Solution to Midterm Exam
October 17, 2023

This test consists of three parts. For the first and second parts, you may write your answers directly on the exam, if you wish. For the other parts, use separate sheets of paper.

Part I: Multiple Choice Everyone: Answer all questions.
For each question, choose the best answer (2 points each)

1. Which of the following is probably **false** about a typical open cluster of stars?
   A) The stars will mostly all remain in this cluster indefinitely
   B) The stars will all be moving in approximately the same direction and speed
   C) The stars might have metallicity comparable to ours
   D) The cluster is typically somewhere in the disk
   E) Actually, all of these are true about open clusters

2. The driving engine that makes active galaxies so bright is assumed to be
   A) O and B stars **B) A black hole** C) A supernova D) Molecular cloud E) Antimatter

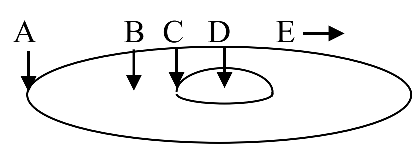
3. Spectroscopic parallax is useful for finding the distance to which types of stars?
   A) Main sequence
   B) Red giants
   C) White dwarfs
   D) Type Ia supernovae
   E) Neutron stars

4. The galaxy pictured at right is approximately what galaxy classification?
   A) E0 B) E7 **C) SAc** D) SB0 E) Im

5. Radar distancing has limited application because
   A) Peculiar velocities add random (unknown) errors to the distance measurement
   B) The time is so short for measuring to nearby objects that you can't get it accurately
   C) Methods like Cepheid variable stars work even better at the relevant distance
   D) Blurring by our atmosphere messes up the measurements
   E) You can only get radar reflections back in reasonable times from nearby objects

6. List the three elements Carbon (C), Hydrogen (H) and Helium (He) in order from most common to least common in a typical star
   A) He, H, C **B) H, He, C** C) H, C, He D) He, C, H E) C, H, He

7. At right is a crude sketch of our galaxy. Where are we in this sketch?
   A) A **B) B** C) C D) D E) E



8. The best way to detect HI regions (neutral hydrogen atoms) in our galaxy is by detecting
   A) X-rays from the hot gas
   B) Spectral lines from molecular vibrations
   C) Dimming of light from stars behind them caused by absorption
   D) Gravitational lensing by these clouds
   E) The 21 cm line from electrons flipping their spin

9. The cause of tidal friction, that slows down the relative speed of two galaxies that pass each other, is
   A) Magnetic attraction between the galaxies
   B) Light pressure from one galaxy pushing on the other
   C) Different acceleration of different parts of the galaxies due to the passing galaxy's gravity
   D) Collision of gas clouds surrounding the galaxies
   E) Collisions of dark matter fluid as the galaxies interact

10. Hubble's Law fails at large distances because
   A) "Distance" is ambiguous for sources moving at high velocity (only)
   B) Relativistic effects need to be taken into account (only)
   C) You are looking into the past, when Hubble's "constant" may have been different (only)
   D) All of the above
   E) None of the above

11. Why is it that there are often very massive elliptical galaxies at the centers of galaxy clusters?
   A) The gas all flows to the center to make these massive galaxies
   B) Spirals in the center have their spiral arms stripped away, making them into ellipticals
   C) Mergers of large numbers of galaxies formed these giant ellipticals
   D) Galaxies in the middle can most easily transfer their angular momentum to other galaxies, converting them to ellipticals
   E) I have no idea; please mark this one wrong

12. The name of the galaxy supercluster we live in is
   A) Milky Way B) Virgo C) Andromeda **D) Laniakea** E) Coma

13. Most of the dark matter in a galaxy is in the
   A) Nucleus B) Bulge C) Disk **D) Halo** E) None of these

14. We now know that dark matter is made mostly of
   A) Black holes B) Neutron stars C) White dwarfs D) Planets **E) None of these**

15. What is the approximate fraction of a typical galaxy's mass that is made of ordinary matter?
   A) 15% B) 50% C) 75% D) 85% E) 99%

Part II: Short Answer PHY 310: Choose three of the four questions PHY 610: Answer all four questions. Write 2-4 sentences about each of the following [10 each]

16. Explain qualitatively how Cepheid variable stars can be used to measure distances to other galaxies.

There is a known relationship between the period T at which a Cepheid variable star pulses and its luminosity (or absolute magnitude M). By measuring the star over time, you can measure the period T and hence deduce the absolute magnitude M. Then you measure the apparent magnitude m, and (after compensating for extinction due to dust, if necessary), you can deduce the distance from the formula $d = 10^{1+\frac{m-M}{5}}$ pc.

