

General Relativity

Quantum Field Theory – also known as particle physics – is wonderful, and has had a host of successes. It successfully accounts for all known effects at the microscopic level, and a good many effects at the macroscopic level as well. There is, however, one glaring flaw: it does not include the effects of gravity. The problem is that we have no experimental measurements of gravity at the microscopic level. The gravitational attraction between a pair of protons is many orders of magnitude smaller than their electrostatic repulsion, and as a consequence, we have no clue how to build a theory of gravity involving fundamental particles.

As a first step, however, it seems like it would be a good idea to try to at least build a relativistic theory of gravity; that is, one that combines Einstein's special theory of relativity with gravitational theory. This goal was achieved by Einstein, and the resulting theory is known as the general theory of relativity, usually referred to as general relativity or even just GR. This theory is rather complicated, and there are several subtleties which need to be discussed, but I hope that I can impart at least some of the flavor of what is going on in this short handout.

A. The Principle of Equivalence

One of the keys that helped Einstein understand what was going on in GR was the principle of equivalence. Consider first the formula for the force between a pair of objects, which is given by

$$F = -\frac{GMm}{r^2}$$

Where G is Newton's constant, M and m are the two masses, and r is their separation. Now, suppose we have an object of mass m , and we let this force act on it. The acceleration that the mass m feels will be given by

$$a = \frac{F}{m} = -\frac{GM}{r^2}$$

Note that the result is independent of the mass m . Hence all objects fall at exactly the same rate, as first pointed out by Galileo. This property of gravity is unlike other forces; for example, electric and magnetic forces do not behave this way at all. Indeed, the fact that objects fall at the same rate has been tested with extreme precision, and works extremely well.

Can we think of any other cases where force is proportional to mass? Consider centrifugal force, the apparent force felt by a mass m as viewed in a rotating frame of reference. This force is given by

$$F = \frac{mv^2}{r}$$

Hence, for example, if you are on a merry-go-round, and you release a ball, you will find that it apparently accelerates away from you. Furthermore, if you release two balls of different masses, you will find that they accelerate away from you at exactly the same rate. The reason, of course, is that the balls aren't accelerating at all, it is you who are accelerating, and the balls are simply going in a straight line. If you were unaware of your rotation, you might attribute the acceleration of the balls to some sort of weird gravitational effect, but this is an illusion because you are simply viewing the balls in an accelerating reference frame.

Now, if you were on the merry-go-round, and you wish to see that the balls are not really accelerating, all you need to do is let go. You, like the balls, will move away from the merry-go-round in a straight line, and you won't see them accelerating at all. In this frame of reference, there will be no centrifugal force at all.

