December 15, 2023 Name ________________

Solutions to Final Exam PHY 310/610

This test consists of three parts. In parts II and III, PHY 310 students can skip one question of those offered, while PHY 610 students must answer all questions.

Part I: Multiple Choice (mixed new and review questions) [40 points] (2 points each)
PHY 310/610: For each question, choose the best answer

1. The James Webb space telescope is focused on the infrared, and will probably be the best for observing extremely distant galaxies because
   A) These galaxies are often shrouded in dust, which infrared light will penetrate
   B) The stars in these galaxies are so cold they only emit infrared light
   C) We are unable to see stars at this distance, but we can detect H-I regions in infrared
   D) These galaxies are so red-shifted that their light will have shifted from visible to infrared
   E) We hope to see planets in these galaxies, and planets produce infrared light, not visible

2. Which force is responsible for holding the quarks together inside the protons and electrons?
   A) Weak **B) Strong** C) Gravity D) Electromagnetism E) Rubber bands

3. Which is believed to be the earliest stage in the evolution of the Universe?
   A) Planck era
   B) Nucelosynthesis
   C) Recombination
   D) Electron-positron annihilation
   E) Electroweak scale

4. The particle that carries the electromagnetic force is called the
   A) W boson B) Z boson C) Gluon **D) Photon** E) Graviton

5. Which process keeps neutrinos at the same temperature as everything else in the current universe?
   A) Collisions with photons
   B) Collisions with electrons
   C) Collisions with protons and neutrons
   D) The interactions with the Higgs field
   E) Nothing; they are not in thermal equilibrium with other particles

6. Our galaxy is probably of which classification?
   A) Elliptical B) Irregular C) Lenticular D) Spiral **E) Barred spiral**

7. The brightest main sequence stars have which spectral classification?
   A) A B) B C) F D) G **E) O**

8. At the center of our galaxy is a giant
   A) Black hole B) Star C) Supernova D) Gas cloud E) Neutron star

9. The proton has baryon number +1, spin ½, and charge +1. The anti-proton has baryon number ___ spin ___ and charge ___
   A) –1, –½, –1 **B) –1, ½, –1** C) +1, –½, –1 D) –1, ½, +1 E) –1, –½, +1

10. Which of the following quarks might be found in a proton or neutron?
    A) Top B) Bottom C) Charm D) Strange **E) Down**

11. The name of the galaxy cluster or group we live in is the
    A) Andromeda **B) Local group** C) Laniakeia D) Coma E) Virgo

12. The largest galaxies are which classification?
    A) Elliptical B) Irregular C) Lenticular D) Spiral E) Barred spiral

13. Which of the following was not one of the scenarios discussed as the reason why there is more matter than anti-matter in the universe?
    A) GUT scale baryon violation
    B) Supersymmetry combined with sphaleron conversions
    C) Neutrino effects combined with sphaleron conversions
    D) Left over baryons from the previous cycle of the universe
    E) Actually, all of these were discussed

14. In the distant future, what is believed will be the largest contribution to the energy density of the universe?
    A) Baryons B) Dark matter **C) Dark energy** D) Radiation E) None of these

15. If we knew both the angular size α (in radians) and the physical size s of an object, how could we find the distance d?
    A) $d = \alpha s$ B) $d = \frac{\alpha}{s}$ **C)** $d = \frac{s}{\alpha}$ D) $d = \frac{1}{\alpha s}$ E) None of these

16. For small red-shifts, the relationship between radial velocity and red-shift is
    A) $\frac{v_r}{c} = z+1$ B) $v_r = \frac{z+1}{c}$ C) $\frac{v_r}{c} = z-1$ D) $v_r = \frac{z-1}{c}$ **E)** $\frac{v_r}{c} = z$

17. Which of the following is a reason it might make sense to believe in multiple universes?
    A) In chaotic inflation, a "universe" appears as a bubble, and it could happen again (only)
    B) If the universe could be created from nothing, it could happen again (only)
    C) In the many worlds interpretation of quantum mechanics, the universe can "split" into many different alternative realities that coexist simultaneously (only)
    D) All of the above were discussed
    E) None of the above were discussed

18. According to our best estimate, the current universe is about ___ billion years old
    A) 12.8 **B) 13.8** C) 14.8 D) 15.8 E) 16.8

19. Inflation explains all of the following except
    A) Why the universe is so flat
    B) Why the universe is so similar everywhere (isotropic and homogeneous)
    C) Why there are more baryons than anti-baryons in the universe
    D) What the origin of the small fluctuations in the universe is
    E) Actually, it accounts for all of these