17. At a naïve level, the Sun moves in a circular orbit in the plane of our galaxy around the center of our galaxy. Describe two types of motion that Sun actually has which differ from this simple circular motion.

The Sun also bobs up and down compared to the galactic plane. This is governed by the local density of mass in our neighborhood. It also alternately increases and decreases its distance from the center of our galaxy due to the overall distribution of mass in the galaxy. This process is called "epicycles." This motion is accompanied by an alternating increase and decrease in its angular velocity, and therefore this means that it is alternately ahead of or behind where it should be compared to the trivial circular motion.

18. Give at least four differences between a typical elliptical galaxy and a typical spiral galaxy

Elliptical galaxies don't have a disk or spiral structure (obviously), they consist almost exclusively of old stars, and they don't have gas and dust in the galaxy themselves. Instead, the gas tends to be very hot (so no molecular clouds), and spread throughout the halo. Finally, the majority of stars are in very non-circular orbits, perhaps with a slight bias for one direction over the other, but not systematically the same.

19. What is Hubble's Law? Assuming Hubble's Law works perfectly, explain what one would have to measure about a particular galaxy to use Hubble's Law to get the distance to that galaxy.

Hubble's law says that the radial velocity v_r is proportional to the distance d, so $v_r = H_0 d$. Hubble's constant is somewhere in the neighborhood of $H_0 = 70$ km/s/Mpc (there is some dispute on its exact value). To find the distance to an object, one would measure the spectrum, find spectral lines at wavelength λ_0, compare to the laboratory wavelength λ, use the Doppler shift formula to find the radial velocity v_r, and then use Hubble's Law to deduce the distance from $d = v_r/H_0$.

Physical Constants	Units	Distance/Magnitude	Black Body Radiation
$k_B = 1.381\times10^{-23}$ J/K	pc $= 3.086\times10^{16}$ m	$d = 10^{1+\frac{m-M}{5}}$ pc	$u = \frac{\pi^2}{15}\frac{(k_B T)^4}{(\hbar c)^3}$
$\hbar = 1.055\times10^{-34}$ J·s	$M_\odot = 1.988\times10^{30}$ kg	$m - M = 5\log(d) - 5$	$\lambda_{\max} T = 0.00290$ m·K
$h = 6.626\times10^{-34}$ J·s	$R_\odot = 6.96\times10^{8}$ m	**Brightness/Magnitude**	**Tip of the Red Giant**
$G = 6.673\times10^{-11}\,\mathrm{m^3/kg/s^2}$	$T_\odot = 5772$ K	$F = 2.518\times10^{-8}\,\mathrm{W/m^2}\left(10^{-\frac{2}{5}m}\right)$	$M_t = -4.10$

Part III: Calculation:
PHY 310: Choose four of the five problems; PHY 610: Do all five problems.
For each of the following problems, give the answer, explaining your work. [20 points each]

20. An important event in the early universe was matter-radiation equality, when the energy density in matter (including dark matter) matched the energy density in radiation. It is estimated that the black-body radiation at this time had an energy density of 4.56 J/m³.

(a) What was the approximate temperature *T* at this time?

We take the formula for the energy density *u* and solve it for the temperature *T*, namely

$$(k_B T)^4 = \frac{15(\hbar c)^3}{\pi^2} u = \frac{15\left[(1.055\times10^{-34}\,\mathrm{J\cdot s})(2.998\times10^{8}\ \mathrm{m/s})\right]^3}{\pi^2}(4.56\ \mathrm{J/m^3}) = 2.190\times10^{-76}\ \mathrm{J^4},$$

$$T = \frac{(2.190\times10^{-76}\ \mathrm{J^4})^{1/4}}{k_B} = \frac{1.216\times10^{-19}\ \mathrm{J}}{1.381\times10^{-23}\mathrm{J/K}} = 8811\ \mathrm{K}.$$

(b) What wavelength $\lambda_{\max}$ would have been the peak of the electromagnetic spectrum at this time?