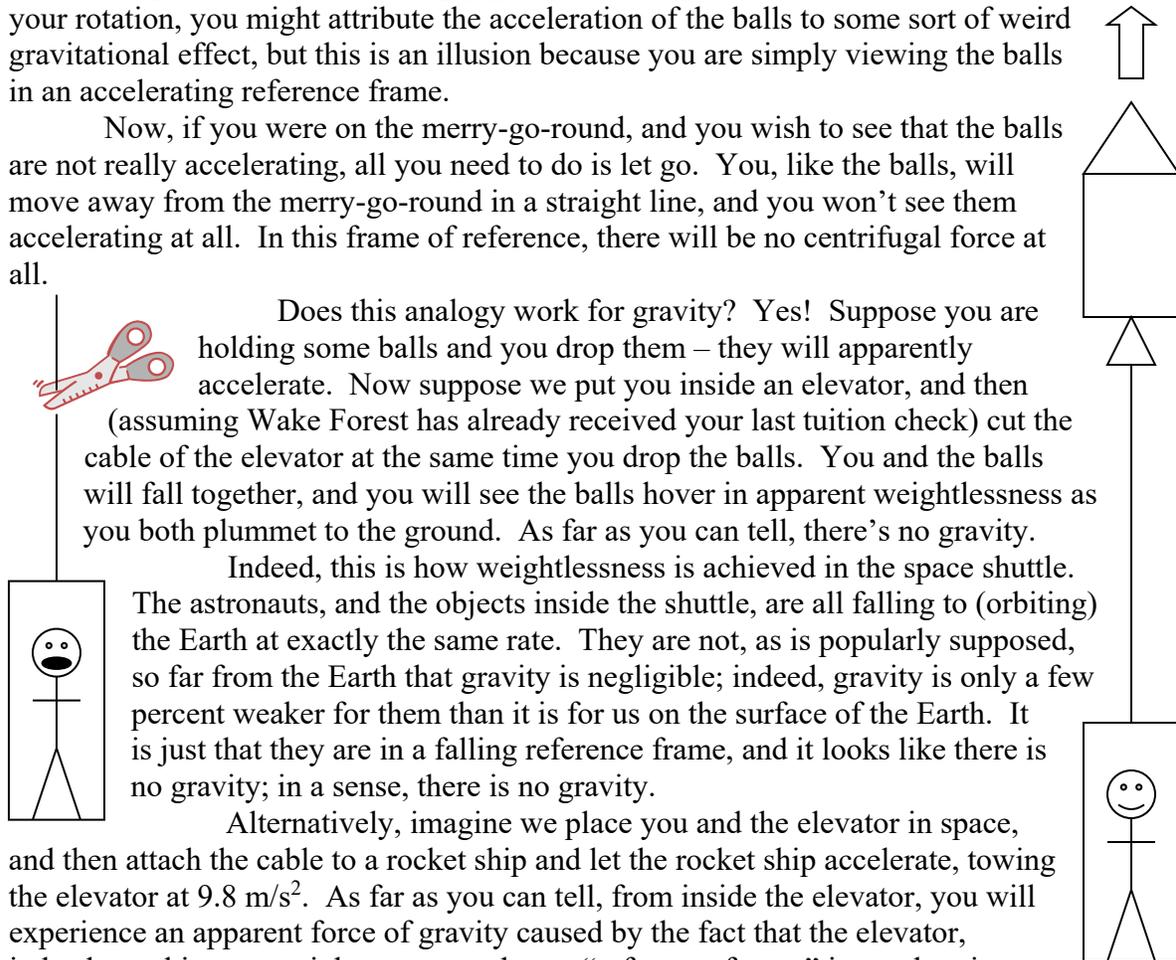
Does this analogy work for gravity? Yes! Suppose you are holding some balls and you drop them – they will apparently accelerate. Now suppose we put you inside an elevator, and then (assuming Wake Forest has already received your last tuition check) cut the cable of the elevator at the same time you drop the balls. You and the balls will fall together, and you will see the balls hover in apparent weightlessness as you both plummet to the ground. As far as you can tell, there's no gravity.

Indeed, this is how weightlessness is achieved in the space shuttle. The astronauts, and the objects inside the shuttle, are all falling to (orbiting) the Earth at exactly the same rate. They are not, as is popularly supposed, so far from the Earth that gravity is negligible; indeed, gravity is only a few percent weaker for them than it is for us on the surface of the Earth. It is just that they are in a falling reference frame, and it looks like there is no gravity; in a sense, there is no gravity.

Alternatively, imagine we place you and the elevator in space, and then attach the cable to a rocket ship and let the rocket ship accelerate, towing the elevator at 9.8 m/s^2 . As far as you can tell, from inside the elevator, you will experience an apparent force of gravity caused by the fact that the elevator, indeed, anything you might use to produce a "reference frame," is accelerating at a uniform rate. This principle is known as the principle of equivalence:

The effects of gravity are indistinguishable from the effects of being in an accelerated reference frame

Einstein was able to use the idea of the principle of equivalence to learn a great deal about gravity, and accurately predicted several observed phenomena, but ultimately these simply led him to the general theory of relativity, in which this idea is fully developed. We will therefore try to move towards this general theory. There is, however, a problem: we have only discussed ordinary, boring, Cartesian coordinates in special relativity. We need to expand our view of special relativity so that we can work in other coordinates. Specifically, we will work in spherical coordinates, but in fact the mathematics of general relativity is so powerful that it can work in virtually *any* coordinate system, no matter how skewed or distorted it may be.



