Name ___________________

Solutions to Midterm Exam
October 18, 2019

This test consists of three parts. For the first and second parts, you may write your answers directly on the exam, if you wish. For the other parts, use separate sheets of paper.

Part I: Multiple Choice Everyone: Answer all questions.
For each question, choose the best answer (2 points each)

1. Type Ia supernovae became a popular distance method for measuring distances in the late 20th century. Why don't we just always use it to measure distances?
   A) These supernovae are not bright enough to measure at large distances
   B) Light extinction due to dust causes huge problems, since these mostly occur in the plane of our galaxy
   C) Parallax is a much more accurate way to measure the distance to typical galaxies
   D) Their enormous gravitation causes the light to be red-shifted, distorting our views
   E) These events are rare, so you can't automatically use it for most objects

2. Which of the elements iron (Fe), carbon (C), neon (Ne), and helium (He), according to astronomers, contributes to the metallicity of a star?
   A) None of them B) Fe only C) Fe and C **D) Fe, C, and Ne** E) All of them

3. Why do clusters of stars tend to get redder as they age?
   A) Dust accumulates around the cluster, reddening it
   B) The cluster moves away from us, and the reddening is due to red shift
   C) Blue supergiant stars turn red after a few billion years
   D) All the high mass (blueish) stars are dead, and the brightest remaining stars are red giants
   E) Red stars get gradually brighten over time, while blue stars gradually get dimmer

4. The galaxy pictured at right is approximately what galaxy classification?
   A) E0 B) E7 C) SAa D) SBd E) Im

5. The name of the galaxy we live in is
   A) Milky Way B) Coma C) Virgo D) Andromeda E) Norma

6. Approximately what percent of our galaxy's mass is in the form of dark matter?
   A) 1% B) 15% C) 50% **D) 85%** E) 99%

7. If an astronomer says they are studying O and B stars, what types of stars are they studying?
   A) Hottest B) Coolest C) Most luminous D) Least luminous E) Brightest

8. The main way we know that most of the dark matter in the galaxy is not solar mass black holes (for example) is because
   A) We should see these black holes forming from stars blowing up
   B) All other stars, including ours, would have been sucked into them long ago
   C) They should cause gravitational lensing events from stars behind them more often than we see them
   D) Large portions of the sky should look black from the black holes sucking up all the light
   E) The gravitational effects of nearby black holes should be obvious

9. In what way were galaxies different a long time ago?
   A) They often had high mass short-lived stars, now there are only old stars
   B) They were often smaller and there were more irregular galaxies
   C) They hadn't accumulated any gas yet
   D) They had much higher metallicities back then
   E) None of the above; they are pretty much the same as modern galaxies

10. When two galaxies nearly collide, energy is conserved. So how can the galaxies slow down as they pass?
    A) Gas clouds outside the galaxies collide, absorbing energy
    B) Gravitational waves are emitted by the nearly colliding galaxies
    C) Gravitational forces convert the energy of the net motion of the galaxies into the internal motion of the stars in each galaxy, in a process called tidal friction
    D) Though the stars don't collide, the dark matter does, and that extends out of the galaxies
    E) Actually, unless they actually collide, the galaxies do not slow down

11. Why are Cepheid variable stars so useful to astronomers?
    A) It is easier to do parallax on them, allowing us to get the distance
    B) They are the brightest stars, so they can be seen at the greatest distance
    C) They produce lots of infrared light, so they can be seen through dust
    D) There have a known relationship between their period and their <u>distance</u>
    E) They have a known relationship between their period and their <u>luminosity</u>

12. At the center of our galaxy lies a
    A) Supernova **B) Black hole** C) Supergiant star D) Nebula E) Void

13. How old, approximately, are the oldest stars in the galaxy?
    A) 9 My B) 13 My C) 17 My D) 9 Gy **E) 13 Gy**

14. Which of the following might be the approximate value of Hubble's constant today?
    A) 50 km/s/Mpc B) 100 km/s/Mpc **C) 70 km/s/Mpc** D) 50 m/s/Gpc E) 70 m/s/Gpc