We simply use Wien's law, which we solve for wavelength to give

$$\lambda_{\max} = \frac{0.00290\ \mathrm{m\cdot K}}{T} = \frac{0.00290\ \mathrm{m\cdot K}}{8811\,\mathrm{K}} = 3.29\times10^{-7}\ \mathrm{m} = 329\ \mathrm{nm}.$$

This is in the near-ultraviolet part of the spectrum.

(c) Find the energy of one photon with the wavelength you found in part (b), in J and in eV (1 eV = 1.602×10^{-19} J).

We use the relationship $\lambda\nu = c$ and $E = h\nu$ to find

$$E = h\nu = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \mathrm{J\cdot s})(2.998\times10^{8}\ \mathrm{m/s})}{3.29\times10^{-7}\ \mathrm{m}} = 6.04\times10^{-19}\ \mathrm{J}$$

$$= \frac{6.04\times10^{-19}\ \mathrm{J}}{1.602\times10^{-19}\ \mathrm{J/eV}} = 3.77\ \mathrm{eV}.$$

21. Sirius is a double star, and the brighter one is the brightest star in the sky (other than the Sun). The star's apparent magnitudes are $m_1 = +8.44$ and $m_2 = -1.46$.

(a) Which of these stars is brighter, and by what factor?

The more negative number is the brighter star, so that's the second star (whose actual name is Sirius A). The apparent magnitude is related to brightness by a relation of the form $F = k \cdot 10^{-\frac{2}{5}m}$, where k is constant, so

$$\frac{F_2}{F_1} = \frac{k \cdot 10^{-\frac{2}{5}m_2}}{k \cdot 10^{-\frac{2}{5}m_1}} = 10^{\frac{2}{5}(m_1 - m_2)} = 10^{\frac{2}{5}(8.44+1.46)} = 10^{\frac{2}{5}\cdot 9.90} = 9120.$$

Essentially, when you look at Sirius with your naked eye, all you see is Sirius A (star 2).

(b) The parallax of the Sirius system is approximately 0.372″. What is the distance to the Sirius system, in pc?

The distance (in parsecs) is the reciprocal of the parallax (in arc-seconds), so

$$d = \frac{1}{p} = \frac{1}{0.372}\ \text{pc} = 2.69\ \text{pc}.$$

(c) What is the absolute magnitude *M* for the brighter of the two stars?

We use the formula $m - M = 5\log(d) - 5$, which we rearrange to yield

$$M = m + 5 - 5\log(d) = -1.46 + 5 - 5\log(2.69) = -1.39.$$

(d) The H-α line normally occurs at a wavelength of 656.279 nm, but the same line coming from Sirius is measured to have a wavelength of 656.267 nm. Approximately how fast is Sirius moving relative to us, and is it towards us or away from us?

The emitted wavelength is $\lambda = 656.279\,\text{nm}$, but it is detected at a shorter wavelength of $\lambda_0 = 656.267\,\text{nm}$. This implies blue shift, so the object is moving towards us. We start by getting the red-shift *z*, defined by

$$1 + z = \frac{\lambda_0}{\lambda} = \frac{656.267\,\text{nm}}{656.279\ \text{nm}} = 0.999982,$$

$$z = -0.000018.$$

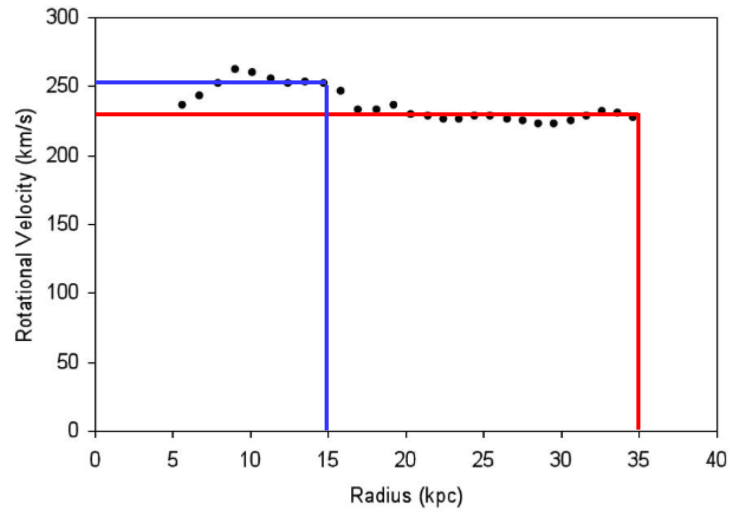
Since this is clearly small, we can use the non-relativistic approximation $v = zc$, so we have

$$v = (-0.000018)(2.998 \times 10^8\ \text{m/s}) = -5480\ \text{m/s},$$

So it is moving towards us at about 5.5 km/s.

