Name ___________________

Solutions to Midterm Exam
October 20, 2017

This test consists of three parts. For the first and second parts, you may write your answers directly on the exam, if you wish. For the other parts, use separate sheets of paper.

Units	Physical Constant	Black Body Radiation
$1\text{ AU} = 1.496\times10^{11}\text{ m}$	$k_B = 1.381\times10^{-23}\text{ J/K}$	$u = \frac{\pi^2}{15}\frac{(k_B T)^4}{(\hbar c)^3}$
$1\text{ pc} = 3.086\times10^{16}\text{ m}$	$k_B = 8.671\times10^{-5}\text{ eV/K}$	$\lambda_{max}T = 2.8978\times10^{-3}\text{ m}\cdot\text{K}$
$R_\odot = 6.955\times10^{8}\text{ m}$	$\sigma = 5.670\times10^{-8}\text{ W/m}^2\text{/K}^4$	**Distance and Magnitudes**
$M_\odot = 1.989\times10^{30}\text{ kg}$	$\hbar = 1.055\times10^{-34}\text{ J}\cdot\text{s}$	$d = 10^{1+\frac{m-M}{5}}$
$L_\odot = 3.839\times10^{26}\text{ W}$	$\hbar = 6.582\times10^{-16}\text{ eV}\cdot\text{s}$	$m - M = 5\log(d) - 5$
$T_\odot = 5777\text{ K}$	$G = 6.673\times10^{-11}\text{m}^3\text{/kg/s}^2$	

Cepheid Period/Luminosity	Type Ia SN	Planetary Nebula
$M = -2.81\log(P) - 1.43$	$M_{max} = -19.3$	$M^* = -4.47$

Part I: Multiple Choice Everyone: Answer all questions
For each question, choose the best answer (2 points each)

1. Which of the following does not typically occur when two galaxies collide or nearly collide?
   A) Individual stars collide with each other
   B) A burst of star formation occurs in the galaxies
   C) Tidal friction transfers kinetic energy into some of the internal motion of the galaxies
   D) Any gas gets heated and can be completely knocked out of the galaxy
   E) The galaxies can become irregular galaxies

2. The two types of stars which are young, hot, and blue, are
   A) A and B B) K and M **C) O and B** D) O and M E) F and G

3. When the supernova SN1987a blew up, a ring of gas around it lit up later on. Why was there this delay?
   A) It takes a long time for the light to heat up the gas in a ring and make it start glowing
   B) The expanding gas from the explosion had to collide with the ring of gas, which took time
   C) The light comes from radioactive decay, which takes a while
   D) The light had to travel the extra distance to reach the ring before traveling to Earth
   E) The ring was illuminated by neutrinos, and it took a long time for the neutrinos to get there

4. What is the approximate age of the oldest known stars?
   A) 23 Gyr **B) 13 Gyr** C) 10 Gyr D) 8 Gyr E) 5 Gyr

5. The spectral lines from quasars often are not sharp lines; instead they tend to be very broad. Why is that?
   A) The gas they come from consists of many different isotopes, which have different wavelengths
   B) They are surrounded by gas and dust, which can shift the wavelengths slightly
   C) They have been gravitationally lensed by intervening galaxy clusters
   D) They are produced by gas under high pressure, which leads to pressure broadening
   E) The gas is orbiting the black hole at high velocity, which causes red and blue shifts in the spectral lines

6. Which of the following is <u>not</u> a characteristic that would help one identify a nearby star as a halo star
   A) The star has a very small metallicity
   B) The star is moving at a speed very different from the orbital velocity of the Sun, 200 km/s
   C) The star is a very young star
   D) The star has a large vertical motion, perpendicular to the plane of the galaxy
   E) All of these actually <u>are</u> signs that they are halo stars

7. The galactic supercluster we live in is called
   A) Laniakea B) Virgo C) Local Group D) Milky Way E) Andromeda

8. What types of galaxies tend to occur at the center of large clusters of galaxies?
   A) Irregular B) Spiral C) Barred Spiral **D) Giant Elliptical** E) Dwarf Spheroidal

