

Name _____

Midterm Exam
March 4, 2010

This test consists of three parts. For the first part, you may write your answers directly on the exam, if you wish. For the other parts, use separate sheets of paper. Useful equations can be found on the last page of this test; feel free to tear it off.

Part I: Multiple Choice [40 points]

For each question, choose the best answer (2 points each)

- Observations indicate that the Sun is currently moving vertically at about 7 km/s northwards, away from the galactic disk. What is the long term effect of this motion?
 - It will move to a positive z -value, and then orbit indefinitely at that z -value
 - It will oscillate up and down, but never straying very far from the disk**
 - It will follow a large orbit which takes it well out into the halo
 - It will settle back down into the plane of the galaxy over time; this is a temporary phenomenon
 - It will leave the galaxy in this direction
- Which of the following is *not* a typical characteristic of a thick disk star, compared to a thin disk star?
 - It will tend to have a larger radius, and hence is thicker**
 - It will tend to be older
 - It will have more random motions; *i.e.* less perfectly circular
 - It will tend to have lower metallicity
 - Actually, all of these *are* characteristics of thick disk stars
- Most of the mass of our galaxy is probably in the form of
 - Black holes in the nucleus
 - Stars, gas, and dust in the disk
 - Globular clusters out in the halo
 - Old stars in the bulge
 - Unknown dark matter spread throughout the halo**
- Although the bulge of our galaxy is obscured by dust, we are still confident it is there because
 - We can detect its gravitational contribution to the orbits of stars
 - Neutrinos from it are observed at Earth
 - We can see stars from it that have been expelled to our neighborhood
 - We can see it with infrared light, which penetrates the gas and dust**
 - We simply infer its presence since other spiral galaxies like ours have bulges

5. If the surface temperature of a star is doubled, but its radius remains the same, its luminosity would increase by a factor of
A) 2 B) 4 C) 8 **D) 16** E) 256
6. Spectroscopic parallax is a technique for measuring distance based on the idea that
A) For main sequence stars, the spectral class more or less tells you the luminosity (absolute magnitude) , and combined with the apparent magnitude you can get the distance
B) The spectrum of a star tells you its velocity; combined with other information, this gives you the distance
C) There is a relationship between the spectrum of a variable star and its period
D) The spectrum of a star tells you its approximate age; this can be combined with how long it took light to get here to get the actual distance
E) The apparent angle of the spectrum as viewed from different points in Earth's orbit tells you the distance to the star
7. Suppose one had a catalog of the apparent magnitude of all the brightest planetary nebulas in a nearby galaxy. What *additional* information would we need to find the distance to this galaxy? Assume there is no dust obscuring the galaxy.
A) The period of at least one Cepheid variable star
B) The parallax to at least one of the planetary nebulas in that galaxy
C) The physical size of at least one of the planetary nebulas
D) The spectral class of one or more planetary nebulas
E) None of the above; you already have sufficient information to find the distance
8. List the types of electromagnetic radiation below in order from longest wavelength to shortest
A) X-rays, infrared, visible, ultraviolet, radio
B) Radio, ultraviolet, visible, infrared, X-rays
C) X-rays, ultraviolet, visible, infrared, radio
D) Radio, infrared, visible, ultraviolet, X-rays
E) None of the above are correct
9. Which of the following is most likely to be found very close to the galactic plane?
A) Thin disk stars
B) Halo stars
C) Cool molecular clouds and star forming regions
D) Hot thin hydrogen clouds
E) Globular clusters
10. The oldest stars can be found in the
A) Thin disk B) Thick disk C) Bulge **D) Globular Clusters** E) Nucleus