22. **At right is a graph of the rotation curve for M31, the Andromeda galaxy, which is at a distance of 765 kpc .**

(a) **Consider the points at 15 kpc and 35 kpc away from the center. How far away in degrees are they from the center?**



The angle in radians is given by $\theta = s/d$, which would simply be

$$\theta_{15} = \frac{s}{d} = \frac{15\,\text{kpc}}{765\text{ kpc}} = \left(0.0196\text{ rad}\right)\frac{180°}{\pi\text{ rad}} = 1.12°.$$

$$\theta_{35} = \frac{s}{d} = \frac{35\,\text{kpc}}{765\text{ kpc}} = \left(0.0458\text{ rad}\right)\frac{180°}{\pi\text{ rad}} = 2.62°.$$

(b) **Measure the rotational velocities at 15 and 35 kpc from the center. Assuming the mass is spherically symmetric, what is the enclosed mass *M* at each of these radii in units of $M_\odot$?**

The appropriate lines have been drawn in. The velocities are approximately $v_{15} = 252$ km/s and $v_{35} = 230$ km/s.

We know that the gravitational force between any object and a mass *M* is given by GMm/R^2 , and the centripetal acceleration is given by mv^2/R . Equating these, we have $v^2 = GM/R$. Solving for the mass, we find

$$M_{15} = \frac{Rv_{15}^2}{G} = \frac{\left(15\times10^3\text{pc}\right)\left(252\times10^3\text{ m/s}\right)^2}{6.674\times10^{-11}\text{ m}^3\text{kg}^{-1}\text{s}^{-2}}\cdot\frac{3.086\times10^{16}\text{ m}}{\text{pc}} = 4.40\times10^{41}\text{ kg}$$

$$= \frac{4.40\times10^{41}\text{ kg}}{1.988\times10^{30}\,\text{kg}/M_\odot} = 2.22\times10^{11}\ M_\odot\,,$$

$$M_{35} = \frac{Rv_{35}^2}{G} = \frac{\left(35\times10^3\text{pc}\right)\left(230\times10^3\text{ m/s}\right)^2}{6.674\times10^{-11}\text{ m}^3\text{kg}^{-1}\text{s}^{-2}}\cdot\frac{3.086\times10^{16}\text{ m}}{\text{pc}} = 8.56\times10^{41}\text{ kg}$$

$$= \frac{8.56\times10^{41}\text{ kg}}{1.988\times10^{30}\,\text{kg}/M_\odot} = 4.31\times10^{11} M_\odot\,.$$

(c) **Does this galaxy show signs of dark matter?**

If there were no dark matter, the mass should be constant at large radii. The fact that it continues to increase (or the flatness of the rotation curves), tells us that there is dark matter.

23. The star Betelgeuse has an estimated mass of 18 $M_\odot$, radius of 300 $R_\odot$, and luminosity of 10,200 $L_\odot$.

(a) Find the surface temperature of Betelgeuse compared to the Sun, and its value in K.

The luminosity of a star is given approximately by $L = 4\pi\sigma R^2 T^4$, where σ is the Stefan-Boltzmann constant. Dividing this by the same values for the Sun, we have

$$\frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{T_\odot}\right)^4,$$

$$\left(\frac{T}{T_\odot}\right)^4 = \frac{L}{L_\odot}\left(\frac{R}{R_\odot}\right)^{-2} = \frac{1.02\times10^4}{300^2} = 0.113,$$

$$T = (0.113)^{1/4} T_\odot = (0.582)(5772\text{ K}) = 3350\text{ K}.$$

(b) Find the estimated surface gravity of Betelgeuse in m/s^2, and in units of Earth's gravity g = 9.8 m/s^2.

