Physics 712
Chapter 3 Problems

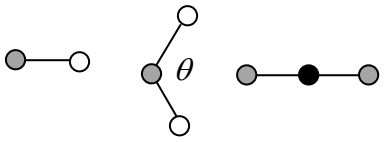
1. On the sphere $r = a$, the potential is given by $\Phi = \lambda x^2$.
   (a) Write the potential in terms of linear combinations of spherical harmonics. You will probably find the solution to quantum mechanics problem 7.5 part (b) helpful, which you can find at http://users.wfu.edu/ecarlson/quantum/solutions/sol7_5.pdf
   (b) Find the potential for $r > a$ and $r < a$. There should only be a finite number of terms in your final answer in each case.

2. A hydrogen atom in the $2P_z$ state has charge density given by

$$\rho(\mathbf{x}) = q\delta^3(\mathbf{x}) - \frac{qr^2}{32\pi a^5} e^{-r/a} \cos^2\theta$$

   Show that this has no $l = 0$ or $l = 1$ multipole moment, but it does have an $l = 2$ moment. Find the leading order contribution to the potential at large r.

3. Consider the three molecules at right. In each case, find only the leading multipole moment (smallest l), and then find the potential far from the molecule, keeping only the leading term. Assume the z-direction is to the right and the x-direction is up. Assume that any gray atom has charge $-2q$, any white atom has charge $+q$, and any black atom has charge $+4q$, and all bonds have length a. The bond angle is θ for the middle molecule; for the last one it is 180.



4. A grounded conducting cube of side a has charge density $\rho(\mathbf{x}) = \lambda x(a-x)\,y(a-y)\,z(a-z)$ inside it. Find the potential everywhere, and numerically at the center. You may keep only the leading term if you want.

5. A semi-infinite cylinder of radius a has potential $\Phi = 0$ on the lateral surface and $\Phi = V$ on the surface at $z = 0$. Write the potential in the interior as an infinite series. Assuming the potential does not diverge as $z \to \infty$, which coefficients must vanish? Find the potential everywhere, and numerically at $\rho = 0$ and $z = a$.