11. In the moving cluster method, how does one determine the actual direction that a cluster of stars is moving?
- A) One calculates the transverse and radial velocities, then you project the resulting vector
 - B) The angular motion of all the stars in the cluster are extended to the vanishing point, or convergent point, where they all come together; this is the direction of motion**
 - C) Differential Doppler shifts on different stars throughout the cluster are used to determine the direction of overall motion
 - D) The ages of the stars is estimated, and based on the location of the molecular cloud from which it was born, one can then determine the direction of motion
 - E) The speed of the stars in the cluster is compared with the mass and size, and the Virial theorem then tells you the direction of motion
12. The distance to the center of our galaxy is probably closest to
- A) 80 pc
 - B) 800 pc
 - C) 8 Mpc
 - D) 80 Mpc
 - E) 8 kpc**
13. The abundance of which of the following elements would be included when computing the metallicity of a star?
- A) Iron, but not neon
 - B) Iron and neon, but not carbon
 - C) Iron, neon, and carbon, but not helium nor hydrogen**
 - D) Iron, neon, carbon, and helium, but not hydrogen
 - E) Iron, neon, carbon, and hydrogen, but not helium
14. The hottest spectral type of those listed is
- A) A
 - B) B**
 - C) K
 - D) M
 - E) F
15. The O and B main sequence stars are the first to die because
- A) They have the least mass, and therefore run out of fuel the fastest
 - B) They burn their fuel the fastest, and therefore run out of fuel the fastest**
 - C) They are the coolest, and therefore they soon lose temperatures sufficient to sustain fusion
 - D) They have the most mass, and attract more mass making them unstable
 - E) The question is false; these stars live the longest, not the shortest
16. Which of the following is probably closest to the speed of the Sun around the center of the galaxy?
- A) 200 km/s**
 - B) 600 km/s
 - C) 2,000 km/s
 - D) 6,000 km/s
 - E) 20,000 km/s
17. The mass of the black hole at the center of our galaxy was estimated by
- A) Observing the Schwarzschild radius
 - B) Sending a probe and measuring the gravitational pull
 - C) Studying the orbit of some stars that were very close to it**
 - D) Measuring the Doppler shift of gas falling into it
 - E) Studying the bending of light from objects behind it

18. In galaxies like ours, how does the approximate rotational velocity depend on distance as you move away from the center of the galaxy?
- A) At large radii, it falls off as about $1/\sqrt{R}$
 - B) At large radii, it falls off as about $1/R$
 - C) At large radii, it rises as about \sqrt{R}
 - D) At large radii, it rises as about R
 - E) **It is roughly constant at large radii**
19. The disk of our galaxy is in some ways more difficult to study than that of other galaxies, primarily because
- A) Our galaxy's disk is very dim, making it difficult to see
 - B) The motion of our disk is exceptionally chaotic, making patterns hard to spot
 - C) **We are in the plane of our galaxy, where the dust blocks our view of much of the disk**
 - D) The stars in our galaxy are so close they overexpose in most telescopes
 - E) The stars are so close that we can't use the usual assumption that the distance to all the stars in the galaxy is identical
20. The spiral arms of a galaxy, according to current theory,
- A) **Are self-propagating waves of higher density**
 - B) Contains a fixed set of stars and gas that orbits together
 - C) Is actually a magnetic phenomenon, connected by magnetic field lines
 - D) Represents a series of exploding stars, which set off formation of new stars in a spiral pattern
 - E) Is actually a giant pinwheel toy made by ultrapowerful aliens for their amusement

Part II: Short essays [60 points]

Write approximately a paragraph or two explaining the following (15 points each)

- 21. Explain what a Cepheid Variable star is, and how it can be used to determine the distance to an object containing one. Include all appropriate equations, explaining how they are relevant.**

A Cepheid Variable is a variety of giant star which is luminous and pulsates with a regular period. There is a relationship between the rate at which it pulsates and the luminosity, called the *period-luminosity* relationship. It is given among the formulas below:

$$M = -2.81 \log(P) - 1.43$$

where the period P is in days and M is the absolute magnitude. If we then measure the apparent magnitude m of the star, we can then get the distance using the standard distance formula

$$d = 10^{1 + \frac{m-M}{5}}$$

where the distance d comes out in parsecs. Cepheids are useful distance indicators because they are bright, consistent, and not so rare as to be hard to find.