20. Photons are bosons with spin $g = 2$. But for some reason, I kept using $g_{\text{eff}} = 3.36$ when discussing recent events in the universe. What other particle is contributing to g_{eff}?
    A) Neutrinos B) Electrons C) Protons D) Gluons E) Higgs field

Part II Short Answer [40 points/50 points] (10 points each)
PHY 310: Choose **four** of the following five questions and give a short answer (1-3 sentences)
PHY 610: Answer **all five** questions

21. Qualitatively, list at least three aspects of how the Sun moves around our galaxy. For example, your answer might be, "It moves approximately in a square, but it goes a little faster on the top and bottom sides, and the corners are a little bit rounded, and it's tilted slightly compared to the galactic plane."

The main motion of the Sun is a circular motion, at approximately constant speed, moving around in the plane of the galaxy. However, it oscillates in and out a little bit, so it isn't quite a circle (epicycles), which also causes it to speed up (when closer than average) and slow down (when farther) a little bit. Finally, it bobs up and down compared to the plane of the galaxy.

22. List the following five ingredients in order of contribution to the current mass/energy density of the universe, from largest contribution to smallest:

ordinary matter, dark energy, dark matter, electromagnetic radiation, neutrinos

The correct order is:

1. Dark energy
2. Dark matter
3. Ordinary matter
4. Neutrinos
5. Electromagnetic radiation

23. Primordial nucleosynthesis is the buildup of simple nuclei from protons and neutrons. Why didn't this process occur much earlier, say just after quark confinement? Why weren't all the protons and neutrons incorporated into helium? Why didn't it go all the way to making heavy elements, ultimately ending in iron?

The temperatures in the early universe were so hot that if any proton combined with a neutron to make deuterium, it would immediately get blasted apart by the high energy photons. Once it got cool enough, the protons and neutrons *did* join to make deuterium, and through further processes then got incorporated into ^{4}He, but since there were more protons than neutrons, the neutrons were used up (plus there were a few stray other isotopes that couldn't find anything to pair up with). The process stopped at helium because the next step requires the combination of three ^{4}He nuclei to make a ^{12}C nucleus, and the densities were too low for them to find each other in this short amount of time.

24. Suppose you are an up quark that existed from the beginning of time and you are lucky enough to survive all the way to the present day as helium. For each of the events listed below, tell me what you would become part of. Be careful to distinguish between nuclei and atoms as necessary. Some entries might have more than one correct answer; simply give one of the options.

Event	Is part of
Beginning	Up quark
Quark confinement	Proton or Neutron
Nucleosynthesis	^{4}H nucleus (or ^{3}He)
Recombination	Helium atom
Structure formation	Galaxy (or star, or cloud)

Quark confinement caused the quarks to be assembled into baryons (typically protons and neutrons) and mesons, but the ones that are going to survive to today are in baryons. In nucleosynthesis, they get combined into nuclei, so if it is going to become helium, it is probably a ^{4}He nucleus. At the time of recombination (or slightly before, for helium), it gains electrons and becomes an atom. The helium, like the rest of the atoms, ultimately form part of the structure, namely, galaxies, or on a smaller scale stars.

25. List the following five events in the future in the correct order from first to last:

black holes evaporate, death of the Sun, killer asteroid, last stars born, matter decays

The correct order is

1. Killer asteroid
2. Death of the Sun
3. Last stars born
4. Matter decays
5. Black holes evaporate

Units and Constants	Physical Constants	Age of Universe	Distance / Magnitudes
$\text{pc} = 3.086\times10^{16}\ \text{m}$	$k_B = 8.617\times10^{-5}\ \text{eV/K}$	Matter	$d = 10^{1+\frac{m-M}{5}}\ \text{pc}$
$\text{eV} = 1.602\times10^{-19}\ \text{J}$	$k_B = 1.381\times10^{-23}\ \text{J/K}$	$t = \dfrac{17.3\ \text{Gyr}}{(z+1)^{3/2}}$	$m - M = 5\log(d) - 5$
$M_\odot = 1.989\times10^{30}\ \text{kg}$	$\hbar = 6.582\times10^{-16}\ \text{eV}\cdot\text{s}$	Radiation	**Quark Charges**
$\text{y} = 3.156\times10^{7}\ \text{s}$	$\hbar = 1.055\times10^{-34}\ \text{J}\cdot\text{s}$	$t = \dfrac{2.42\ \text{s}}{\sqrt{g_{\text{eff}}}}\left(\dfrac{\text{MeV}}{k_B T}\right)^2$	Up: $\frac{2}{3}$, Down: $-\frac{1}{3}$
$G = 6.674\times10^{-11}\ \text{m}^3/\text{kg/s}^2$	$\hbar c = 1.973\times10^{-7}\ \text{eV}\cdot\text{m}$	**Temperature**	Strange: $-\frac{1}{3}$
		$T_0 = 2.725\ \text{K}$	

