Physics 780 – General Relativity

Homework Set L

28. Download the program **grcalc** from the class website and run it on the metric $ds^2 = h(r)\,\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)$ (or find any other program you can use to calculate it). Compare to the posted answers (or your answers) to problem 25, parts (b) and (c). Do they agree? As your submission, you may send me the Maple worksheet, or any other evidence you did the work.

29. The Einstein equations in 4D, $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$ can be rewritten in the form $R_{\mu\nu} = 8\pi G\left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right)$. Redo this process if we have a cosmological constant, so starting from $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$, rewrite it in the form $R_{\mu\nu} = \cdots$, where there is no curvature on the right side.

30. Assume a static, spherically symmetric spacetime as given below, outside a spherical source (so $T_{\mu\nu} = 0$), but include a cosmological constant. The metric is

$$ds^2 = -f(r)\,\mathrm{d}t^2 + h(r)\,\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)$$

(a) Using your formula from problem 29, write down equations for R_{tt}, R_{rr}, and $R_{\theta\theta}$, substituting the explicit forms for $R_{\mu\nu}$ using the handout **sssst.pdf**

(b) Combining $hR_{tt} + fR_{rr}$, show that in this case you *still* can show that fh is a constant, which can be chosen to be 1.

(c) Rewrite the $R_{\theta\theta}$ equation by replacing $h = f^{-1}$.

(d) Solve the $R_{\theta\theta}$ equation to find $f(r)$.