22. Two clusters have identical metallicity, but one is much older than the other. Explain how one might observationally determine which cluster is older.

As clusters age, the main sequence stars will cease being main sequence stars, going on to be giant stages. If we plot the Hertzsprung-Russell diagram for a star cluster, many of the stars will follow the diagonal path of the main sequence, but towards the top the stars will have “turned off” of the main sequence. The O and B stars are the first to turn off, then the cooler A, F, and eventually G stars (K and M stars are never old enough to have left the main sequence). The turn off point, the place where the stars leave the main sequence, gives you an estimate of the age.

As the young, hot, bluish stars leave the main sequence, the cluster will evolve in color, first from a blueish tinge (representing the hottest O and B stars) and then turning more neutral and ending up with a red tinge (when only the cooler main sequence and a few giant stars dominate the light). Hence the overall color can also give some indication of age.

23. Most of the stars in our neighborhood are members of the thin or thick disk, but a very small number are halo stars. Explain how these stars might noticeably differ from ordinary stars in our neighborhood. Give at least two ways they generally differ.

Halo stars have a variety of differences. Most noticeable is their metallicity, which can be one percent or even less of the solar abundance. Halo stars also move very far from the plane of the galaxy. Nearby halo stars, of course, are quite close to the plane, but they will likely have considerable vertical motion. They also don't move particularly close to circular orbits. Hence their velocity relative to the Sun will commonly have significant backwards motion, and probably have a lot of radial motion as well. Finally, Halo stars are always old.

24. What is the source of the 21 cm radio line? What type of gas does it come from? Why is it important in understanding the rotation of the galaxy?

The 21 cm line comes from neutral hydrogen atoms. It is produced when the electron in a hydrogen atom flips its spin compared to the spin of the proton. The magnetic energy released comes out in the form of electromagnetic radiation.

Unlike visible light, the radio waves at 21 cm can penetrate the gas and dust in the plane of our galaxy. We can peer through this murk using this radio wave, and see the neutral hydrogen atoms circling the galaxy. By measuring Doppler shift, we can work out the relative rotation rate at various radii compared to our own. The recessional velocity compared to us is given by

$$v_r = R_0 \sin l \left(\frac{V}{R} - \frac{V_0}{R_0} \right)$$

where V and R are the rotational speed and radius of the observed cloud, V_0 and R_0 are the same thing for us, and l is the galactic longitude, the angle relative to the direction of the center of the galaxy.

Part III: Calculation: [100 points]

For each of the following problems, give the answer, explaining your work. The value of each portion appears in square brackets. Some possibly useful equations appear below. [20 points each]

Units	Physical Constant	Gravitational Potential Energy	
1 AU = 1.496×10^{11} m 1 pc = 3.086×10^{16} m 1 eV = 1.602×10^{-19} J 1 y = 3.155×10^7 s $R_{\odot} = 6.955 \times 10^8$ m $M_{\odot} = 1.989 \times 10^{30}$ kg $L_{\odot} = 3.839 \times 10^{26}$ W $T_{\odot} = 5777$ K	$k_B = 1.381 \times 10^{-23}$ J/K $k_B = 8.671 \times 10^{-5}$ eV/K $\sigma = 5.670 \times 10^{-8}$ W/m ² /K ⁴ $\hbar = 1.055 \times 10^{-34}$ J·s $\hbar = 6.582 \times 10^{-16}$ eV·s $G = 6.673 \times 10^{-11}$ m ³ /kg/s ²	$E_p = -\frac{1}{2} \int d^3\mathbf{r} \int d^3\mathbf{r}' \frac{G\rho(\mathbf{r})\rho(\mathbf{r}')}{ \mathbf{r}-\mathbf{r}' }$ $E_p = \frac{1}{2} \int d^3\mathbf{r} \rho(\mathbf{r}) \Phi(\mathbf{r})$	
		Black Hole	Relative speed, Circular Orbits
		$R_s = \frac{2GM}{c^2}$	$v_r = R_0 \sin l \left(\frac{V}{R} - \frac{V_0}{R_0} \right)$
	Black Body Radiation		Doppler Shift
Brightness/Magnitude	$u = \frac{\pi^2 (k_B T)^4}{15 (\hbar c)^3}$ $\lambda_{\max} T = 2.8978 \times 10^{-3}$ m·K		$1+z = \frac{\lambda}{\lambda_0} = \sqrt{\frac{1+v_r/c}{1-v_r/c}}$
$b = 2.52 \times 10^{-8}$ W/m ² ($10^{-\frac{2}{5}m}$)			
Moving Cluster	Planetary Nebula	Type Ia SN	Cepheid Period/Luminosity
$d = \frac{v_r \tan \theta}{\mu}$	$M^* = -4.47$	$M_{\max} = -19.3$	$M = -2.81 \log(P) - 1.43$

