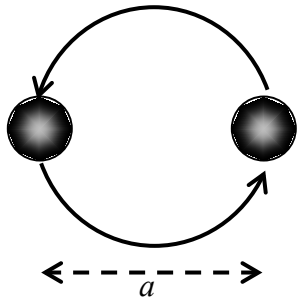
Physics 780 – General Relativity

Homework Set X

56. Suppose we have two non-relativistic objects of mass M orbiting each other in circular orbits with a separation a (so the radius of the orbit is $a/2$).



(a) Since the objects are non-relativistic, we can use Newtonian approximations. By equating the gravitational force to the centripetal force $F = M\omega^2 (a/2)$, find a formula for the angular velocity for the orbit ω.

(b) Write the position of each particle, assuming they are orbiting in the xy-plane about the origin, as a function of time.

(c) Find the moments $Q_{ij} = \sum_a m_a x_a^i x_a^j$ as a function of time.

(d) Rewrite $Q_{ij}(t)$ as a constant term plus oscillatory terms. What is the frequency of the oscillatory terms? Write the oscillating terms as $Q_{ij}(t) = (\text{constant}) + Q_{ij} e^{-i\omega t} + Q_{ij}^* e^{i\omega t}$.

(e) Find the power radiated $P = \frac{2}{5} G\omega^6 c^{-5} Q_{ij}^* Q_{ij}$.

(f) Find total energy $E = K + V$, where the potential energy is $V = -GM^2/a$, and K is the sum of the two potential energies, each of which is $K_a = \frac{1}{2} M (\omega a/2)^2$. You should find that the potential energy is exactly twice as big as the kinetic term (and of opposite sign).

(g) Find a formula for the characteristic time $\tau = |E|/P$ it will take for the orbit to decay to radius zero. Evaluate it for $M = M_{Sun} = 1.989 \times 10^{30}$ kg and $a = 2 \times 10^4$ km.