B. The Distance Formula Again

We will begin at the beginning, the very foundation of special relativity, namely, the distance formula. Specifically, we will work with the formula for proper time, which was given by*

$$c^2 \tau^2 = c^2 (\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

This tells us how much time is perceived by an object as it moves from one point to another in space over a period of time – provided that object moves at a steady speed and in a straight line. If one moves in some more erratic fashion, we must imagine breaking up the trip into several small steps, each of which is so short that we can treat each segment as a straight line. Mathematically, we just replace each of the differences above by infinitesimal differences, $\Delta x \rightarrow dx$ for example, to find a formula for the proper time for an infinitesimal line segment:

$$c^2 d\tau^2 = c^2 (dt)^2 - (dx)^2 - (dy)^2 - (dz)^2$$

If you are familiar with infinitesimals, this formula may already make sense, but if you aren't rest assured that this formula can be massaged into formulas that make more sense for you. For example, if we are given an arbitrary path through spacetime, denoted by functions $x(t)$, $y(t)$, and $z(t)$, we can use this formula to find the proper time for the whole trip. A little mathematical manipulation will show you that

$$c\tau = \int dt \sqrt{c^2 - \left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2 - \left(\frac{dz}{dt}\right)^2}$$

Yes, I know it looks messy, but we won't be using this to do any actual problems. You're welcome.

Now, in the absence of forces, we know that objects move at a steady pace in a straight line. We normally think of a straight line as the shortest distance between two points, but in relativity, it turns out that a constant velocity motion along a straight line is actually the *longest* proper time between two events in space time. This path, which maximizes the proper time between two events, is called a geodesic, and I will state the following “geodesic principle”:

An object with no forces acting on it will always follow a geodesic, which is the longest proper time path between two points in spacetime.

This principle will remain the same as we consider different coordinate systems, whether we switch to spherical coordinates, or rotating coordinates, or even work with coordinates in an intrinsically curved spacetime.

* Whether one should work with the proper time formula or the distance formula is a matter of disagreement between physicists, somewhat akin to the religious wars between Protestants and Catholics. They claim to worship the same deity, but that doesn't stop them from fighting. My view is that working with the time formula is somewhat superior, but most general relativists use the alternative. The convention used here is called the “mostly minus” convention, since most of the terms in the distance formula (the “metric”) have minus signs in them.

C. Other Coordinate Systems

We now want to consider changing coordinates. You might think we should be switching to rotating coordinates, or accelerating coordinates, or something like that, but my goal can be achieved much more modestly: we will attempt to change to spherical coordinates. Spherical coordinates are related to Cartesian coordinates by the relationships:

$$x = r \sin \theta \cos \phi, \quad y = r \sin \theta \sin \phi, \quad z = r \cos \theta$$

We now simply want to rewrite the infinitesimal distance formula in terms of the new coordinates. We will do so by simply calculating differentials like this:

$$\begin{aligned} dx &= (dr) \sin \theta \cos \phi + r(d \sin \theta) \cos \phi + r \sin \theta (d \cos \phi) \\ &= \sin \theta \cos \phi (dr) + r \cos \theta \cos \phi (d\theta) - r \sin \theta \sin \phi (d\phi). \end{aligned}$$

Similarly, we can show

$$\begin{aligned} dy &= \sin \theta \sin \phi (dr) + r \cos \theta \sin \phi (d\theta) + r \sin \theta \cos \phi (d\phi), \\ dz &= \cos \theta (dr) - r \sin \theta (d\theta). \end{aligned}$$

We then plug all three of these formulas into our distance formula, and after considerable work (see homework), we find that

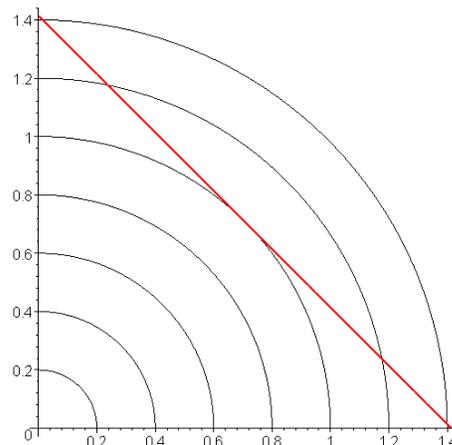
$$c^2 d\tau^2 = c^2 (dt)^2 - (dr)^2 - r^2 (d\theta)^2 - r^2 \sin^2 \theta (d\phi)^2$$

This formula doesn't really contain any information that wasn't already in the original coordinate system, it just is rewriting the distance formula.

