Physics 780 – General Relativity

Homework Set G

17. Consider the flat FLRW metric, $ds^2 = -\mathrm{d}t^2 + a^2(t)\left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right)$.

    (a) Consider first the case of a radiation-dominated universe, $a(t) = \sqrt{t}$, with a big bang singularity at $t = 0$. In time t, how far can a light beam travel, starting at the origin? Give your answer in the form $s = kt$, where k is a simple constant.

    (b) Now consider an exponentially expanding universe, with $a(t) = e^{Ht}$, with H a constant. In this case, nothing special happens at $t = 0$, so let's define $t = 0$ as now. Imagine a light beam starting at us at $x = 0$ and traveling in the x-direction. Find $x(t)$, and show that there is a limiting value x_∞ that cannot be reached by the light beam, even as $t \to \infty$.

18. In this problem we will find the 2D "volume" of two similar metrics. Note that the answer is not guaranteed to be finite.

    (a) First consider the metric $ds^2 = \dfrac{\mathrm{d}x^2 + \mathrm{d}y^2}{\left(1 + x^2 + y^2\right)^2}$, where x and y are unrestricted real numbers. As a first step, rewrite this metric in polar coordinates, $(x, y) = (\rho\cos\phi, \rho\sin\phi)$. What is the appropriate range of ρ and ϕ?

    (b) Calculate the volume of the metric described in part (a).

    (c) Repeat parts (a) and (b) for the metric $ds^2 = \dfrac{\mathrm{d}x^2 + \mathrm{d}y^2}{\left(1 - x^2 - y^2\right)^2}$, where now x and y are restricted to the disk $x^2 + y^2 < 1$.