Physics 745 - Group Theory

Homework Set 31

Due Friday, April 24

1. The Σ^{*0} is part of the 10 of SU(3). There are four possible decays that conserve charge and hypercharge and are kinematically allowed (that means they don't violate conservation of energy and momentum): $\Sigma^0\pi^0$, $\Sigma^+\pi^-$, $\Sigma^-\pi^+$, and $\Lambda\pi^0$. Indeed, these decay modes represent the overwhelming majority of the decay modes for the Σ^{*0}.
   a) In each of the four cases, work out the corresponding matrix element $\left\langle BM \middle| H \middle| \Sigma^{*0} \right\rangle$.
   b) Which of the four "allowed" decays does not actually occur? For each of the other three cases, make a naïve prediction of the relative rate for the decay $\Gamma\left(\Sigma^{*0} \to BM\right)$, and predict the fraction that each decay occurs, which is the decay rate for a given channel divided by the total. (This is called the branching ratio. Because the Λ is noticeably lighter than the Σ's, the $\Lambda\pi^0$ mode actually is enhanced a bit compared to the naïve prediction).

2. The η_{c0} is a heavy, neutral, SU(3) singlet meson. Among its many decay modes, it can decay to two light mesons, $\eta_{c0} \to M'M$.
   a) Suppose we write the matrix elements for the *M* and *M'* as $\left|M\right\rangle = w^i_j \left|M^j_i\right\rangle$ and $\left|M'\right\rangle = u^i_j \left|M^j_i\right\rangle$. Write down the form of all possible non-vanishing terms that appear in
   $$\left\langle M'M \middle| H \middle| \eta_{c0} \right\rangle .$$
   The η_{c0} has no indices associated with it, because it is an SU(3) singlet.
   b) Calculate the relative size of the matrix element for $\left|M'M\right\rangle = \left|\pi^0\pi^0\right\rangle$, $\left|\pi^\pm\pi^\mp\right\rangle$, $\left|K^\pm K^\mp\right\rangle$, $\left|K^0\bar{K}^0\right\rangle$, $\left|\bar{K}^0 K^0\right\rangle$, and $\left|\eta\eta\right\rangle$ (eight cases in all).
   c) The mesons are so light that their relative masses are irrelevant. Predict the relative decay rates for $\Gamma\left(\eta_{c0} \to \pi^0\pi^0\right)$, $\Gamma\left(\eta_{c0} \to \pi^+\pi^-\right)$, $\Gamma\left(\eta_{c0} \to K^0\bar{K}^0\right)$, $\Gamma\left(\eta_{c0} \to K^+K^-\right)$, and $\Gamma\left(\eta_{c0} \to \eta\eta\right)$. In some cases, you will have to add the results of two different decay rates, since $\Gamma\left(A \to BC\right)$ is really the sum of $\Gamma\left(A \to BC\right)$ and $\Gamma\left(A \to CB\right)$.