Physics 215– Elementary Modern Physics

Equations for Test 3

The following equations you should have memorized, and understand how to use them:

For one dimension

Expectation Value of Operator

$$\langle \mathcal{O} \rangle = \int_{-\infty}^{\infty} \psi^*(x)\,\mathcal{O}\,\psi(x)\,dx$$

Uncertainty

$$(\Delta\mathcal{O})^2 = \langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2$$

Time-dependence

$$\Psi(x,t) = e^{-iEt/\hbar}\psi(x)$$

Schrödinger Equations

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi + V\Psi \quad \text{or} \quad i\hbar\frac{\partial}{\partial t}\Psi = H\Psi$$

$$E\psi = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V\psi \quad \text{or} \quad E\psi = H\psi$$

Momentum Operator

$$p_{op} = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

Hamiltonian

$$H = \frac{1}{2m}p_{op}^2 + V$$

$$\bar{E} = \langle H \rangle$$

For three dimensions

Momentum Operators (3D):

$$p_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$p_y = \frac{\hbar}{i}\frac{\partial}{\partial y}$$

$$p_z = \frac{\hbar}{i}\frac{\partial}{\partial z}$$

Schrödinger Equations in 3D:

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Psi + V\Psi \quad \text{or} \quad i\hbar\frac{\partial}{\partial t}\Psi = H\Psi$$

$$E\psi = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi + V\psi \quad \text{or} \quad E\psi = H\psi$$

The Hamiltonian

$$H = \frac{1}{2m}\vec{p}_{op}^{\,2} + V$$

$$\bar{E} = \langle H \rangle$$

Restrictions on Quantum Numbers:

$$n = 1, 2, 3, \ldots$$

$$l = 0, 1, 2, 3, \ldots, n-1$$

$$m = -l, -l+1, \ldots, 0, \ldots, l$$

$$m_s = \pm\tfrac{1}{2}$$

Spin Values

$$s = \tfrac{1}{2}$$

$$S^2 = \hbar^2\left(s^2 + s\right) = \tfrac{3}{4}\hbar^2$$

$$S_z = \hbar m_s$$

Angular Momentum Values:

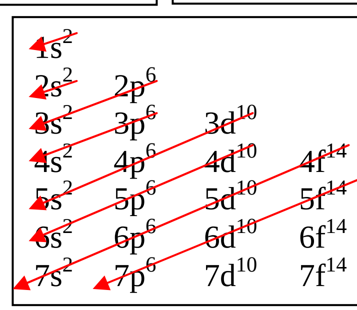
$$L^2 = \hbar^2\left(l^2 + l\right)$$

$$L_z = \hbar m$$

Hydrogen Wave Functions

$$\psi_{n,l,m}(r,\theta,\phi) = R_{n,l}(r)\,Y_{l,m}(\theta,\phi)$$

Filling up Atoms: $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2 \ldots$



The following equations you need not memorize, but you should know how to use them if given to you:

$$h = 6.626\times10^{-34}\ \text{J}\cdot\text{s} = 4.136\times10^{-15}\ \text{eV}\cdot\text{s}$$
$$\hbar = 1.055\times10^{-34}\ \text{J}\cdot\text{s} = 6.582\times10^{-16}\ \text{eV}\cdot\text{s}$$

Harmonic Osc.

$$E_n = \hbar\omega\left(n+\tfrac{1}{2}\right)$$
$$n = 0,1,2,\ldots$$

Hydrogen

$$E_n = -\frac{(13.6\ \text{eV})Z^2}{n^2}$$

1D square well:

$$E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$$
$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi n x}{L}\right)$$
$$n = 1,2,3,\ldots$$

Barrier penetration:

$$T \approx 16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)e^{-2\alpha L},$$
$$\alpha = \frac{\sqrt{2m(V_0-E)}}{\hbar}$$

Reflection off a step:

$$R = \begin{cases} \left(\dfrac{\sqrt{E}-\sqrt{E-V_0}}{\sqrt{E}+\sqrt{E-V_0}}\right)^2 & \text{if } E > V_0 \\ 1 & \text{if } E < V_0 \end{cases}$$

Layout of the exam: Below is an outline of the exam

This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered.

Part I: Multiple Choice [20 points]
For each question, choose the best answer (2 points each)

[questions 1-10]

Part II: Short answer [20 points]
Choose **two** of the following three questions and give a short answer (1-3 sentences) (10 points each).

[questions 11-13]

Part III: Calculation: [60 points]
Choose **three** of the following four questions and perform the indicated calculations (20 points each)

[questions 14-17]