Physics 745 - Group Theory

Solution Set 15

The crystal momentum **k** forms a cubic lattice. In units of the **G**'s, the basis of the reciprocal lattice, the lines/points listed have the coordinates listed, where *a* denotes an arbitrary real number (though as drawn, *a* would be between 0 and ½).

For each type of point, we have to determine which of the 48 symmetry operations of O_h keep them the same, and then figure out what subgroup this represents. Keep in mind that since adding a basis vector **G** to a reciprocal lattice is physically meaningless, so ½ and - ½ are the same thing.

One thing I found very helpful, in making my own tables, was that the only group with only four elements of which two are mirror planes is C_{2v}.

Γ: Clearly unchanged by all symmetry operations, so O_h.

X: Unchanged by symmetry operations that mix/reverse *y* and *z*, and either leaves *x* alone or reverses it, this is D_{4h}.

M: Unchanged by symmetry operations that mix/reverse *x* and *y*, and either leaves *z* alone or reverses it, this is D_{4h}.

R: Unchanged by all symmetry operations, so O_h.

Δ: Unchanged by all symmetry operations that mix/reverse *y* and *z*, but don't touch *x*, this is C_{4v}.

Σ: Unchanged by anything that swaps *x* and *y*, or reverses *z*, or both, this is C_{2v}.

Λ: Unchanged by any permutation of *x*, *y*, and *z*, this has a C_3 axis and three mirror planes, so it's C_{3v}.

Z: Unchanged by things that reverse *x* or *z* (or both), this is C_{2v}.

S: Unchanged by things that swap *y* and *z*, or reverse *x*, this is C_{2v}.

T: Unchanged by things that mix/reverse *x* and *y*, but don't touch *z*, this is C_{4v}.

Pt	Coords	Sym
Γ	$(0,0,0)$	O_h
X	$(\frac{1}{2},0,0)$	D_{4h}
M	$(\frac{1}{2},\frac{1}{2},0)$	D_{4h}
R	$(\frac{1}{2},\frac{1}{2},\frac{1}{2})$	O_h
Δ	$(a,0,0)$	C_{4v}
Σ	$(a,a,0)$	C_{2v}
Λ	(a,a,a)	C_{3v}
Z	$(\frac{1}{2},a,0)$	C_{2v}
S	$(\frac{1}{2},a,a)$	C_{2v}
T	$(\frac{1}{2},\frac{1}{2},a)$	C_{4v}