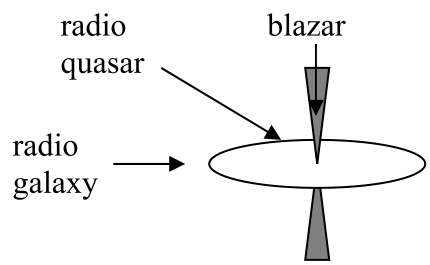
15. For the stars in a galaxy or cluster, the potential energy E_P is negative and the kinetic energy E_K is positive. If the galaxy or cluster is in a state of equilibrium, so it satisfies the virial theorem, how are the magnitudes of these two quantities related?
    A) $|E_K| = 2|E_P|$ **B)** $|E_P| = 2|E_K|$ C) $|E_K| = |E_P|$ D) $|E_K| = 4|E_P|$ E) $|E_P| = 4|E_K|$

Part II: Short Answer PHY 310: Choose three of the four questions PHY 610: Answer all four questions. Write 2-4 sentences about each of the following [10 each]

16. Galaxy collisions are common, and yet the stars within them rarely collide. Give at least three reasons why galaxy collisions still have significant effects.

Although the stars in the galaxies do not collide, the gas can collide, causing a burst of star formation (a starburst galaxy), a heating of the gas (expelling it from the galaxy) or feeding gas into the black hole engine at the center (causing an active galaxy). Also, the stars can interact gravitationally, which may cause the galaxies to become distorted (making them irregular) or even merging.

17. Astronomers think that radio galaxies, radio quasars, and blazars may all possibly be the same thing. Explain how this might be possible. A simple sketch might help.



A galaxy with a jet might look like a radio galaxy when viewed edge on (when we can't see the center), a radio quasar from an angle (where we can see the center and the jets), or a blazar if we are looking straight along the jet, as illustrated at right.

18. Explain qualitatively how we can estimate the total mass of a cluster of galaxies, including all the dark matter, etc.

The most reliable way to measure the total mass of a cluster of galaxies is to use gravitational lensing, where the gravity of the cluster causes it to lens the objects behind it, causing multiple images. Measuring those images allows one to find the mass of the whole cluster.

19. Hubble's Law states that the distance and velocity of galaxies are given by $v = H_0 d$. Explain why this formula is imprecise (a) at small distances, and (b) at large distances.

At small distances, this formula tends to fail because galaxies have peculiar velocities, velocities caused by gravitational effects, on top of the general Hubble flow. These velocities can be comparable to or even larger than the Hubble flow velocities. On large distances, we are looking so far away and so far back in time that we are seeing the universe as it was, not as it is, and the changes must be taken into account. There are also other effects, such as red shift and the curvature of the universe, that become relevant at these distances.

Physical Constant	**Units**	**Distance and Magnitudes**	**Galactic Orbits**
$k_B = 1.381\times10^{-23}$ J/K	$\text{pc} = 3.086\times10^{16}$ m	$d = 10^{1+\frac{m-M}{5}}$ pc	$\Omega = \frac{V_0}{R_0}$
$\hbar = 1.055\times10^{-34}$ J$\cdot$s	$M_\odot = 1.989\times10^{30}$ kg	$m - M = 5\log(d) - 5$	$\kappa^2 = \frac{2V_0^2}{R_0^2} + \frac{1}{R_0}\frac{d}{dR}V^2\Big\vert_{R_0}$
$h = 6.626\times10^{-34}$ J$\cdot$s	$\text{y} = 3.156\times10^{7}$ s		$\nu = \sqrt{4\pi G\rho_0}$
$G = 6.673\times10^{-11}\,\text{m}^3/\text{kg/s}^2$	$\text{rad} = 206,265''$		

Part III: Calculation:
PHY 310: Choose four of the five problems
PHY 610: Do all five problems.
For each of the following problems, give the answer, explaining your work. [20 points each]

Black Body Radiation

$$u = \frac{\pi^2}{15}\frac{(k_B T)^4}{(\hbar c)^3}$$

$$\lambda_{\max} T = 0.00290 \text{ m}\cdot\text{K}$$

Planetary Nebula

$M^* = -4.47$

20. One important event in the early universe is called *recombination*, and occurred when the temperature of the universe was approximately $T = 2970$ K. The universe was filled with almost perfect black body radiation at this time.

(a) Find the energy density of the blackbody radiation at this time in J/m^3.

