Physics 215 – Elementary Modern Physics

Hydrogen-Like Wave Functions

Angular Momentum

$$L^2 = \hbar^2\left(l^2 + l\right)$$
$$L_z = \hbar m$$
$$S^2 = \tfrac{3}{4}\hbar^2$$
$$S_z = \hbar m_s$$

Quantum Numbers

$$\left(n, l, m, m_s\right)$$
$$n = 1, 2, 3, \ldots$$
$$l = 0, 1, 2, 3, \ldots, n-1$$
$$m = -l, -l+1, \ldots, 0, \ldots, l$$
$$m_s = \pm\tfrac{1}{2}$$

Energy

$$E_n = -\frac{k^2 Z^2 e^4 \mu}{2\hbar^2 n^2} = -\frac{\left(13.6\text{ eV}\right)Z^2}{n^2}$$

Characteristic radius:

$$a = \frac{\hbar^2}{kZe^2\mu} = \frac{a_0}{Z} = \frac{0.0529\text{ nm}}{Z}$$
$$\langle r \rangle \sim an^2$$

Wave function (ignoring spin):

$$\psi_{n,l,m}\left(r,\theta,\phi\right) = R_{n,l}\left(r\right)Y_{l,m}\left(\theta,\phi\right)$$

Spherical Harmonics

$$Y_{0,0} = \frac{1}{\sqrt{4\pi}}$$
$$Y_{1,\pm1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\phi}$$
$$Y_{1,0} = \sqrt{\frac{3}{4\pi}}\cos\theta$$
$$Y_{2,\pm2} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{\pm 2i\phi}$$
$$Y_{2,\pm1} = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{\pm i\phi}$$
$$Y_{2,0} = \sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta - 1\right)$$
$$Y_{3,\pm3} = \mp\sqrt{\frac{35}{64\pi}}\sin^3\theta e^{\pm 3i\phi}$$
$$Y_{3,\pm2} = \sqrt{\frac{105}{32\pi}}\sin^2\theta\cos\theta e^{\pm 2i\phi}$$
$$Y_{3,\pm1} = \mp\sqrt{\frac{21}{64\pi}}\sin\theta\left(5\cos^2\theta - 1\right)e^{\pm i\phi}$$
$$Y_{3,0} = \sqrt{\frac{7}{16\pi}}\left(5\cos^3\theta - 3\cos\theta\right)$$

Radial Wave Functions

$$R_{1,0} = \frac{2}{\sqrt{a^3}}e^{-r/a}$$
$$R_{2,0} = \frac{1}{\sqrt{2a^3}}\left(1 - \frac{r}{2a}\right)e^{-r/2a}$$
$$R_{2,1} = \frac{r}{2\sqrt{6a^5}}e^{-r/2a}$$
$$R_{3,0} = \frac{2}{3\sqrt{3a^3}}\left(1 - \frac{2r}{3a} + \frac{2r^2}{27a^2}\right)e^{-r/3a}$$
$$R_{3,1} = \frac{4\sqrt{2}r}{27\sqrt{3a^5}}\left(1 - \frac{r}{6a}\right)e^{-r/3a}$$
$$R_{3,2} = \frac{2\sqrt{2}r^2}{81\sqrt{15a^7}}e^{-r/3a}$$
$$R_{4,0} = \frac{1}{4\sqrt{a^3}}\left(1 - \frac{3r}{4a} + \frac{r^2}{8a^2} - \frac{r^3}{192a^3}\right)e^{-r/4a}$$
$$R_{4,1} = \frac{\sqrt{5}r}{16\sqrt{3a^5}}\left(1 - \frac{r}{4a} + \frac{r^2}{80a^2}\right)e^{-r/4a}$$
$$R_{4,2} = \frac{r^2}{64\sqrt{5a^7}}\left(1 - \frac{r}{12a}\right)e^{-r/4a}$$
$$R_{4,3} = \frac{r^3}{768\sqrt{35a^9}}e^{-r/4a}$$