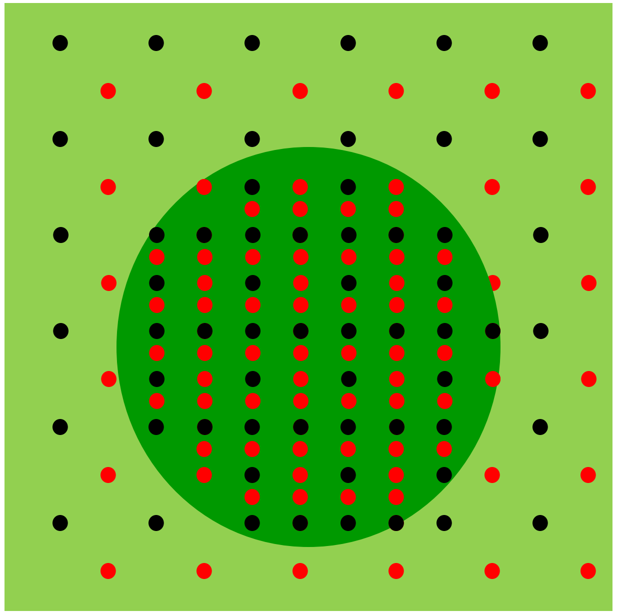


Outline of History of Universe

Time	T or $k_B T$	Events
10^{-43} s	10^{18} GeV	Planck Era; time becomes meaningless?
10^{-39} s	10^{16} GeV	Inflation begins; forces unified
10^{-35} s	10^{15} GeV	Inflation ends; reheating; forces separate; baryosynthesis (?)
10^{-13} s	1500 GeV	Supersymmetry breaking, LSP (dark matter)
10^{-11} s	160 GeV	Electroweak symmetry breaking
14 μs	150 MeV	Quark Confinement
0.4 s	1.5 MeV	Neutrino Decoupling
1.5 s	0.7 MeV	Neutron/Proton freezeout
20 s	170 keV	Electron/Positron annihilation
200 s	80 keV	Nucleosynthesis
57 ky	0.76 eV	Matter-Radiation equality
370 ky	0.26 eV	Recombination
600 My	30 K	First Structure/First Stars
13.8 Gy	2.725 K	Today

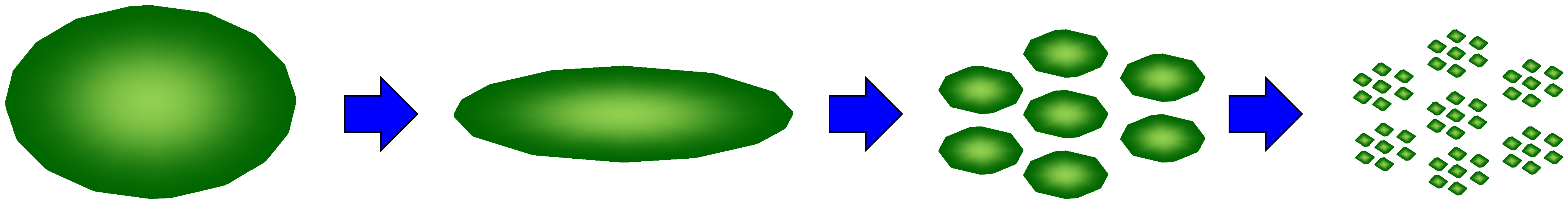
Structure Formation

- Up to now, the density fluctuations have been assumed small $\delta\rho/\rho \ll 1$
- As universe expands, can show in general that $\delta\rho \propto a^{-2}$
- But the density itself is falling as $\rho \propto a^{-3}$
- Therefore, $\delta\rho/\rho \propto a$

- Eventually, the perturbation becomes large (non-perturbative)
- To some extent, must rely on computer simulations

Qualitatively:

- Collapses in some direction first
- Then it fragments
- Fragments fragment further
- Form successively smaller structures



Which Structures Form First?

- For “scale invariant” density perturbations, all scales start about the same
  - But inflation implies smaller structures have a slight advantage
- During radiation era, smaller structures can grow a little
  - Another slight advantage for small structures

- Structures larger than the size of the universe *ct* at matter/radiation equality have a big disadvantage
  - This sets the largest scales at which structures form
  - This is a homework problem

- There is also a smallest structure that can form, set by the Jeans Mass
  - When pressure overcomes gravity
  - Homework problem

- Conclusion – smallest structures form first, but not by a lot
  - Globular Clusters/Dwarf Ellipticals – around $z = 8 – 12$
  - Galaxies/quasars/etc. – around $z = 5–8$
  - Galaxy Clusters – around $z = 2–5$
  - Galaxy Superclusters – around $z = 0–2$