This is a straightforward application of the formula for the energy density of a blackbody,

$$u = \frac{\pi^2}{15}\frac{(k_B T)^4}{(\hbar c)^3} = \frac{\pi^2\left[\left(1.381\times10^{-23}\text{ J/K}\right)\left(2970\text{ K}\right)\right]^4}{15\left[\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)\left(2.998\times10^8\text{ m/s}\right)\right]^3} = 0.0589\text{ J/m}^3 .$$

(b) Find the wavelength where the blackbody radiation was strongest at this time, in nm.

We simply use the formula $\lambda_{\max} T = 0.00290 \text{ m}\cdot\text{K}$, which we rewrite as

$$\lambda_{\max} = \frac{0.00290\text{ m}\cdot\text{K}}{T} = \frac{0.00290\text{ m}\cdot\text{K}}{2970\text{ K}} = 9.76\times10^{-7}\text{ m} = 976\text{ nm}.$$

(c) Find the energy for a single photon with the wavelength you found in part (b).

We use the formulas $\nu\lambda = c$ and $E = h\nu$ to find the energy, so

$$E = h\nu = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(2.998\times10^8\text{ m/s}\right)}{9.76\times10^{-7}\text{ m}} = 2.035\times10^{-19}\text{ J}.$$

(d) Using the energy density you found in part (a) and the energy you found in part (c), estimate the number of photons per unit volume (in m^{-3}) at this time.

The energy density would be the number density times the energy per particle, $u = nE$. This isn't quite right, since the energy found in part (c) is not the true average, but it's a good estimate. So we have

$$n = \frac{0.0589\text{ J/m}^3}{2.035\times10^{-19}\text{ J}} = 2.894\times10^{17}\text{ m}^{-3} .$$

21. Barnard's Star has a parallax of $p = 0.547''$ and is moving with proper motion $\mu_x = -0.803''/\mathrm{y}$ and $\mu_y = 10.36''/\mathrm{y}$. The Lyman-α line, normally at a wavelength of $\lambda_0 = 121.567\ \mathrm{nm}$, is detected from this star at $\lambda = 121.522\ \mathrm{nm}$.

(a) What is the distance to Barnard's Star in pc?

We use the formula $d = 1/p$ to find

$$d = \left(\frac{1''}{p}\right)\ \mathrm{pc} = \frac{1\ \mathrm{pc}}{0.547} = 1.83\ \mathrm{pc}.$$

(b) What is the velocity perpendicular to our line of sight v_x and v_y compared to us in km/s?

We use the formula $v = \mu d$ for each of the two directions. However, it is important that we write μ in radians, so we write

$$\mu_x = \frac{-0.803''/\mathrm{y}}{206,265''/\mathrm{rad}} = -3.893\times10^{-6}\ \mathrm{rad/y},$$
$$\mu_y = \frac{10.36''/\mathrm{y}}{206,265''/\mathrm{rad}} = 5.023\times10^{-5}\ \mathrm{rad/y}.$$

We then multiply each of these by the distance, to get

$$v_x = \mu_x d = \left(-3.893\times10^{-6}/\mathrm{y}\right)\left(1.83\ \mathrm{pc}\right)\cdot\frac{3.086\times10^{13}\ \mathrm{km/pc}}{3.156\times10^{7}\ \mathrm{s/y}} = -6.97\ \mathrm{km/s},$$
$$v_y = \mu_y d = \left(5.023\times10^{-5}/\mathrm{y}\right)\left(1.83\ \mathrm{pc}\right)\cdot\frac{3.086\times10^{13}\ \mathrm{km/pc}}{3.156\times10^{7}\ \mathrm{s/y}} = 89.9\ \mathrm{km/s},$$

(c) What is the radial velocity v_r of the star compared to us in km/s?