9. The approximate diameter of the disk of the galaxy we live in is probably about
   A) 300 Mpc B) 30 Mpc C) 3 Mpc D) 300 kpc **E) 30 kpc**

10. Globular clusters are mostly found in
    A) The nucleus **B) The halo** C) The disk D) The spiral arms E) Satellite galaxies

Part II: Short Answer PHY 310: Choose two of the questions PHY 610: Answer all three questions

Write 2-4 sentences about each of the following [10 each]

PHY 310: Answer two of the three questions

PHY 610: Answer all three questions

11. The following three words all are reasons why a galaxy might appear especially red. Explain why in each case: Dust, Motion, Age

Dust absorbs short wavelength light, while letting long wavelength (red) light through more easily, so dust reddens the appearance of stars. A galaxy that is moving away from us will have a red shift, which shifts all the visible light towards the red end of the spectrum. Finally, galaxies containing old stars have none of the hot, blue stars, just the small red main sequence stars plus some red giants, so that also makes galaxies look red.

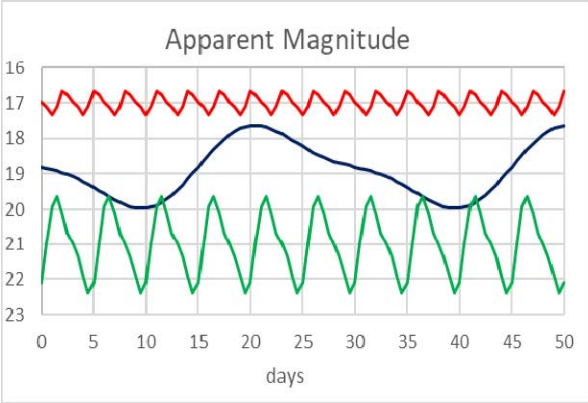
12. The Sun is currently moving both vertically, perpendicular to the plane of the galaxy, and slightly inwards, towards the center of the galaxy. Explain over the long term what the effects of these two types of motion are.

The vertical motion will cause the Sun to bob up and down within the galaxy, with a motion very similar to a harmonic oscillater. The radial inwards motion will also cause the Sun to oscillate between smaller and larger radii. Both types of motion are periodic, though they have different periods.

13. Explain qualitatively how we are pretty confident that the dark matter is not primarily brown dwarfs, Jupiters, white dwarfs, neutron stars, or black holes.

All of these compact massive halo objects (MACHOs) have enough mass to gravitationally lens objects behind them. As a MACHO passes between us and a distant star, the star will briefly brighten due to lensing, in a predictable, and achromatic way (all colors treated equally). By studying a large number of halo stars this way, we can confidently conclude that MACHOs DO in fact, exist, but there aren't nearly enough of them to account for the dark matter.

Part III: Calculation: PHY 310: Choose three of the problems PHY 610: Do all four problems
For each of the following problems, give the answer, explaining your work. [20 points each]



14. Three Cepheid variable stars all in the direction of a nearby galaxy have their apparent magnitude *m* plotted as a function of time.

(a) Find the period and absolute magnitude for each of the three stars

The period for the three stars is just the time between peaks, which looks like it is approximately:

$$P_r = 3.0\text{ d},\quad P_b = 30\text{ d},\quad P_g = 5.0\text{ d}.$$

We then use the period-luminosity relationship to get the absolute magnitudes, which are

$$M_r = -2.81\log(3) - 1.43 = -2.77,$$
$$M_b = -2.81\log(30) - 1.43 = -5.58,$$
$$M_g = -2.81\log(5) - 1.43 = -3.39.$$

(b) Find the distance to each of the three stars

We first find the apparent magnitude, which is approximately the central value of each of the three curves. By eyeball, we estimate these as

$$m_r = 17.0,\quad m_b = 18.8,\quad m_g = 21.0.$$

We then simply substitute these into the distance-magnitude equation to find

$$d_r = 10^{1+\frac{m_r - M_r}{5}}\text{ pc} = 10^{1+\frac{17.0+2.27}{5}}\text{ pc} = 71{,}400\text{ pc} = 71.4\text{ kpc},$$
$$d_b = 10^{1+\frac{m_b - M_b}{5}}\text{ pc} = 10^{1+\frac{18.8+5.58}{5}}\text{ pc} = 752{,}000\text{ pc} = 752\text{ kpc},$$
$$d_g = 10^{1+\frac{m_g - M_g}{5}}\text{ pc} = 10^{1+\frac{21.0+3.39}{5}}\text{ pc} = 755{,}000\text{ pc} = 755\text{ kpc}.$$

(c) In fact, two of the stars are in the galaxy, and one is just coincidentally in the same direction. Which one is not in the galaxy?

