Physics 780 – General Relativity

Homework Set J

25. In homework set H, problem 20, you had to work out all the components if $\Gamma^{\nu}_{\alpha\beta}$ for the metric $ds^2 = h(r)\,\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2$.
    (a) Use these to get all non-zero components of the Riemann tensor of the form $R^{\mu}{}_{\nu\mu\nu}$ (no sums). There should be six in total. As a check, note that they must all vanish if $h(r) = 1$.
    (b) Find the diagonal components of the Ricci tensor, $R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}$, for the three components R_{rr}, $R_{\theta\theta}$, and $R_{\phi\phi}$. If you have made no mistakes so far, you should find $R_{\phi\phi} = \sin^2\theta R_{\theta\theta}$.
    (c) Find the Ricci scalar and show that it equals $R = \dfrac{2h'}{h^2 r} + \dfrac{2}{r^2} - \dfrac{2}{r^2 h}$.

26. Assume that the metric found in question 25 is homogenous, and in particular, the Ricci scalar is a constant given by 6*C*, so $R = 6C$.
    (a) Find a simple formula for the combination $\dfrac{1}{r^2}\dfrac{d}{dr}\left(\dfrac{r}{h}\right)$.
    (b) Multiply this equation by r^2 and integrate it. The constant of integration can be found if we insist that $h(r)$ does not diverge at the origin. Solve the equation for h.