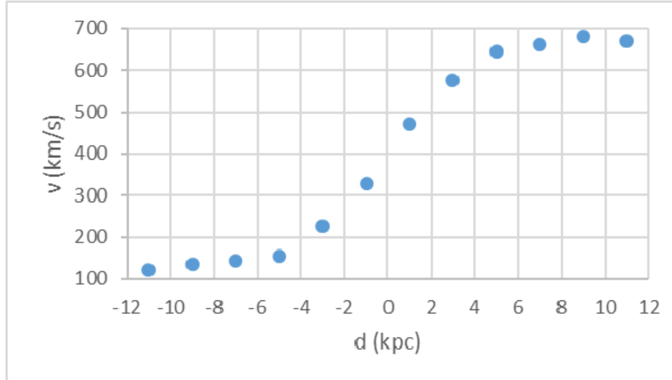
We first need to find the red-shift z, defined as $1 + z = \lambda/\lambda_0$, so we have

$$z = \frac{\lambda}{\lambda_0} - 1 = \frac{121.522\ \mathrm{nm}}{121.567\ \mathrm{nm}} - 1 = 0.999630 - 1 = -0.000370.$$

This number is small enough that we can use the non-relativistic formula $v_r = zc$, so we have

$$v_r = zc = -0.000370\left(2.998\times10^{8}\ \mathrm{m/s}\right) = -1.11\times10^{5}\ \mathrm{m/s} = -111\ \mathrm{m/s}.$$

22. A spiral galaxies has stars in various places measured compared to us via Doppler shift. The resulting velocities are as plotted at right.



(a) Estimate the rotational velocities (ignoring the net motion of the galaxy) at distances of 5 kpc and 10 kpc from the center of the galaxy.

The rotational velocities should simply be measured compared to the average velocity, the same as the velocity near the center, which looks to be very close to $v_c = 400\text{ km/s}$. If we look at $\pm 5\,\text{kpc}$ from the center, I'd say the speeds are about 650 km/s on the right and 170 km/s on the right. These differ from the central speed by about +250 km/s on the right, and –230 km/s on the left, which represents rotation (right side away from us) of about

$$V(5\text{ kpc}) \approx 240\text{ km/s}\,.$$

At 10 kpc, the numbers look close to 680 km/s on the right and 120 km/s on the left, which compared to the center is about +280 km/s on the right and –280 km/s on the left, so

$$V(10\text{ kpc}) \approx 280\text{ km/s}\,.$$

(b) Assume the mass of the galaxy is distributed in a spherical symmetric manner. What is the total mass of the galaxy contained within spheres of radius 5 kpc and 10 kpc of the center of the galaxy?

For a spherically symmetric mas distribution, the gravitational attraction of the galaxy on a mass at a distance *R* will be the same as the gravitational pull from a point mass located at the center whose mass *M* is just the amount of mass contained within the radius *R*. Treating the mass of the pulled on object as *m*, we would have the gravitational force matching the centrifugal force $GMm/R^2 = mV^2/R$, so $M = V^2R/G$. We now just substitute this formula at each radius, to find

$$M(5\text{ kpc}) = \frac{\left(2.40\times10^5\text{ m/s}\right)^2\left(5\times10^3\text{ pc}\right)}{6.674\times10^{-11}\text{ m}^3\text{ kg}^{-1}\text{s}^{-2}}\cdot\frac{3.086\times10^{16}\text{ m}}{\text{pc}} = \frac{1.33\times10^{41}\text{ kg}}{1.989\times10^{30}\text{ kg}/M_\odot} = 6.70\times10^{10}\,M_\odot,$$

$$M(10\text{ kpc}) = \frac{\left(2.80\times10^5\text{ m/s}\right)^2\left(10^4\text{ pc}\right)}{6.674\times10^{-11}\text{ m}^3\text{ kg}^{-1}\text{s}^{-2}}\cdot\frac{3.086\times10^{16}\text{ m}}{\text{pc}} = \frac{3.63\times10^{41}\text{ kg}}{1.989\times10^{30}\text{ kg}/M_\odot} = 1.82\times10^{11}\,M_\odot.$$

(c) Does this galaxy show evidence for dark matter?

The fact that the rotation curves do not fall off at large radius indicates that there is dark matter. Also, the fact that the mass contained within 10 kpc is much more than at 5 kpc.

23. A group of galaxies has their brightest planetary nebulas (PN) and brightest red giants (RG) apparent magnitudes measured, as shown at right.

(a) For galaxies A, B, and C, estimate the distance using planetary nebulae.