Two of them have such similar distances that it is easy to believe they belong to the same galaxy. The red one, however, is clearly too close to be in the same galaxy, since it is a factor of ten closer.

15. Suppose a galaxy has a luminosity comparable to the Milky Way ($L = 1.20\times10^{10} L_\odot$), and is a <u>sphere</u> comparable in size to our galaxy ($R = 16$ kpc). It is completely

surrounded by dust, which completely thermalizes the radiation, so it radiates at a uniform temperature *T*.

(a) What is the power in W coming out of this galaxy? What is the flux *F* of power per unit area (W/m²) coming out of this galaxy?

The power is just

$$L = 1.20\times10^{10} L_\odot = \left(1.20\times10^{10}\right)\left(3.839\times10^{26}\ \text{W}\right) = 4.61\times10^{36}\ \text{W}.$$

The flux per unit area is this divided by the area of a sphere, which is about $4\pi R^2$, so we have

$$F = \frac{L}{A} = \frac{L}{4\pi R^2} = \frac{4.61\times10^{36}\ \text{W}}{4\pi\left(16{,}000\ \text{pc}\right)^2}\cdot\left(\frac{\text{pc}}{3.086\times10^{16}\ \text{m}}\right)^2 = 1.505\times10^{-6}\ \text{W/m}^2.$$

(b) What will be the temperature of the thermal radiation coming from the galaxy, in K?

The flux is given by $F = \sigma T^4$, so we solve this for the temperature:

$$T^4 = \frac{F}{\sigma} = \frac{1.505\times10^{-6}\ \text{W/m}^2}{5.670\times10^{-8}\ \text{W/m}^2/\text{K}^4} = 26.5\ \text{K}^4,$$

$$T = \left(26.5\ \text{K}^4\right)^{1/4} = 2.27\ \text{K}.$$

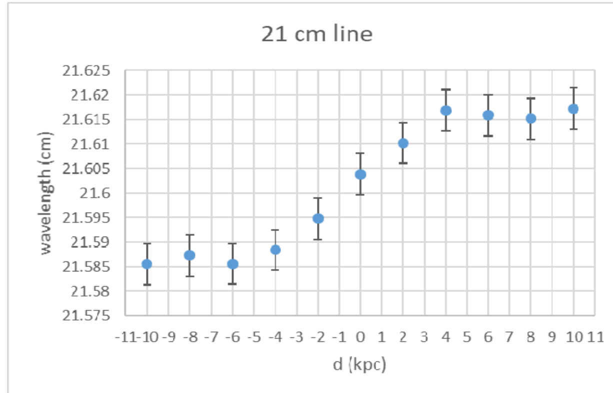
I now realize this entire problem is invalid because we have neglected the heating coming from the cosmic microwave background radiation. But that's okay, it's just a problem.

(c) At what wavelength will this radiation be strongest?

We simply use Wien's Law, $\lambda_{max} T = 2.8978\times10^{-3}\ \text{m}\cdot\text{K}$

$$\lambda_{max} = \frac{2.8978\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.8978\times10^{-3}\ \text{m}\cdot\text{K}}{2.27\ \text{K}} = 1.277\times10^{-3}\ \text{m} = 1.277\ \text{mm}.$$

16. A certain galaxy is edge on to us, and the 21 cm line (whose actual wavelength is 21.106 cm) is measured at various distances from the center of the galaxy.



(a) Find the velocity of the hydrogen at the right edge of the galaxy, and from the left edge of the galaxy.

