Physics 745 - Group Theory

Homework Set 19
Due Monday, March 23

1. In the defining representation, the four generators of the group $U(2)$ can be given by

$$T_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad T_2 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad T_4 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

   (a) These generators are orthogonal, but not orthonormal. Assume the even ones T_2 and T_4 are normalized correctly, fix the other two by multiplying by an appropriate constant such that they will be orthonormal as well.

   (b) Using your "fixed" generators, work out all commutators $[T_a, T_b]$ for every pair $a < b$ (6 total). Write your answer in terms of other generators.

   (c) Find all the non-zero structure constants f_{abc} for this group. You may use the complete anti-symmetry of f_{abc} to save work or check your answers, if you wish.