Gal-axy	m (PN)	m (RG)	d (Mpc)	M (RG)
A	20.21	20.68	0.86	–3.99
B	22.30	22.76	2.26	–4.01
C	24.86	25.34	7.35	–3.99
D	?	21.63	1.34	–4.00

The brightest planetary nebulas have absolute magnitude about $M^* = -4.47$. The distances are calculated using the formula

$$d = 10^{1+\frac{m-M}{5}}\text{ pc} = 10^{1+0.2(m+4.47)}\text{ pc}.$$

We now just substitute the value of m for the planetary nebulas into the distance formula. The results are included in the table above.

(b) For the same three galaxies, find the absolute magnitude of the red giant stars.

For these, we use the reverse relations, namely $m - M = 5\log(d) - 5$, so we have

$$M = m + 5 - 5\log(d).$$

We simply substitute the values given and find M in each case. The results are included in the table above.

(c) Are the brightest red giants a decent standard candle? Why or why not?

A standard candle is something that has consistently the same luminosity (or absolute magnitude). With a typical variation of only 0.01 magnitudes, it looks like the brightest red giants in a galaxy have absolute magnitudes that are very consistent, with $M = -4.00$.

(d) Galaxy D hasn't had its planetary nebulas measured. Estimate the distance anyway.

We use the value $M = -4.00$ together with the apparent magnitude $m = 21.63$ to find

$$d = 10^{1+\frac{m-M}{5}}\text{ pc} = 10^{1+0.2(21.63+4.00)}\text{ pc} = 1.34\times 10^{6}\text{ pc} = 1.34\text{ Mpc}.$$

24. A long time ago in a faraway spiral galaxy, the star system Tatoo orbits at an average distance of $R_0 = 12.00\ \text{kpc}$ from the center of its galaxy. The velocity rotation curve for the galaxy fits the formula $V^2 = AR$, where $A = 8{,}800\ \text{km}^2\text{s}^{-2}\text{kpc}^{-1}$. The mass density in the neighborhood of Tatoo is $\rho = 0.132\ M_\odot\text{pc}^{-3}$.

(a) What is the orbital velocity V_0 at the distance R_0? What is the angular velocity Ω at this radius in My^{-1}, and what is the period T_Ω to complete one orbit?

We simply use the formula to find the velocity, so

$$V_0^2 = AR_0 = \left(8{,}800\ \text{km}^2\text{s}^{-2}\text{kpc}^{-1}\right)\left(12\ \text{kpc}\right) = 1.056\times10^5\ \text{km}^2/\text{s}^2\ ,$$

$$V_0 = \sqrt{1.056\times10^5\ \text{km}^2/\text{s}^2} = 325\ \text{km/s}.$$

The angular velocity and period are given by

$$\Omega = \frac{V_0}{R_0} = \frac{3.25\times10^5\ \text{m/s}}{1.200\times10^4\ \text{pc}}\cdot\frac{\left(3.156\times10^7\ \text{s/y}\right)\left(10^6\ \text{y/My}\right)}{3.086\times10^{16}\ \text{m/pc}} = 0.0277\ \text{My}^{-1},$$

$$T_\Omega = \frac{2\pi}{\Omega} = \frac{2\pi}{0.0277\ \text{My}^{-1}} = 227\ \text{My}.$$

(b) Tatoo doesn't stay exactly at R_0, but also wanders in and out compared to the center of the galaxy. What is the frequency κ for these epicycles in My^{-1}, and the corresponding period T_κ?

We use the formula

$$\kappa^2 = \frac{2V_0^2}{R_0^2} + \frac{1}{R_0}\frac{d}{dR}V^2\Big|_{R_0} = \frac{2AR_0}{R_0^2} + \frac{1}{R_0}\frac{d}{dR}\left(AR\right)\Big|_{R_0} = \frac{2A}{R_0} + \frac{A}{R_0} = \frac{2V_0^2}{R_0^2} + \frac{V_0^2}{R_0^2} = 3\Omega^2\ ,$$

$$\kappa = \Omega\sqrt{3} = \left(0.0277\ \text{My}^{-1}\right)\sqrt{3} = 0.0480\ \text{My}^{-1}$$

$$T_\kappa = \frac{2\pi}{\kappa} = \frac{2\pi}{0.0480\ \text{My}^{-1}} = 131\ \text{My}.$$