Now, recall the geodesic principle: objects without forces on them will always follow the path which maximizes the proper time. This statement is true no matter what coordinates we use, because the proper time is independent of coordinates. It is possible (though somewhat more difficult) to work directly in spherical coordinates. I won't go through the math, but one can find, for example, that the radius satisfies the equation

$$\frac{d^2 r}{d\tau^2} = r \left(\frac{d\theta}{d\tau} \right)^2 + r \sin^2 \theta \left(\frac{d\phi}{d\tau} \right)^2$$

If you look at the left side, it looks as if there is some sort of acceleration going on, but in fact, all this equation is telling you is that if you are moving in the θ or ϕ direction, there will be changes in r that simply have to do with the curving of the coordinates, as I have attempted to illustrate at right. The red curve is straight, but it "accelerates" in the r direction. This acceleration will, of course, be independent of mass, because it is not real, it is just compensating for the curved coordinates.

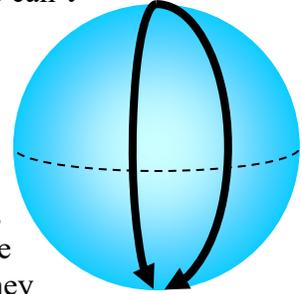


Of course, just because the equations seem to indicate that some coordinate is accelerating, doesn't mean there is any real acceleration going on. We could always go

back to ordinary Cartesian coordinates, and then it would be obvious that no acceleration occurs. This is because the coordinates are curved, but the spacetime itself is not. This leads us to a discussion of curvature of spacetime.

D. Curvature

Consider an ancient explorer on the surface of the Earth. As far as he is concerned, the Earth looks flat (ignoring mountains and so on), and he can't travel in the vertical direction, so the Earth is effectively two dimensional. Nonetheless, it is possible, without him leaving the Earth, to notice that the Earth is curved. Consider, for example, two explorers that leave the North Pole simultaneously, traveling at the same speed, and walk in a "straight" line. At first, these two explorers will be traveling apart, their separation gradually increasing, but soon they will find their rate of separation slowing, and by the time they reach the Equator, they will not be separating at all. Indeed, as they continue southward, they will start moving closer together, and ultimately will meet at the South pole. Their paths have apparently curved, but in fact, they both traveled in the straightest line they possibly can, it is space itself that has curved.



It is pretty easy to work out the distance formula on the surface of a sphere. Since the radius R is fixed, a position on the sphere is described simply by the longitude and latitude, which we will describe by standard spherical coordinates θ and ϕ . Then one can show that the distance between nearby points on the sphere is given by

$$ds^2 = R^2 (d\theta)^2 + R^2 \sin^2 \theta (d\phi)^2$$

Furthermore, there is no way of changing these two coordinates to any other two coordinates in such a way as to "flatten out" the sphere – because it isn't flat. The surface of a sphere has curvature, and though it is not obvious, the curvature can be deduced directly from the metric, or formula for the distance.

Of course, you have doubtless seen flat maps of the Earth, or at least portions of the Earth. These maps always involve a distortion of distances, so that the straightest lines possible – the geodesics – will appear curved on the maps. If you look at the paths airplanes take for international travel – say, across the Atlantic – they will often appear curved. Their curvature is not because the airlines like flying north, they are simply taking the shortest path between two points on the surface of a sphere, and that path appears curved when drawn on a flat map.

What has this to do with gravity? Well, in the presence of matter, it turns out that spacetime itself gets curved. The curvature can be deduced from the metric, or distance formula, though the equation is incredibly complicated. The point is, given *only* the distance formula, it is possible (though not easy) to deduce the fact that spacetime is curved.