25. A certain galaxy has an unusual distribution of matter. The density of matter takes the form

$$\rho(r) = \frac{A}{r}$$

at a distance r from the center of the galaxy.

(a) [6] What is the total amount of mass contained within a radius R from the center of the galaxy?

We divide the space into thin spherical shells, each having thickness dr and area $4\pi r^2$. We then multiply the volume by the density, to yield

$$M(R) = \int_0^R \frac{A}{r} 4\pi r^2 dr = 4\pi A \int_0^R r dr = 2\pi A r^2$$

(b)[8] What is the magnitude of the gravitational acceleration g as a function of R ? As demonstrated with the help of Gauss's law, the gravitational acceleration from a spherically symmetric mass distribution is simply given by

$$\mathbf{g}(R) = -\frac{GM(R)}{R^2} \hat{\mathbf{r}} = -\frac{2\pi GAr^2}{r^2} \hat{\mathbf{r}} = -2\pi GA \hat{\mathbf{r}}$$

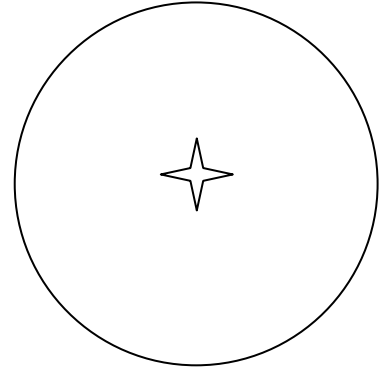
(c) [6] What is the velocity V for circular orbits as a function of R ?

The centripetal acceleration, given by $a = V^2/R$, must match the gravitational acceleration, so

$$V^2 = gR = 2\pi GAR,$$

$$V = \sqrt{2\pi GAR}.$$

26. A distant star is a recurrent nova. By sheer luck, there is a circular ring of gas surrounding the star, as sketched at right, exactly perpendicular to our line of sight, with 1 cm representing 1 arc-second. Exactly 30 days after the nova goes off, the ring is suddenly illuminated. (1 d = 86,400 s)
 (a) [8] What is the radius of the ring, in AU?



The light that from the ring first went to the ring, then to our eyes. The distance of the ring and the star are equal, and so the time delay for the light to reach us is just the time it took to get to the ring. The radius therefore is the speed of light c times the time we had to wait, or

$$r = ct = (2.998 \times 10^8 \text{ m/s})(30 \text{ d}) \frac{86400 \text{ s}}{\text{d}} \frac{\text{AU}}{1.496 \times 10^{11} \text{ m}} = 5200 \text{ AU}$$

(b) [12] What is the distance to the nova/ring, in pc?

