Tinkham Problem 2-1

$\cdot$	***E***	***A***	***B***	***C***	***D***	***F***	***G***	***H***
E	*E*	*A*	*B*	*C*	*D*	*F*	*G*	*H*
A	*A*	*E*	*G*	*H*	*F*	*D*	*B*	*C*
B	*B*	*G*	*E*	*F*	*H*	*C*	*A*	*D*
C	*C*	*F*	*H*	*E*	*G*	*A*	*D*	*B*
D	*D*	*H*	*F*	*G*	*E*	*B*	*C*	*A*
F	*F*	*C*	*D*	*B*	*A*	*G*	*H*	*E*
G	*G*	*B*	*A*	*D*	*C*	*H*	*E*	*F*
H	*H*	*D*	*C*	*A*	*B*	*E*	*F*	*G*

a) The multiplication table is given at right.
b) The classes are *E*, *G*, *AB*, *CD*, and *FH*.
c) The proper subgroups are *E*, *EG*, *EA*, *EB*, *EC*, *ED*, *EABG*, *ECDG*, and *EFGH*. Of these, *E*, *EG*, *EABG*, *ECDG*, and *EFGH* are normal.
d) The cosets for the subgroup *E* are, of course, all the elements of the group. The cosets for the other proper subgroups are: *EG*: *AB*, *CD*, *FH*; *EABG*: *CDFH*; *ECDG*: *ABFH*; *EFGH*: *ABCD*
e) Yuck! They are given in the tables at right. Unlike Tinkham, I define the subgroup itself as one of the cosets, so I've labeled the identity element as the subgroup.

$\cdot$	***EG***	***AB***	***CD***	***FH***
EG	*EG*	*AB*	*CD*	*FH*
AB	*AB*	*EG*	*FH*	*CD*
CD	*CD*	*FH*	*EG*	*AB*
FH	*FH*	*CD*	*AB*	*EG*

$\cdot$	***EABG***	***CDFH***
EABG	*EABG*	*CDFH*
CDFH	*CDFH*	*EABG*

$\cdot$	***ECDG***	***ABFH***
ECDG	*ECDG*	*ABFH*
ABFH	*ABFH*	*ECDG*

$\cdot$	***EFGH***	***ABCD***
EFGH	*EFGH*	*ABCD*
ABCD	*ABCD*	*EFGH*

f) The multiplication tables for the classes are listed in the table below. From these the coefficients can be readily deter-mined.

$\cdot$	***E***	***G***	***AB***	***CD***	***FH***
E	*E*	*G*	*AB*	*CD*	*FH*
G	*G*	*E*	*AB*	*CD*	*FH*
AB	*AB*	*AB*	2*E*+2*G*	2*FH*	2*CD*
CD	*CD*	*CD*	2*FH*	2*E*+2*G*	2*AB*
FH	*FH*	*FH*	2*CD*	2*AB*	2*E*+2*G*

Homework Set 2

1. Matrix $S = S^{-1} = S^{\dagger} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ -i & -1 \end{pmatrix}$, $d = S^{-1}MS = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix}$

2. Matrix $S = S^{-1} = S^{\dagger} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, $d = S^{-1}MS = \begin{pmatrix} \cos\theta + \sin\theta & 0 \\ 0 & \cos\theta - \sin\theta \end{pmatrix}$

3. Matrix $S = \begin{pmatrix} 1 & 5 & 1 \\ 0 & 2 & 0 \\ 0 & -1 & 3 \end{pmatrix}$, $S^{-1} = \begin{pmatrix} 1 & -\frac{8}{3} & -\frac{1}{3} \\ 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} \end{pmatrix}$, $d = S^{-1}MS = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$