There are, in fact, many different measures of the curvature of a given spacetime. The most general one, called the Riemann tensor, is a complicated object with 256 components (though some of them are zero)! Einstein, in developing his general theory

of relativity, found that this one didn't work. Instead, he found he could work with a much simpler object, called the Einstein tensor, which is denoted by $G_{\mu\nu}$.

G is just the name of the tensor, and the little indices μ and ν specify the components of the tensor. For example, if you are working in spherical coordinates, the indices take on the values t , r , θ , and ϕ , the four coordinates we are working with, so there is a G_{tt} component, a G_{rr} component, a $G_{\theta\phi}$ component, and so on, for a total of sixteen possible combinations.

The Einstein tensor is a measure of the curvature of spacetime. If you work out the Einstein tensor for any flat spacetime, you will always get zero. This is true whether you use Cartesian coordinates, or spherical coordinates, or rotating coordinates, or any coordinates you can think of. You can't hide curvature by changing coordinates; if space is curved, it will show up in the equations no matter what coordinates you use. In fact, Einstein's equations (see section F) work in *any* coordinate system. But they are so messy that I won't write them out; I'll simply fool you into thinking they are simple by burying the complexity in symbols like $G_{\mu\nu}$.

E. The Stress-Energy Tensor

We have half of the material in place for writing Einstein's equations, we now need to put in place the other half. As was realized by Newton, mass causes gravity, and it makes sense that this would somehow be true in general relativity as well. However, in relativity, we already know that mass and energy are related, so logically it should be energy, not mass, that causes gravity. As we know, however, energy is just one component of a four-component four-dimensional momentum, so perhaps we should be including momentum as well.

In fact, it turns out that not only do the momentum and energy come into the equations, but also the way they are flowing or being transferred plays a role as well. The whole source is not just the energy, or even the energy and the momentum, but something nastier, denoted by the symbol $T_{\mu\nu}$. This object should properly be called the stress-strain-energy-momentum tensor, but this intimidating phrase serves no purpose other than to impress your parents over Thanksgiving with the high falutin' stuff you are learning in modern physics, and that their money is going to a good cause. Because even physicists can't get their mouths around this phrase, it is more commonly called the stress-energy tensor or energy-momentum tensor or even just the stress tensor (or sometimes, just $T_{\mu\nu}$). I'll call it the stress-energy tensor.

The stress-energy tensor again, contains many components, sixteen in all. Just one of these components, the time-time component, is the actual energy density (which is proportional to the mass density), $T_{tt} = U$. The space-time components are proportional to the momentum density, and the space-space components are related to mechanical concepts like stress and strain (which are sort of like pressure). Don't worry about it. Just remember that $T_{\mu\nu}$ somehow is aware of all the matter, and what its gravitational influence is.

F. Einstein's Equations

We now need to put these two ideas together. If spacetime is curved, then geodesics (the longest proper

Einstein tensor: $G_{\mu\nu}$ measures curvature
Stress-Energy tensor: $T_{\mu\nu}$ measures presence of matter

time paths between two points) will be curved. And I mean really curved, like the paths on the surface of the Earth, not just apparently curved, like we found in spherical coordinates. When there is gravity present, the paths of objects will be curved as well. Einstein speculated that the matter caused the gravity. In other words, the stress-energy tensor $T_{\mu\nu}$ somehow causes the curving of spacetime $G_{\mu\nu}$. He speculated that the relationship was as follows:

$$G_{\mu\nu} = 8\pi Gc^{-4}T_{\mu\nu},$$

where G is Newton's constant, c is the speed of light, and we have already described $G_{\mu\nu}$ and $T_{\mu\nu}$. Of course, you can't really understand this equation unless you know how to calculate $G_{\mu\nu}$ (which you don't) and you understand what $T_{\mu\nu}$ is (which I have only vaguely described). Suffice it to say that the left side describes the curvature of spacetime, the right side describes the presence and nature of the matter, and the equation relates them. Enough said.