The ring as drawn is 4.8 cm wide, or 2.4 cm in radius, so this means the angular size is 2.4". The distance is related to the apparent size by $R = d\theta$, so we have

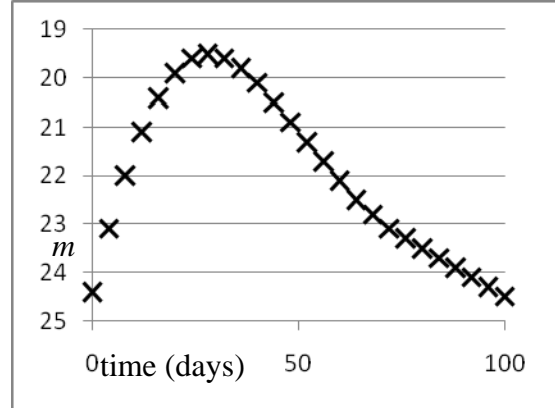
$$d = \frac{R}{\theta} = \frac{5200 \text{ AU}}{2.4''} \cdot \frac{1 \text{ pc} \cdot 1''}{1 \text{ AU} \cdot 1 \text{ rad}} = 2200 \text{ pc}$$

This places it well within the galaxy.

27. A type Ia supernova goes off in a distant galaxy! The apparent magnitude as a function of time is sketched at right.

(a) [5] What is the approximate distance to this galaxy?

The brightest point for this supernova occurs at about $m = 19.5$. But according to our simplified formulas, it should have an absolute magnitude of about $M_{\max} = -19.3$. We therefore use the distance formula to find



$$d = 10^{1 + \frac{m-M}{5}} = 10^{1 + (19.5 + 19.3)/5} = 5.75 \times 10^8 \text{ pc} = 575 \text{ Mpc}$$

(b) [7] With a good telescope, one can observe planetary nebulas that are as dim as about $m = 30$. Would planetary nebulas be a good way to check the distance to this galaxy? Why or why not?

We need to figure out how dim a planetary nebula would be at this distance. Using an alternate version of the distance formula, we have

$$m - M = 5 \log d - 5 = 5 \log (5.75 \times 10^8) - 5 = 38.8$$

Actually, we could have gotten this more quickly by noting that this combination will be identical for the supernova and for the planetary nebulae. The planetary nebulae have absolute magnitudes no brighter than about $M = -4.47$, so the brightest apparent magnitude they can have is about $m = M + 38.8 = -4.47 + 38.8 = 34.3$. This is about four magnitudes too dim to use the planetary nebula method on.

(c) [8] A study of the spectrum from this galaxy shows that the Lyman-alpha line, normally at a wavelength of 121.57 nm, has been shifted to a wavelength of 153.28 nm. What is the speed of this galaxy, and is it towards us or away from us? You may leave your answer as a fraction of c if you wish.

We use the Doppler shift formula and solve for the velocity:

$$\begin{aligned} \sqrt{\frac{1 + v_r/c}{1 - v_r/c}} &= \frac{\lambda}{\lambda_0} = \frac{153.28}{121.57} = 1.2608, \\ (1 + v_r/c) &= 1.2608^2 (1 - v_r/c) = 1.5897 (1 - v_r/c), \\ (v_r/c)(1 + 1.5897) &= 1.5897 - 1, \\ \frac{v_r}{c} &= \frac{0.5897}{2.5897} = 0.2277 \quad \text{or} \quad v_r = 6.83 \times 10^7 \text{ m/s} \end{aligned}$$

Since the result came out positive, it is moving away from us.

28. A cluster of stars has total mass M and radius R . The mass distribution is such that the gravitational potential energy is given by

$$E_p = -\frac{2GM^2}{R}$$

(a) [8] According to the Virial theorem, what is the kinetic energy of this cluster? What is the total energy of this cluster?

The Virial theorem says that the $E_p + 2E_K = 0$, so the kinetic energy is about

$$E_K = -\frac{1}{2}E_p = \frac{GM^2}{R}$$

and the total energy is $E_T = E_K + E_p = -GM^2/R$

(b) [8] Due to an amazing coincidence, half of the stars suddenly explode, causing them to completely expel their mass from the cluster. Hence the mass suddenly is reduced, so $M \rightarrow \frac{1}{2}M$. The speed and position of all remaining stars are unchanged. Immediately after the explosion, what is the new potential, kinetic, and total energy of the cluster?