Part III: Calculation [100/120 points] (20 each)
PHY 310: Answer **five** of the following six problems
PHY 610: Answer **all six** of the following problems

26. The muon is a spin ½ particle with four total spin states with a mass of 105.7 MeV/c^2.

(a) What was the approximate temperature k_BT at the time when the muon disappeared? Would this have been in the matter or radiation dominated era?

The approximate temperature is when the thermal energy $3k_BT$ matches the rest energy of the muon, so $3k_BT = m_\mu c^2 = 105.7\ \text{MeV}$, so $k_BT = 35.2\ \text{MeV}$. This is much higher than 1 eV, so it's in the radiation dominated era.

(b) The other particles around at this time were the electrons (spin ½, 4 spin states), the neutrinos (spin ½, 6 spin states) and the photons (spin 1, 2 spin states). What was the value of g_{eff} if we also include the muon?

The spin ½ particles are all fermions. Including the muons, we would have $g_F = 4+4+6$ while the bosons would have $g_B = 2$, so

$$g_{eff} = g_B + \tfrac{7}{8} g_F = 2 + \tfrac{7}{8}\cdot 14 = 14.25$$

(c) How old was the universe at this time? The muon decays at a rate $\Gamma = 4.55\times10^5\ \text{s}^{-1}$. Would this decay have kept the muon in thermal equilibrium?

We simply use the time formula given above, so

$$t = \frac{2.42\ \text{s}}{\sqrt{g_{\text{eff}}}}\left(\frac{\text{MeV}}{k_BT}\right)^2 = \frac{2.42\ \text{s}}{\sqrt{14.25}}\left(\frac{1}{35.2}\right)^2 = 5.16\times10^{-4}\ \text{s}.$$

To determine if the muon is in equilibrium, we look at the combination Γt, which yiels

$$\Gamma t = \left(5.16\times10^{-4}\ \text{s}\right)\left(4.55\times10^{5}\ \text{s}^{-1}\right) = 235.$$

This is significantly larger than one, so it should keep it in equilibrium. In fact, there are much faster processes that make it even more in equilibrium.

27. You might think the universe is transparent to *all* photons, but this is not necessarily the case. A pair of photons colliding head-on can create an electron/positron pair, $\gamma\gamma \to e^+e^-$, if the energy of the two photons satisfies the inequality $E_1E_2 > \left(m_ec^2\right)^2$, where $m_ec^2 = 0.511\text{ MeV}$.

(a) The universe has a thermal bath of photons at temperature $T = 2.725$ K and a number density of $n = 4.11\times10^8\text{ m}^{-3}$. What is the typical energy of a CMBR photon in eV?

The typical energy of a particle in thermal equilibrium is $3k_BT$, which in this case is

$$3k_BT = 3\left(8.617\times10^{-5}\text{ eV/K}\right)\left(2.725\text{ K}\right) = 7.04\times10^{-4}\text{ eV}.$$

(b) What is the minimum energy E_2 such that a photon of this energy can pair create by hitting a cosmic ray photon?

Since this high-energy photon is hitting a CMBR photon, we use $E_1 = 7.04\times10^{-4}\text{ eV}$ and solve for E_2, so we have

$$E_2 > \frac{\left(m_ec^2\right)^2}{E_1} = \frac{\left(5.11\times10^5\text{ eV}\right)^2}{7.04\times10^{-4}\text{ eV}} = 3.71\times10^{14}\text{ eV} = 3.71\times10^5\text{ GeV} = 371\text{ TeV}.$$

(c) For the purpose of this problem, assume the cross-section is $\sigma = 7.94\times10^{-30}\text{ m}^2$. What is the rate at which a high energy photon will collide with a background photon, in s^{-1}?