In addition, we will continue to assume that particles follow geodesics; that is, they will take the path which maximizes the proper time going from one place to another. To restate this geodesic principle, we have

An object with no non-gravitational forces acting on it will always follow a geodesic, which is the longest proper time path between two points in spacetime.

Got that? I have added the words "non-gravitational" to make an important point: Particles in the presence of gravity *still* follow geodesics. Because of the curvature of spacetime, these geodesics are curved, just as the curvature of the Earth forces explorers to follow curved paths on the surface of the Earth. In general relativity, objects do *not* curve because there are gravitational forces on them. They curve because they are moving through curved spacetime.

Einstein's equation, together with the geodesic principle, constitutes the essence of general relativity. The former is difficult to solve; indeed, only a few exact solutions of Einstein's equations are known. But there is ample evidence for the correctness of these equations.

Probably the best way to end this short section is with a quote from John Wheeler, which summarizes these two principles:

Matter tells space how to curve, and space tells matter how to move

G. The Schwarzschild Solution

By far the most important exact solution to Einstein's equation is the Schwarzschild solution. This describes the metric (distance formula) around a spherically symmetrical source, such as a planet, star, neutron star, or black hole. The formula for the metric, in spherical coordinates, is

$$c^2 d\tau^2 = c^2 \left(1 - \frac{2GM}{c^2 r}\right) (dt)^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} (dr)^2 - r^2 (d\theta)^2 - r^2 \sin^2 \theta (d\phi)^2,$$

Where c is the speed of light, M is the mass of the source, and G is Newton's constant again. If you compare this with the metric we found before, you will see that it is the same, other than the factors of $1 - 2GM/c^2 r$ and its reciprocal in the time and radial parts.

What does this equation tell us? Well, the change in the coefficient of dr is kind of interesting; it can be shown, for example, that a circle of radius R around the Earth will have a circumference that is *not* simply $2\pi R$. Far more interesting, however, is the fact that there is a coefficient in front of the time factor. For example, suppose someone stays at a constant position compared to the Earth ($dr = d\theta = d\phi = 0$), then this formula can be shown, without too much difficulty, to lead to the following remarkable equation:

$$\tau = t \sqrt{1 - \frac{2GM}{c^2 r}}$$

Since the factor in the square root is always less than one, this means that the time measured by someone near a mass is less than someone not near it, or

Time slows down when you get near a massive object

The effect is pretty small, probably around 1 part in 10^9 near the surface of the Earth, for example. You might think this is too small effect to be measured, and certainly too small to worry about, but in fact, Global Positioning System (GPS) satellites (which include atomic clocks) need to keep *extremely* accurate track of time, and such an effect, if unaccounted for, would quickly accumulate into large errors in the position measured by GPS equipment. This is not a problem, since GR was well understood by the time we had GPS satellites, so everything is fine.

The effect is much larger if you use a smaller and more massive object, such as a neutron star. A typical neutron star weighs something like 1.4 times the mass of the Sun, and might typically have a radius as small as 15 km. Because the mass is large and the radius is small, atoms on the surface will oscillate more slowly (as observed by us), and we will see radiative spectral lines that are red-shifted to longer wavelengths by an amount

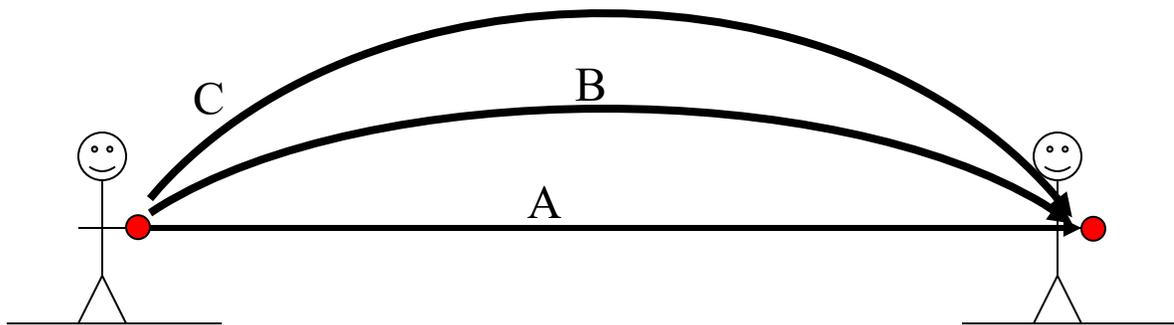
$$\lambda = \lambda_0 \left(1 - 2GM/c^2 r\right)^{-1/2},$$

Where λ_0 is the natural wavelength of the light, λ is the wavelength we observe, and r is the radius of the neutron star.