Since the wavelength shifts are relatively small, we can use the nonrelativiistic approximation, $z = v/c$, together with $1+z = \lambda/\lambda_0$. We find

$$1+\frac{v_r}{c} = 1+z_r = \frac{21.617\text{ cm}}{21.106\text{ cm}} = 1.02421,$$
$$v_r = (1.02421-1)c = 0.02421\left(2.998\times10^8\text{ m/s}\right) = 7.258\times10^6\text{ m/s} = 7{,}258\text{ km/s},$$
$$1+\frac{v_l}{c} = 1+z_l = \frac{21.587\text{ cm}}{21.106\text{ cm}} = 1.02279.$$
$$v_l = (1.02279-1)c = 0.02279\left(2.998\times10^8\text{ m/s}\right) = 6.832\times10^6\text{ m/s} = 6{,}832\text{ km/s}.$$

(b) Find the speed at which the galaxy is moving towards or away from us, and the speed at which it is rotating. Which side is rotating towards us?

We estimate the speed of the galaxy as the average of these, namely

$$v = \tfrac{1}{2}\left(v_r + v_l\right) = \tfrac{1}{2}\left(7{,}258\text{ km/s} + 6{,}832\text{ km/s}\right) = 7{,}045\text{ km/s}.$$

The rotational speed is the difference between this and either side; for example

$$v_{rot} = v_r - v = \left(7{,}258\text{ km/s}\right) - \left(7{,}045\text{ km/s}\right) = 213\text{ km/s}.$$

So it is moving a little faster than 7,000 km/s <u>away from us</u>, and the right side is rotating away an additional 210 km/s or so. Obviously, <u>the left side is rotating towards us</u>.

(c) Does this galaxy show evidence for dark matter?

In the absence of dark matter, we would expect the rotation velocities to fall off as one over the square root of distance at large velocities. But there is no evidence of a fall off, so there must be some additional contribution to the mass, presumably dark matter.

17. A group of stars called Carlson Stars have just been discovered! Listed at right are the parallaxes *p* and apparent magnitudes *m* of a group of Carlson Stars. You may put some of your answers in the box at right if you wish.

Star	p (″)	m	d (pc)	M
A	0.0530	16.04	18.9	14.66
B	0.0280	17.43	35.7	14.67
C	0.0130	19.10	76.9	14.67
D	?	27.56	3,780	

(a) For stars A, B, and C, use the parallax to find the distance to these stars in pc.

We use the simple formula $d = 1/p$, where p is in arc-seconds and d is in parsecs. We find

$$d_A = \frac{1}{p_A} = \frac{1}{0.0530} = 18.9\text{ pc},\quad d_B = \frac{1}{p_B} = \frac{1}{0.0280} = 35.7\text{ pc},\quad d = \frac{1}{p_C} = \frac{1}{0.0130} = 76.9\text{ pc}$$

It is implausible that such close stars have just been discovered, but it is equally unlikely that they would be named after me.

(b) For the same three stars, find the absolute magnitude of these stars.

We rearrange the formula $m - M = 5\log(d) - 5$ to yield $M = m - 5\log(d) + 5$, then use this on all three stars:

$$M_A = m_A - 5\log(d_A) + 5 = 16.04 - 5\log(18.9) + 5 = 14.66,$$
$$M_B = m_B - 5\log(d_B) + 5 = 17.43 - 5\log(35.7) + 5 = 14.67,$$
$$M_C = m_C - 5\log(d_C) + 5 = 19.10 - 5\log(76.9) + 5 = 14.67.$$

I've entered this into the table as well.

(c) Is there evidence, based on part (b), that Carlson Stars are good standard candles?

A standard candle is something that consistently has the same luminosity. The three numbers are almost identical, which means that Carlson Stars are an (unrealistically) excellent Standard Candle, with M = 14.67. Unfortunately, they are rather dim, so they actually aren't going to be much help at long distances.

(d) Star D is too distant to use parallax. Estimate the distance to star D anyway.

We assume that star D has the same absolute magnitude, M = 14.67, and then use the distance formula

$$d_D = 10^{1+\frac{m_D - M_D}{5}}\text{ pc} = 10^{1+\frac{27.56-14.67}{5}}\text{ pc} = 3,780\text{ pc}.$$

At this distance, the parallax would be about 0.00026″, which is difficult to measure with any accuracy.