The potential energy is easy to calculate. We simply substitute the new formula for the mass, and we find

$$E'_p = -\frac{2G\left(\frac{1}{2}M\right)^2}{R} = -\frac{GM^2}{2R}$$

However, the kinetic energy takes some thought. Since the process happens instantaneously, the system will no longer satisfy the Virial theorem. Now, the kinetic energy of each star is given by $E_i = \frac{1}{2}m_i v_i^2$, so if we simply eliminate some fraction of the mass, but keep the velocities the same, the kinetic energy will decrease proportional to the mass. Hence the new kinetic energy is down by a factor of $\frac{1}{2}$, and the resulting kinetic energy is

$$E'_K = \frac{1}{2}E_K = \frac{GM^2}{2R}$$

The new total energy is the sum of these two, which is $E'_T = E'_p + E'_K = 0$.

(c) [4] Does the cluster now have enough gravity to hold it together? Or is it too little energy? Or is it right at the boundary?

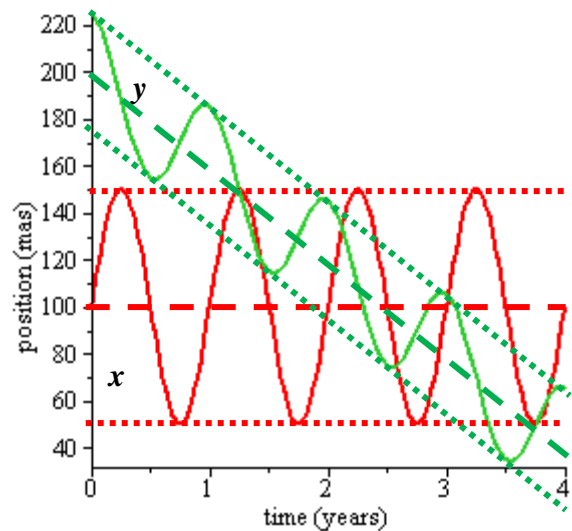
The cluster has enough energy to hold together *if* its total energy is negative; if it's positive, the cluster dissolves. We're right at the boundary, so it will spread out, slowly and gradually. Effectively, the cluster will dissolve.

29. A star at $\beta = 30^\circ$ is tracked over the course of four years. Plotted at right is the position of the star in mas, with x denoting the motion parallel to the plane of the Earth's orbit around the Sun, and y denoting the apparent motion perpendicular to that orbit.

(a) [7] Explain qualitatively the two types of motion seen; *i.e.*, what is causing the oscillatory motion, and what is causing the gradual drift?

The position of the star seems to oscillate, drifting back and forth over time, or more precisely, moving in an apparent oval. This does not represent real motion of the star, but rather motion of the Earth. As we orbit the Sun, the *apparent* direction of the star will vary cyclically.

The apparent position of the star *also* shows some real motion, at least in the y direction. This represents actual motion of the star, or at least, motion of the star relative to the Sun.



(b) [5] What is the angular velocity μ_x and μ_y for this star?

Ignoring the oscillatory motion, it is apparent that it is drifting downwards in the y -direction, though there is no obvious motion in the x -direction. I have drawn in some lines to guide the eye, and we estimate that the y -curve has dropped from 200 to 40 in four years, so we have

$$\mu_x = 0, \quad \mu_y = -40 \text{ mas/y}.$$

(c) [8] What is the approximate parallax p and distance d in parsecs for this star?

The parallax is the amount it moves in the x -direction due to the orbit of the Earth. Reading from the graph, it is moving from a position of 100 mas up and down to 150 mas and 50 mas respectively, for a parallax of about $p = 50 \text{ mas} = 0.050''$. We can also deduce it from the y -motion, but this motion is reduced by a factor of $\sin \beta = 0.50$. This motion appears to be from $y = 200 \text{ mas}$ up and down to 225 mas and 175 mas, which is a motion of 25 mas, but we need to divide by $\sin \beta = 0.50$ to get the parallax, which again is $p = 50 \text{ mas} = 0.050''$. So it looks like it's working out. We then find the distance from

$$d = \frac{1}{p} = \frac{1}{0.050''} = 20.0 \text{ pc}.$$