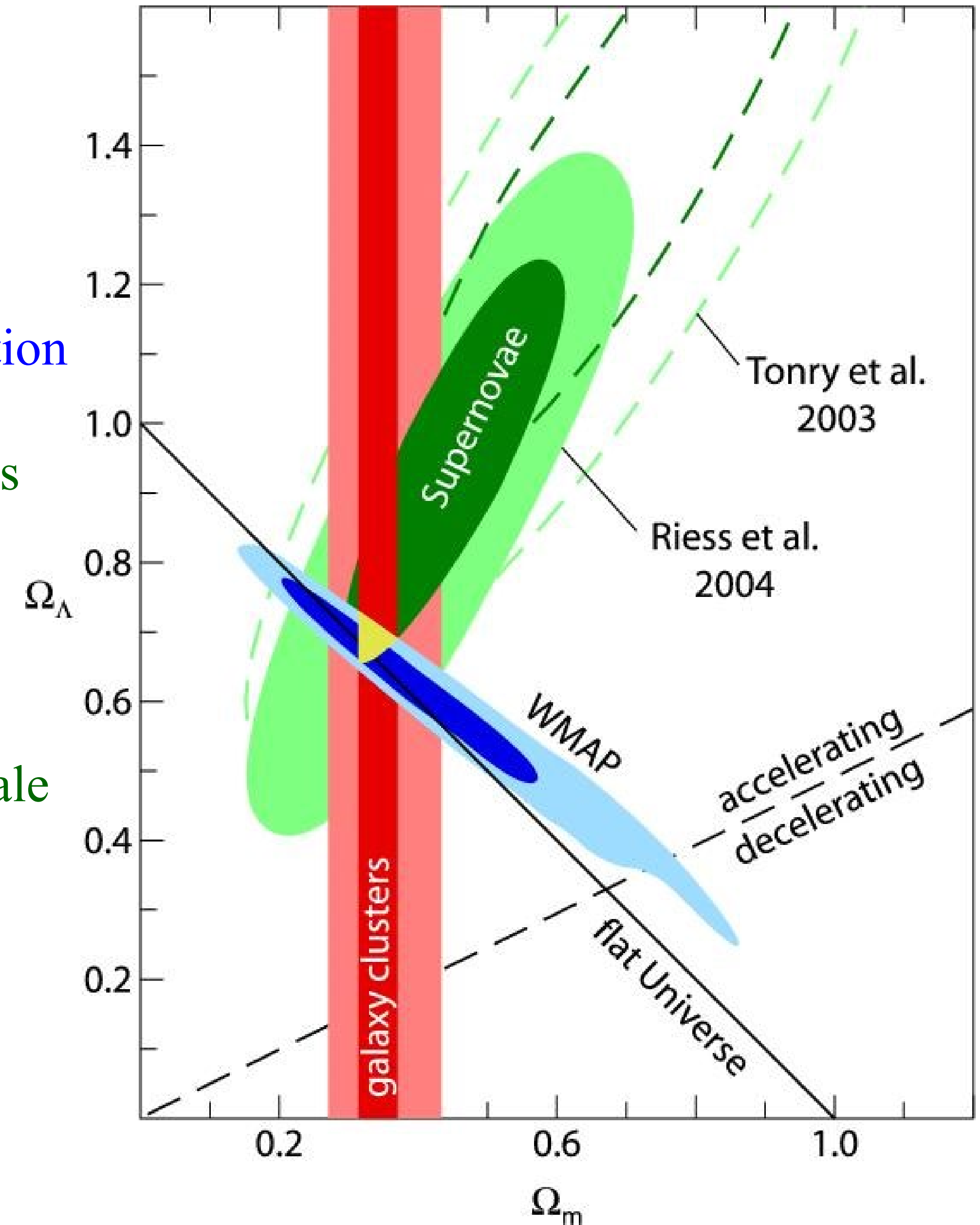
Cosmological Parameters (1)

- We have three independent ways of estimating cosmological parameters
  - Cosmic background
  - Structure in the current universe
  - Supernovae red shift
- Collectively, these give lots of independent information

- The fact that these all work *consistently* together tells us we are on the right track
  - And note that $\Omega = 1$
- To extract the *best* values, do a full fit to *everything* simultaneously
- Indications are that inflation (especially not quite scale invariant structure) yields best fit

- *Most* cosmologists now think inflation is right

$$n_s = 0.9665 \pm 0.0038$$



Cosmological Parameters (2)