H. Gravitational Forces

I claimed previously that particles follow geodesics, and that can account for the fact that objects accelerate downwards when we release them. Let me try to convince you of how this works.

Suppose I am standing still on the surface of the Earth, and am holding a ball. I want to let go of the ball, and recapture it 0.5 second later at the exact place I released it. In other words, I am throwing the ball to myself. How am I going to do this? Well, the goal is to find a path through space time that starts and ends at the same *place*, but at a different time. Let's draw a spacetime diagram, and consider three possible paths from the initial point to the final point. This will be a highly qualitative diagram, where time will be increasing to the right.



Remember that our goal is to *maximize* the proper time, as measured by the ball, as the ball moves from the throwing incident (on the left) to the catching incident (on the right). Consider first the path A, in which the ball stays near me all the time; the ball is floating in space at a constant height for one second. Because time slows down when the ball is near the Earth, this will have a relatively small proper time. We can do better. To increase the proper time, the ball needs to move away from the Earth, and get to a greater height above the Earth.

Consider the path C, which represents a path where the ball rushes upwards, so as to get away from the Earth. This is good, because being farther from the Earth, there is more proper time involved, so more proper time passes for the ball. However, this ball had to move fast to get so high, and according to relativity, when you move fast, time slows down. So, even though path C is farther away from the gravitational source (good), it is moving faster (bad), and may have no more proper time than path A.

The ideal path is a compromise, path B. In fact, one can show that in Earth's gravitational field, the ideal path is one that accelerates downwards at about 9.8 m/s^2 . General relativity predicts gravitational forces on physical particles, just as does Newton's theory.

Of course, the curvature of spacetime affects everything that moves through it, including light. One of the first measurements of general relativity was the gravitational deflection of starlight by the Sun. The exact magnitude is predicted by relativity. Indeed, the deflection of light from distant sources is now one of the principal methods used to measure the masses of distant galaxies and galaxy clusters, by seeing how much the light is deflected by the intervening galaxy or cluster.

I. Black Holes

Let's look at the Schwarzschild metric again:

$$c^2 d\tau^2 = c^2 \left(1 - \frac{2GM}{c^2 r}\right) (dt)^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} (dr)^2 - r^2 (d\theta)^2 - r^2 \sin^2 \theta (d\phi)^2$$

Recall, this is the metric *outside* of any gravitational source. If you look at the formula, something pretty interesting happens when $r = R_s$, called the Schwarzschild radius, given by

$$R_s = \frac{2GM}{c^2}$$

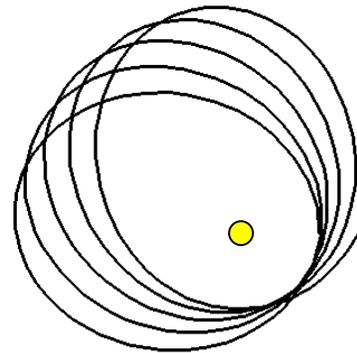
For something with the mass of the Sun, this is about 3 km, and is well inside the Sun, so it is not a concern.

But suppose we took the entire mass of the Sun, and somehow compressed it into a space only about 3 km in radius. What would happen? If you look at the equation naively, time would stop and space would get infinitely stretched at the Schwarzschild radius. Actually, it's a bit more complicated than this, and you have to change coordinates to figure out what is really going on. What happens is that all of the mass would (theoretically) get crushed down to an infinitely small point. Furthermore, anything that gets sucked closer than the Schwarzschild radius would continue falling in towards the center of the black hole. Nothing – not even light – can escape a black hole. This is the point of no return – if you reach the Schwarzschild radius, you are doomed to fall into the black hole.