The actual formula for the cross-section is complicated, but using the value given, the rate is $\Gamma = n\sigma\left(\Delta v\right)$. Since the two particles are described as colliding head-on, and they are both moving at the speed of light, we let $\Delta v = 2c$, so we have

$$\Gamma = n\sigma\left(\Delta v\right) = 2n\sigma c = 2\left(4.11\times10^8\text{ m}^{-3}\right)\left(7.94\times10^{30}\text{ m}^2\right)\left(2.998\times10^8\text{ m/s}\right) = 1.96\times10^{-12}\text{ s}^{-1}.$$

(d) On what time scale *t* in years will such a photon undergo on average one collision? Compare with the time it takes photons to cross our galaxy, about 10^5 y.

The number of collisions is Γt. Since we want $\Gamma t = 1$, we have

$$t = \frac{1}{\Gamma} = \frac{1}{1.957\times10^{-12}\text{ s}^{-1}} = \left(5.11\times10^{11}\text{ s}\right)\cdot\frac{\text{yr}}{3.156\times10^7\text{ s}} = 1.62\times10^4\text{ yr}.$$

It is therefore expected that these extremely high energy photons must originate within our galaxy, since the universe becomes opaque beyond this distance.

28. I don't know about you, but I like temperatures around $T = 295$ K. What was the universe like when it was at this temperature?

(a) Is this during the radiation or matter dominated eras? What was the value of the red-shift z at this time? What was the average energy of a photon, in eV?

This during the matter-dominated era, because as we will see, the temperature is well below an eV. The red shift can be found from

$$1+z = \frac{T}{T_0} = \frac{295\text{ K}}{2.725\text{ K}} = 108.3,$$

so we have roughly $z = 107$. The average energy of a photon is $3k_BT$, or

$$3k_BT = 3\left(8.617\times10^{-5}\text{ eV/K}\right)\left(295\text{ K}\right) = 0.0763\text{ eV}.$$

(b) How old was the universe at this time?

We simply use the given relationship for age in the matter dominated era, so

$$t = \frac{17.3\text{ Gyr}}{\left(z+1\right)^{3/2}} = \frac{17.3\text{ Gyr}}{\left(108.3\right)^{3/2}} = 0.0153\text{ Gyr} = 15.3\text{ Myr}.$$

This is well after the radiation-dominated era, and well before the formation of first structure.

(c) At present there are about 0.250 baryons/m^3. What was the density of baryons then? Assume each baryon corresponds to a hydrogen atom with mass 1.674×10^{-27} kg. What was the mass density of baryons at that time?

The universe has expanded by a factor of 1+z since then, and therefore the volume has increased by $\left(1+z\right)^3$. This means that the number density of particles has *decreased* by this factor since then, so to get the density then, we multiply by this factor. We have

$$n = n_0\left(1+z\right)^3 = 0.25\left(108.3\right)^3 = 3.18\times10^5\text{ m}^{-3}.$$

We then multiply this by the mass of the hydrogen atoms to give

$$\rho_b = mn = \left(1.674\times10^{-27}\text{ kg}\right)\left(3.18\times10^5\text{ m}^{-3}\right) = 5.32\times10^{-22}\text{ kg/m}^3.$$

29. One of the isotopes that is produced in primordial nucleosynthesis is ^{7}Be. In the laboratory, ^{7}Be decays to ^{7}Li with a mean lifetime of 76.78 days (day = 86,400 s).

(a) Is 76.78 days after the era of nucleosynthesis? When the universe was 76.78 days old, was this in the matter or radiation dominated era?

Nucleosynthesis happened in the first couple hundreds of seconds, so it is well after the era of nucleosynthesis. The matter/radiation equality happened many thousands of years after the start, so we are definitely in the radiation dominated era.

(b) What is the temperature k_BT at this time in eV? If you need it, use $g_{eff} = 3.36$.

We first convert the time to seconds, so

$$t = (76.78 \text{ d})(86,400 \text{ s/d}) = 6.634 \times 10^6 \text{ s}$$

We then substitute this into the age of the universe formula to give

$$t = \frac{2.42 \text{ s}}{\sqrt{g_{eff}}}\left(\frac{\text{MeV}}{k_BT}\right)^2,$$

$$\left(\frac{k_BT}{\text{MeV}}\right)^2 = \frac{2.42\text{s}}{t\sqrt{g_{eff}}} = \frac{2.42 \text{ s}}{(6.634 \times 10^6 \text{ s})\sqrt{3.36}} = 1.99 \times 10^{-7},$$

$$k_BT = \sqrt{1.99 \times 10^{-7}} \text{ MeV} = 4.46 \times 10^{-3}\text{MeV} = 446 \text{ eV}.$$

(c) The binding energy of a single electron in a beryllium atom is about 218 eV. Would a typical photon have enough energy to dissociate a beryllium atom by removing the electron?