Parameter	TT+lowE 68% limits	TE+lowE 68% limits	EE+lowE 68% limits	TT,TE,EE+lowE 68% limits	TT,TE,EE+lowE+lensing 68% limits	TT,TE,EE+lowE+lensing+BAO 68% limits
$\Omega_b h^2$	0.02212 ± 0.00022	0.02249 ± 0.00025	0.0240 ± 0.0012	0.02236 ± 0.00015	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1206 ± 0.0021	0.1177 ± 0.0020	0.1158 ± 0.0046	0.1202 ± 0.0014	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$	1.04077 ± 0.00047	1.04139 ± 0.00049	1.03999 ± 0.00089	1.04090 ± 0.00031	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0522 ± 0.0080	0.0496 ± 0.0085	0.0527 ± 0.0090	$0.0544^{+0.0070}_{-0.0081}$	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10} A_s)$	3.040 ± 0.016	$3.018^{+0.020}_{-0.018}$	3.052 ± 0.022	3.045 ± 0.016	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9626 ± 0.0057	0.967 ± 0.011	0.980 ± 0.015	0.9649 ± 0.0044	0.9649 ± 0.0042	0.9665 ± 0.0038
H_0 [km s^{-1} Mpc^{-1}] . .	66.88 ± 0.92	68.44 ± 0.91	69.9 ± 2.7	67.27 ± 0.60	67.36 ± 0.54	67.66 ± 0.42
Ω_Λ	0.679 ± 0.013	0.699 ± 0.012	$0.711^{+0.033}_{-0.026}$	0.6834 ± 0.0084	0.6847 ± 0.0073	0.6889 ± 0.0056
Ω_m	0.321 ± 0.013	0.301 ± 0.012	$0.289^{+0.026}_{-0.033}$	0.3166 ± 0.0084	0.3153 ± 0.0073	0.3111 ± 0.0056
$\Omega_m h^2$	0.1434 ± 0.0020	0.1408 ± 0.0019	$0.1404^{+0.0034}_{-0.0039}$	0.1432 ± 0.0013	0.1430 ± 0.0011	0.14240 ± 0.00087
$\Omega_m h^3$	0.09589 ± 0.00046	0.09635 ± 0.00051	$0.0981^{+0.0016}_{-0.0018}$	0.09633 ± 0.00029	0.09633 ± 0.00030	0.09635 ± 0.00030
σ_8	0.8118 ± 0.0089	0.793 ± 0.011	0.796 ± 0.018	0.8120 ± 0.0073	0.8111 ± 0.0060	0.8102 ± 0.0060
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$.	0.840 ± 0.024	0.794 ± 0.024	$0.781^{+0.052}_{-0.060}$	0.834 ± 0.016	0.832 ± 0.013	0.825 ± 0.011
$\sigma_8\Omega_m^{0.25}$	0.611 ± 0.012	0.587 ± 0.012	0.583 ± 0.027	0.6090 ± 0.0081	0.6078 ± 0.0064	0.6051 ± 0.0058
z_{re}	7.50 ± 0.82	$7.11^{+0.91}_{-0.75}$	$7.10^{+0.87}_{-0.73}$	7.68 ± 0.79	7.67 ± 0.73	7.82 ± 0.71
$10^9 A_s$	2.092 ± 0.034	2.045 ± 0.041	2.116 ± 0.047	$2.101^{+0.031}_{-0.034}$	2.100 ± 0.030	2.105 ± 0.030
$10^9 A_s e^{-2\tau}$	1.884 ± 0.014	1.851 ± 0.018	1.904 ± 0.024	1.884 ± 0.012	1.883 ± 0.011	1.881 ± 0.010
Age [Gyr]	13.830 ± 0.037	13.761 ± 0.038	$13.64^{+0.16}_{-0.14}$	13.800 ± 0.024	13.797 ± 0.023	13.787 ± 0.020
z_*	1090.30 ± 0.41	1089.57 ± 0.42	$1087.8^{+1.6}_{-1.7}$	1089.95 ± 0.27	1089.92 ± 0.25	1089.80 ± 0.21
r_* [Mpc]	144.46 ± 0.48	144.95 ± 0.48	144.29 ± 0.64	144.39 ± 0.30	144.43 ± 0.26	144.57 ± 0.22
$100\theta_*$	1.04097 ± 0.00046	1.04156 ± 0.00049	1.04001 ± 0.00086	1.04109 ± 0.00030	1.04110 ± 0.00031	1.04119 ± 0.00029
z_{drag}	1059.39 ± 0.46	1060.03 ± 0.54	1063.2 ± 2.4	1059.93 ± 0.30	1059.94 ± 0.30	1060.01 ± 0.29
r_{drag} [Mpc]	147.21 ± 0.48	147.59 ± 0.49	146.46 ± 0.70	147.05 ± 0.30	147.09 ± 0.26	147.21 ± 0.23
k_D [Mpc^{-1}]	0.14054 ± 0.00052	0.14043 ± 0.00057	0.1426 ± 0.0012	0.14090 ± 0.00032	0.14087 ± 0.00030	0.14078 ± 0.00028