Of course you can't see a black hole, because any light you shine on it will only be soaked up. But black holes have such enormous gravity that as gas falls into them, the gas is accelerated to relativistic velocities, and before it falls in, it will radiate superhot radiation (X-rays) that can give us clues about what the gravitational effect of the black hole is. Such observations have convinced us that the universe is peppered with black holes ranging in mass from a few times the Sun's mass up to many millions of times the Sun's mass.

J. Other Interesting Effects

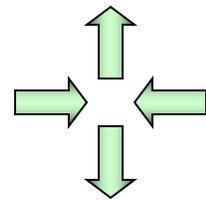
Many other effects of Einstein's equations have experimental consequences that either have been observed or will soon be searched for. For example, although the acceleration of gravity predicted by Einstein is *close* to that predicted by Newton, when objects are moving at high velocity, they will not be *exactly* right. Hence, for example, whereas Newton's theory predict Kepler's laws, including the claim that planetary orbits are ellipses, Einstein's equations show that they won't be quite perfect ellipses, as sketched at right. Instead, as a planet (say Mercury) orbits the Sun,



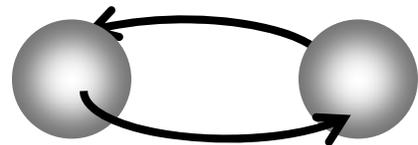
the orbit will slowly but inexorably change the orientation of its principal axis. This “precession of the perihelion,” in the case of Mercury, was actually observed but unexplained *before* Einstein developed his theory. The precession is quite tiny, working out to 43 arc-seconds per century (an arc second is 1/3600 of a degree), but Einstein’s theory accounted for it; Newton’s did not.

Another interesting effect is called “frame dragging.” Recall that the source of gravitational fields, according to Einstein, is not just the mass/energy density, but also includes the *motion* of that mass. Consider the Earth, which is not just a sphere, but instead a *rotating* sphere, so that all that mass is moving (although not very fast, by relativistic standards). This moving mass should produce an effect where spacetime is, in a sense, dragged around a little by the rotating Earth. Gravity Probe B, a NASA experiment recently completed, looked for this effect for three years, ending in early 2007. Their final analysis looking for this effect should be available any day now.

Yet another effect is gravitational waves. It is possible to produce distortions in spacetime that travel through space at the speed of light, much like light waves. Such distortions would cause distances to stretch in one dimension, while shrinking in the perpendicular directions. Cataclysmic effects throughout the universe, such as the collision of black holes or neutron stars, should create these ripples, which should produce small but potentially observable effects here on Earth. In 2015, gravitational waves from the Laser Interferometer Gravitational-Wave Observatory (LIGO) detected gravitational waves caused by the merger of two black holes of mass $35 M_{\odot}$ and $30 M_{\odot}$. The result was published in 2016. As of late 2018, four or five other black hole mergers have produced measurable gravitational waves. In 2017 a merger of two neutron stars was found, and this was accompanied by light from across the electromagnetic spectrum in the same direction.



Even before gravitational waves had been directly observed, their indirect effect had been seen. It turns out that accelerating masses, such as orbiting astronomical objects, should produce small but steady quantities of gravitational waves, which slowly drain energy from the system. As a consequence, orbiting systems should slowly lose energy, and in general the objects will spiral towards each other. In the solar system, this effect is too small to see, but pairs of closely orbiting neutron stars have been observed that are moving so quickly and feel so much gravity that this effect can easily be detected. These systems also exhibit very fast precession of the perihelion of the orbit (similar to Mercury, but much larger), as well as other effects consistent with general relativity.



Indeed, we may well be entering the Golden Age of general relativity experiments. When I was studying “modern physics” in the 1970’s, the tests of GR were so few and the effects so small that there was still room to doubt the theory. Now we have several accurate experiments, with prospects of many more coming in the next few years. General relativity is finally maturing, and entering the realm of precision experiments, where it can take its place with special relativity, quantum mechanics, and other well-tested theories.