The force of gravity is $F = GMm/R^2$, which will cause an acceleration of $a = F/m$, so

$$a = \frac{GM}{R^2} = \frac{\left(6.674\times10^{-11}\text{ m}^3\text{kg}^{-1}\text{s}^{-2}\right)\left(18\times1.988\times10^{30}\text{ kg}\right)}{\left(300\times6.96\times10^8\text{ m}\right)^2} = 0.0548\text{ m/s}^2.$$

Dividing by 9.8 m/s^2, this is only $0.0056g$. So the surface gravity is very low for Betelgeuse.

(c) Escape velocity is the velocity v such that the kinetic energy of an object of mass m matches the magnitude of the gravitational potential energy in the presence of a gravitational source. Find the escape velocity from the surface of Betelgeuse.

The gravitational potential energy of an object of mass m near a mass M is given by $E_P = GMm/R$. The kinetic energy of the same object at velocity v is $E_K = \frac{1}{2}mv^2$. Equating these, we have

$$\frac{mv^2}{2} = \frac{GMm}{R},$$

$$v^2 = \frac{2GM}{R} = \frac{2\left(6.674\times10^{-11}\text{m}^3/\text{kg/s}^2\right)\left(18\times1.988\times10^{30}\text{ kg}\right)}{300\left(6.96\times10^8\text{ m}\right)} = 2.288\times10^{10}\text{ m}^2\text{s}^{-2},$$

$$v = \sqrt{2.288\times10^{10}\text{ m}^2\text{s}^{-2}} = 1.51\times10^5\text{ m/s} = 151\text{ km/s}.$$

24. Three galaxies have their brightest red giants (RG) and brightest globular cluster (GC) apparent magnitudes measured, as shown at right.

Gal.	*m* (RG)	*m* (GC)	*d* (Mpc)	*M* (GC)
A	19.75	16.36	0.589	–7.49
B	21.63	18.22	1.40	–7.51
C	23.47	20.07	3.27	–7.50
D	?	22.46	9.82	–7.50

(a) For galaxies A, B, and C, estimate the distance using the tip of the red giant method

We assume they all have the same approximate absolute magnitude $M_t = -4.1$, and then use the formula for distance

$$d_A = 10^{1+\frac{1}{5}(m_A - M_t)}\ \text{pc} = 10^{1+\frac{1}{5}(19.75+4.1)}\ \text{pc} = 5.80\times 10^5\ \text{pc} = 0.589\ \text{Mpc},$$

$$d_B = 10^{1+\frac{1}{5}(m_B - M_t)}\ \text{pc} = 10^{1+\frac{1}{5}(21.63+4.1)}\ \text{pc} = 1.40\times 10^6\ \text{pc} = 1.40\,\text{Mpc},$$

$$d_C = 10^{1+\frac{1}{5}(m_C - M_t)}\ \text{pc} = 10^{1+\frac{1}{5}(23.47+4.1)}\ \text{pc} = 3.27\times 10^6\ \text{pc} = 3.27\,\text{Mpc}.$$

These numbers have been added to the table above.

(b) For the same three galaxies, find the absolute magnitude of the brightest globular clusters.

We use the formula $m - M = 5\log(d) - 5$, rearranged to give $M = m - 5\log(d) + 5$, which yields

$$M_A = m_A - 5\log(d_A) + 5 = 16.36 - 5\log(5.89\times 10^5) + 5 = -7.49,$$

$$M_B = m_B - 5\log(d_B) + 5 = 18.22 - 5\log(1.40\times 10^6) + 5 = -7.51,$$

$$M_C = m_C - 5\log(d_C) + 5 = 20.07 - 5\log(3.27\times 10^6) + 5 = -7.50.$$

These numbers appear in the table as well.

(c) Are the brightest globular clusters a decent standard candle? Why or why not?

Because the absolute magnitude is always around –7.50, they should make excellent standard candles.

(d) Galaxy D is too far away to see individual red giants, but not too far to see globular clusters. Estimate the distance to galaxy D.

We assume the gloubular cluster for D is also around $M = -7.50$, and then we find the distance from

$$d_D = 10^{1+\frac{1}{5}(m_D - M_{GC})}\ \text{pc} = 10^{1+\frac{1}{5}(22.46+7.5)}\ \text{pc} = 9.82\times 10^5\ \text{pc} = 9.82\ \text{Mpc}.$$