Physics 745 - Group Theory

Homework Set 25
Due Monday, April 6

1. In the electric dipole approximation, the rate at which an atom decaying from one state to another by the emission of a photon is given by $\Gamma\left(I \to F\right) = \frac{4}{3}\alpha\omega_{IF}^3 \left|\mathbf{r}_{FI}\right|^2 / c^2$ where

$$\mathbf{r}_{FI} = \left\langle F \left| \mathbf{r} \right| I \right\rangle$$

The absolute value symbol means that $\left|\mathbf{r}_{FI}\right|^2$ contains not only a sum of the three components of **r**, but also the real and imaginary parts.

(a) Demonstrate first that

$$\left|\mathbf{r}_{FI}\right|^2 = \sum_{q=-1}^{1} \left| \left\langle F \left| r_q^{(1)} \right| I \right\rangle \right|^2$$

where $r_q^{(1)}$ are the three components of the spherical tensor corresponding to the vector operator **r**.

(b) An atom in the state $\left|njm\right\rangle$ with j = 3/2 is about to decay via dipole radiation. What possible j' values might be allowed for the final state $\left|n'j'm'\right\rangle$?

(c) The atom is actually going to decay to a state with j' = ½. Using the Wigner Eckart Theorem, find the *relative* rate of decay

$$\Gamma\left(njm \to n'j'm'\right)$$

for all non-vanishing possible values of m and m'.