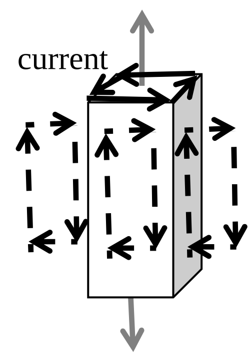
Physics 712
Chapter 5 Problems

1. We are trying to trap a charged particle of mass $q > 0$ and mass m by using a combination of magnetic and electric fields given by $\mathbf{B} = B\hat{\mathbf{z}}$ and $\mathbf{E} = A\left(x\hat{\mathbf{x}} + y\hat{\mathbf{y}} - 2z\hat{\mathbf{z}}\right)$.
   (a) Obviously, $\nabla \cdot \mathbf{B} = \nabla \times \mathbf{B} = 0$. Check that it also satisfies $\nabla \cdot \mathbf{E} = \nabla \times \mathbf{E} = 0$.
   (b) Assume the particle has motion given by $x = R\cos(\omega t)$, $y = R\sin(\omega t)$. Find an equation for ω in terms of A and B.
   (c) Argue that there is a maximum value of A for which circular motion is possible. Also argue that for $A > 0$, the particle will not "wander off" in the z-direction.

2. A wire loop centered on the origin in the xy-plane has perimeter C and carries a current I traveling counter-clockwise as viewed from above. Find the magnetic flux density at the origin if the loop is an equilateral triangle, square, regular hexagon, or circle.

3. Consider a cylinder of arbitrary cross-sectional shape, such as a square, circle, or other similar shape. This cylinder will be infinitely long. It will have a surface current K, with units A/m, running around it in a counter-clockwise direction as viewed from above. Because it is infinitely long, you can use symmetry arguments to show that the magnetic flux density will always be parallel to the axis everywhere inside and outside the cylinder.

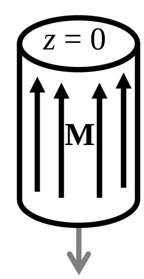


   (a) By considering the Ampere loop wholly inside the cylinder (middle dashed loop), argue that the magnetic field is in fact constant everywhere outside the cylinder. Repeat for the loop outside the cylinder (left dashed loop). If we assume the magnetic field at infinity is zero, what is the magnetic field everywhere outside this cylinder?
   (b) By considering the Ampere loop that is partly inside the cylinder and partly outside it (right dashed loop), find the magnetic flux density inside the cylinder.

4. Consider a localized static current distribution. By considering the expression $\int \nabla \cdot \left[x_i x_j \mathbf{J}(\mathbf{x})\right] d^3\mathbf{x}$, show $\int x_i J_j(\mathbf{x})\, d^3\mathbf{x} = -\int x_j J_i(\mathbf{x})\, d^3\mathbf{x}$.

5. For each of the four loops in problem 2, find the magnetic dipole moment $\mathbf{m}$ and the resulting magnetic flux density far from the loop.

6. A hard magnet is in the shape of a semi-infinite cylinder of radius R with its main axis on the z-axis and running from $z = -\infty$ to $z = 0$. It has magnetization $\mathbf{M} = M\hat{\mathbf{z}}$ uniformly inside it. Find the surface current $\mathbf{K}$ on the surface, and then use the Biot-Savart law for current density to find $\mathbf{B}$ on every point on the z-axis.



7. Repeat problem 6, but this time use the magnetic potential Φ_M approach. Find the magnetic potential on the z-axis, and then find the magnetic field $\mathbf{H}$ at all points $z > 0$. If you have been careful, you will find that $\mathbf{B} = \mu_0 \mathbf{H}$ for $z > L$, but not for $z < L$. Explain why.