A typical photon would have an energy of about three times this, or 1338 eV. This is easily enough energy to knock the electron out of beryllium. So in fact the beryllium atoms will all be completely ionized.

(d) In the laboratory, the decay occurs via electron capture, where a bound electron is captured by the nucleus, so $^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu$. Explain why it is likely that ^{7}Be lasts longer in the early universe than in the laboratory.

The process is greatly enhanced in the laboratory because there is an electron available. In the early universe, the electrons are much harder to find, until the beryllium recombines with electrons (which happens well before regular recombination), but this means the beryllium lasts until this time.

30. Several rich galaxy clusters are measured, and the apparent magnitude of the brightest galaxy is also measured, as shown in the table. In some cases the distance is known.

(a) For clusters *A*, *B*, and *C*, find the absolute magnitude of the brightest galaxy.

Clus-ter	d (Mpc)	m	M
A	35	10.22	–22.50
B	137	13.18	–22.50
C	1200	17.90	–22.49
D	6400	21.53	–22.50

We start with the formula $m - M = 5\log(d) - 5$, which we rearrange to $M = m - 5\log(d) + 5$. Remembering to convert all our distances to Mpc, we have

$$M_A = m_A - 5\log(d_A) + 5 = 10.22 - 5\log(35\times10^6) + 5 = -22.50,$$

$$M_B = m_B - 5\log(d_B) + 5 = 13.18 - 5\log(137\times10^6) + 5 = -22.50,$$

$$M_C = m_C - 5\log(d_C) + 5 = 17.90 - 5\log(1200\times10^6) + 5 = -22.49.$$

These numbers have been incorporated into the table.

(b) Is the brightest galaxy in a galaxy cluster a standard candle? Why or why not?

If this were reality (it's not), it would be a fantastic standard candle, since it works essentially perfectly, with an absolute magnitude essentially exactly at –22.50. I have assumed that this applies to the brightest galaxy in cluster *D* as well.

(c) Estimate the distance to cluster *D*.

We simply use the formula in reverse, and find

$$d_D = 10^{1+\frac{1}{5}(m_D - M_D)}\ \text{pc} = 10^{1+\frac{1}{5}(21.53+22.50)}\ \text{pc} = 6.40\times10^9\ \text{pc} = 6400\ \text{Mpc}.$$

This number has also been incorporated into the table.

31. The Ω^- is a baryon with mass 1672 MeV/c^2 and strangeness –3. Its most common decay is $\Omega^- \to \Lambda^0 K^?$, where Λ^0 and $K^?$ each have strangeness –1, the Λ^0 has a mass of 1116 MeV/c^2, and the $K^?$ is a meson.

(a) Is the Λ^0 a baryon, an anti-baryon, or a meson?

Since the Ω^- is a baryon, and baryon number is conserved, there must be one baryon on the right, and it isn't the *K*. So Λ^0 is a baryon.

(b) What is the charge of the $K^?$? What is its maximum possible mass?

The left side has charge –1, and since charge is conserved, so must the right side. Therefore the *K* must be negatively charged; it is the K^- meson, with charge –1.

Because we have a decay, the mass of the right side must be less than the mass on the left, so

$$m_\Omega > m_\Lambda + m_K ,$$

$$m_K < m_\Omega - m_\Lambda = \left(1672 \text{ MeV}/c^2\right) - \left(1116 \text{ MeV}/c^2\right) = 556 \text{ MeV}/c^2 .$$

(c) Is this interaction, strong, weak, or electromagnetic?

We have been assured this is the most common decay, so it is certainly possible. The fact that strangeness is not conserved is only allowed for weak interactions, so it must be weak.

(d) Only up, down, and strange quarks (or their anti-quarks) are found in these three particles. What is the quark composition of each of these particles?

The strange quark has strangeness minus one, so a negative strangeness tells you the number of strange quarks. The Omega is a baryon (three quarks), and with strangeness –3 , it must have three strange quarks, so its quark composition is [sss]. The Lambda is a baryon (three quarks), and with strangeness –1 it must have one strange quark. To get the charge to come out right, the other two must be one down and one up, so we have [uds] as its quark composition. Finally, the K^- is a meson (quark plus anti-quark), and with strangeness – 1 it must have one strange quark. To get the charge to work out, it must have an anti-up quark, so its composition is [$s\bar{u}$].