Physics 780 – General Relativity

Homework B

5. We can define the four-acceleration as $A^{\mu} = \dfrac{d}{d\tau} U^{\mu} = \dfrac{d^2}{d\tau^2} x^{\mu}$.
   (a) Show that if you are in a frame such that a particle is momentarily at rest, so $U^{\mu}(\tau_0) = (1,0,0,0)$, then $\eta_{\mu\nu} A^{\mu} A^{\nu} = \mathbf{a}^2$, where **a** is the ordinary acceleration $\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$. Since $a = \sqrt{\eta_{\mu\nu} A^{\mu} A^{\nu}}$ is Lorentz invariant, this must be the magnitude of the acceleration as experienced by an object with arbitrary velocity U^{μ}
   (b) An object moves according to the formula $x = \sqrt{b^2 + t^2} - b$, $y = z = 0$. Rewrite these in terms of proper time τ (hint: this was almost done for you in class), then work out the four-velocity $U^{\mu}(\tau)$ the four-acceleration $A^{\mu}(\tau)$ and find a formula for the proper acceleration *a*, which should be constant.
   (c) Determine the value of *b* in years (or light-years) if $a = g = 9.80\ \text{m/s}^2$.
   (d) If you leave Earth, starting at rest, and accelerate at *g*, how much proper time in years would it take you go get to α-Centauri (4.3 *c*·yr), the center of the galaxy (27,000 *c*·yr) and the approximate edge of the Universe (2×10^{10} *c*·yr).

6. The electromagnetic field tensor $F_{\mu\nu}$ is given by equation (1.69). If the fields are given by the three components of **E** and **B**, what would be the new values of the electric and magnetic fields **E′** and **B′** if you
   (a) Performed a rotation by an angle θ around the *z*-axis,
   (b) Perform a boost in the *x*-direction by rapidity ϕ.
   The corresponding inverse Lorentz matrices in each case are given below. Note that because $F_{\mu\nu}$ is anti-symmetric, you can calculate just six components of $F_{\mu\nu}$ to get **E′** and **B′**.

$$\Lambda^{-1}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \Lambda^{-1}(\phi) = \begin{pmatrix} \cosh\phi & \sinh\phi & 0 & 0 \\ \sinh\phi & \cosh\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$