Point Groups in Three Dimensions

Name	Restrictions*	Sample Elements	Order	Isomorphisms**	Crystals
$\mathcal{C}_N$		C_N	N		$N = 1,2,3,4,6$
$\mathcal{C}_{Nv}$	$N > 1$	C_N, σ_v	$2N$		$N = 2,3,4,6$
$\mathcal{C}_{Nh}$		C_N, σ_h	$2N$	$= \mathcal{C}_N \times \boldsymbol{\sigma_h}$	$N = 1,2,3,4,6$
$\mathcal{S}_N$	N even	S_N	N	$\mathcal{C}_N$	$N = 2,4,6$
$\mathcal{D}_N$	$N > 1$	C_N , C_2'	$2N$	$\mathcal{C}_{Nv}$	$N = 2,3,4,6$
$\mathcal{D}_{Nh}$	$N > 1$	C_N, C_2', σ_v, σ_h	$4N$	$= \mathcal{C}_{Nv} \times \boldsymbol{\sigma_h}$	$N = 2,3,4,6$
$\mathcal{D}_{Nd}$	$N > 1$	C_N, C_2', σ_d, S_{2N}	$4N$	$\mathcal{C}_{2N,v}$	$N = 2,3$
$\mathcal{C}_\infty$		$C(\theta)$	∞		No
$\mathcal{C}_{\infty v}$		$C(\theta)$, σ_v	∞		No
$\mathcal{C}_{\infty h}$		$C(\theta)$, σ_h	∞	$= \mathcal{C}_\infty \times \boldsymbol{\sigma_h}$	No
$\mathcal{D}_\infty$		$C(\theta)$, C_2'	∞	$\mathcal{C}_{\infty v}$	No
$\mathcal{D}_{\infty h}$		$C(\theta)$, C_2', σ_v, σ_h	∞	$= \mathcal{C}_{\infty v} \times \boldsymbol{\sigma_h}$	No
$\mathcal{T}$		C_3, C_2	12		Yes
$\mathcal{T}_\mathrm{d}$		C_3, C_2, σ, S_4	24		Yes
$\mathcal{T}_\mathrm{h}$		C_3, C_2, i, σ	24	$= \mathcal{T} \times \boldsymbol{i}$	Yes
$\mathcal{O}$		C_2, C_3, C_4	24	$\mathcal{T}_\mathrm{d}$	Yes
$\mathcal{O}_\mathrm{h}$		C_2, C_3, C_4, i, σ, S_4, S_6	48	$= \mathcal{O} \times \boldsymbol{i}$	Yes
$\mathcal{I}$		C_2, C_3, C_5	60		No
$\mathcal{I}_\mathrm{h}$		C_2, C_3, C_5, i, σ, S_6, S_{10}	120	$= \mathcal{I} \times \boldsymbol{i}$	No
SO(3)		$C(\theta)$	∞		No
O(3)		$C(\theta)$, i	∞	$=$ SO(3) $\times \boldsymbol{i}$	No

* $\mathcal{C}_{1v} = \mathcal{C}_{1h}$ $\mathcal{D}_1 = \mathcal{C}_2$ $\mathcal{D}_{1d} = \mathcal{C}_{2h}$ $\mathcal{D}_{1h} = \mathcal{C}_{2v}$ $\mathcal{S}_N = \mathcal{C}_{Nh}$ for N odd

** The = sign means physical identity, in addition to isomorphic

Bravais Lattices in Three Dimensions

Crystallographic Systems	Restrictions	Point Groups	Bravais Lattices
Triclinic	*none*	$\mathcal{C}_1$, $\mathcal{S}_2$	Triclinic-P
Monoclinic	$\alpha = \beta = 90^o$	$\mathcal{C}_{1h}$, $\mathcal{C}_2$, $\mathcal{C}_{2h}$	Monoclinic-P, -B
Orthorhombic	$\alpha = \beta = \gamma = 90^o$	$\mathcal{C}_{2v}$, $\mathcal{D}_2$, $\mathcal{D}_{2h}$	Orthorhombic-P, -C, -I, -F
Tetragonal	$\alpha = \beta = \gamma = 90^o$ $a = b$	$\mathcal{C}_4$, $\mathcal{S}_4$, $\mathcal{D}_4$, $\mathcal{C}_{4h}$, $\mathcal{D}_{2d}$, $\mathcal{C}_{4v}$, $\mathcal{D}_{4h}$	Tetragonal-P, -I
Trigonal	$\alpha = \beta = 90^o$, $\gamma = 120^o$, $a = b$	$\mathcal{C}_3$, $\mathcal{S}_6$, $\mathcal{C}_{3v}$, $\mathcal{D}_3$, $\mathcal{D}_{3d}$	Hexagonal-P, Trigonal-R
Hexagonal	$\alpha = \beta = 90^o$, $\gamma = 120^o$, $a = b$	$\mathcal{C}_6$, $\mathcal{C}_{3h}$, $\mathcal{D}_6$, $\mathcal{D}_{3h}$, $\mathcal{C}_{6h}$, $\mathcal{C}_{6v}$, $\mathcal{D}_{6h}$	Hexagonal-P
Cubic	$\alpha = \beta = \gamma = 90^o$ $a = b = c$	$\mathcal{T}$, $\mathcal{T}_h$, $\mathcal{T}_d$, $\mathcal{O}$, $\mathcal{O}_h$	Cubic-P, -I, -F

Suffix Meanings

Suffix	Elements of Translation Group
-P	$\{(0,0,0)\}$
-I	$\{(0,0,0),(\frac{1}{2},\frac{1}{2},\frac{1}{2})\}$
-B	$\{(0,0,0),(\frac{1}{2},0,\frac{1}{2})\}$
-C	$\{(0,0,0),(0,\frac{1}{2},\frac{1}{2})\}$
-F	$\{(0,0,0),(0,\frac{1}{2},\frac{1}{2}),(\frac{1}{2},0,\frac{1}{2}),(\frac{1}{2},\frac{1}{2},0)\}$
-R	$\{(0,0,0),(\frac{2}{3},\frac{1}{3},\frac{1}{3}),(\frac{1}{3},\frac{2}{3},\frac{2}{3})\}$