Physics 745 - Group Theory

Solution Set 24

1. [5] The *total* angular momentum of an atom actually has three pieces: The orbital angular momentum of the atom, the spin of the electron, and the spin of the nucleus.

(a) [1] Suppose a hydrogen atom has a single electron in a p-wave (that means *l* = 1). What are the possible values for the total angular momentum of the electron (orbit plus electron spin). The electron has spin ½.

The total angular momentum of the electron is the combination of the orbital and spin angular momentum, which ranges from $l+s$ to $|l-s|$ in steps of one. Hence it goes from $\frac{3}{2}$ to $\frac{1}{2}$, so these are the only allowed values: $1\otimes\frac{1}{2}=\frac{1}{2}\oplus\frac{3}{2}$

(b) [2] What are the possible values of the total angular momentum of an entire ordinary hydrogen atom? If there are two ways to make a certain combination, list it twice. Ordinary hydrogen atoms have a nucleus consisting of a proton with spin ½.

To get the total angular momentum, we simply combine with the previous part, and find

$$1\otimes\tfrac{1}{2}\otimes\tfrac{1}{2}=\left(\tfrac{1}{2}\oplus\tfrac{3}{2}\right)\otimes\tfrac{1}{2}=\left(\tfrac{1}{2}\otimes\tfrac{1}{2}\right)\oplus\left(\tfrac{3}{2}\otimes\tfrac{1}{2}\right)=0\oplus1\oplus1\oplus2$$

Note there are two ways to make total angular momentum 1.

(c) [2] Repeat part (b) for heavy hydrogen, an atom containing a nucleus with spin 1.

The procedure is exactly as before,

$$1\otimes\tfrac{1}{2}\otimes1=\left(\tfrac{1}{2}\oplus\tfrac{3}{2}\right)\otimes1=\left(\tfrac{1}{2}\otimes1\right)\oplus\left(\tfrac{3}{2}\otimes1\right)=\tfrac{1}{2}\oplus\tfrac{3}{2}\oplus\tfrac{1}{2}\oplus\tfrac{3}{2}\oplus\tfrac{5}{2}$$

2. [5] Quarks are components of some particles that interact very strongly, so that, for example, different ways of binding the same three quarks can have significantly different energies, enough different that they are given different names. At right is a complete list of those particles lighter than 1500 MeV/c^2 which are a combination of one quark each of type up, down, and strange. Each of these quarks has the same spin. The lowest energy (mass) baryons always have zero orbital angular momentum, but the "spin" of the baryon comes from the spin of the quarks. Explain why the list is the way it is; *i.e.*, why we have these three particles and no others. You will have to guess the spin of the quarks.

Name	Mass	Spin
Λ	1116	1/2
Σ^0	1193	1/2
Σ^{*0}	1384	3/2

Well, we *could* cheat and look up the spin of the quarks, or we could try to intelligently guess. We notice the largest spin is 3/2, which, since we are combining three quarks, might be attributed to adding together three spin ½ quarks. Let's just conjecture this and see what we get. Combining the three spins, we find

$$\tfrac{1}{2}\otimes\tfrac{1}{2}\otimes\tfrac{1}{2}=\left(0\oplus1\right)\otimes\tfrac{1}{2}=\left(0\otimes\tfrac{1}{2}\right)\oplus\left(1\otimes\tfrac{1}{2}\right)=\tfrac{1}{2}\oplus\tfrac{1}{2}\oplus\tfrac{3}{2}$$

Hence, if our conjecture is right, the lightest baryons consisting of these three quarks would have spins of ½, ½, and 3/2, Indeed, this is exactly what is observed.